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GEOMETRY
PLANE TRIGONOMETRY
NATURAL TRIGONOMETRIC FUNCTIONS
LOGARITHMIC TRIGONOMETRIC FUNCTIONS

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PREFACE

The volumes of the International Library of Technology are made up of Instruction Papers, or Sections, comprising the various courses of instruction for students of the International Correspondence Schools. The original manuscripts are prepared by persons thoroughly qualified both technically and by experience to write with authority, and in many cases they are regularly employed elsewhere in practical work as experts. The manuscripts are then carefully edited to make them suitable for correspondence instruction. The Instruction Papers are written clearly and in the simplest language possible, so as to make them readily understood by all students. Necessary technical expressions are clearly explained when introduced.

The great majority of our students wish to prepare themselves for advancement in their vocations or to qualify for more congenial occupations. Usually they are employed and able to devote only a few hours a day to study. Therefore every effort must be made to give them practical and accurate information in clear and concise form and to make this information include all of the essentials but none of the non-essentials. To make the text clear, illustrations are used freely. These illustrations are especially made by our own Illustrating Department in order to adapt them fully to the requirements of the text.

In the table of contents that immediately follows are given the titles of the Sections included in this volume, and under each title are listed the main topics discussed.

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NOTE.—This volume is made up of a number of separate Sections, the page numbers of which usually begin with 1. To enable the reader to distinguish between the different Sections, each one is designated by a number preceded by a Section mark (§), which appears at the top of each page, opposite the page number. In this list of contents, the Section number is given following the title of the Section, and under each title appears a full synopsis of the subjects treated. This table of contents will enable the reader to find readily any topic covered.

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GEOMETRY

(PART 1)

PRELIMINARY DEFINITIONS

NOTE.—The study of Geometry is a process of systematic and orderly reasoning rather than a matter of memory. The student is advised to study the principles and propositions stated until he understands them thoroughly and sees their relation one to another, and, when a proposition is accompanied by an explanation in small type, to read over the explanation carefully one or more times, until he clearly understands the matter, following out the references to the figure when a figure is given. If he will do this he will find Geometry to be of great benefit and assistance to him in his subsequent studies. But he is not required to commit to memory the explanations or any part of the text except a few of the more important principles and propositions, such as those to which the Examination Questions relate.

1. Every material body possesses two general properties without regard to any other condition, namely: **form**, or **shape**, which is due to the relative positions of its parts; and **magnitude**, or **size**, which is due to the distance of its parts from one another.

The form and magnitude of a body can be described by the relative positions of *points*, *lines*, and *surfaces*.

2. A **point** has position without magnitude. A dot is commonly used to represent a point; but a dot, no matter how small, has length, breadth, and thickness, while a theoretical point has position only.

3. A **line** is the path of a point in motion; it has one dimension—length. Thus, if a point is moved from the position A, Fig. 1, to the position B, its path, or trace, is the line AB.

FIG. 1

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FIG. 2

4. A **straight line, or right line,** Fig. 2, is a line that does not change its direction.

5. The **distance** between two points is the length of the straight line joining them.

FIG. 3

6. A **curved line,** Fig. 3, is a line that changes its direction at every point.

FIG. 4

7. A **broken line,** Fig. 4, is a line that changes its direction at only certain points. It is made up wholly of different straight lines.

The word *line*, when not qualified by any other word, is understood to mean a straight line.

8. A **surface** is the path of a line when moved in a direction other than its length. Thus, if a line is moved from the position *A B*, Fig. 5, to the position *C D*, the line describes the surface *A B D C*.

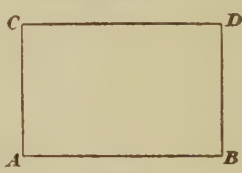

FIG. 5

9. A **flat surface, plane surface,** or simply a **plane,** is a surface such that a straight line between any two of its points lies wholly in the surface. If a straightedge is laid on a plane surface in any direction, every point of the straightedge will touch the surface.

10. A **figure** is any combination of points and lines. A figure that lies entirely in one plane is a **plane figure.**

In referring to a figure, a point is designated by a letter placed conveniently near it; thus, in Fig. 1, the left end of the line is referred to as the point *A*. The entire line is referred to as "the line *A B*," the letters *A* and *B* designating two points, usually the ends of the line. If a line is broken or curved, as many points are named as are considered necessary to designate the line.

11. **Geometry** is that branch of mathematics that treats of the construction and properties of figures.

12. To **produce** a line is to prolong it or to increase its length. A straight line can be prolonged or produced to any extent in either direction. Thus, in Fig. 6, the straight line AB is produced to the points C and D.

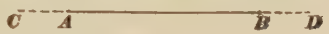

Fig. 6

13. To **bisect** any given magnitude is to divide it into two equal parts. Thus, the straight line AB, Fig. 7, is bisected at the point C if AC is equal to CB. When a given magnitude is bisected, each of the parts into which it is divided is one-half the given magnitude.

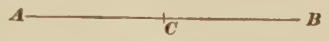

Fig. 7

STRAIGHT-LINE FIGURES

ANGLES AND PERPENDICULARS

14. An **angle**, Fig. 8, is the opening between two straight lines that meet in a point. The two straight lines are the **sides**, and the point where the lines meet is the **vertex**, of the angle. Thus, in Fig. 8, the straight lines OA and OB form an angle at the point O; the lines OA and OB are the sides of this angle, and the point O is its vertex.

Fig. 8

An angle is usually referred to by naming a letter on each of its sides and a third letter at the vertex, the letter at the vertex being placed between the other two. Thus, the angle in Fig. 8 is called angle AOB or angle BOA.

An angle may also be designated by a letter placed between its sides near the vertex. Thus, the two angles XCY and YCZ, Fig. 9, may be referred to as the angles A and B, respectively.

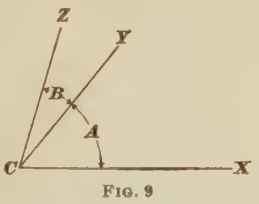

Fig. 9

An isolated angle, that is, an angle whose vertex is not the vertex of any other angle, may be designated by naming the letter at its vertex. For example, the angle in Fig. 8 may be called the angle O.

15. Two angles, as A and B, Fig. 9, having the same vertex and a common side $C\,Y$, are called **adjacent angles.**

16. Two angles are equal when one can be placed on the other so that they will coincide. Thus, in Fig. 10, the angles $A\,O\,B$ and $A'\,O'\,B'$ are equal, because $A'\,O'\,B'$ can be superimposed on $A\,O\,B$, so that with O' upon O and $A'\,O'$ along $A\,O$, $B'\,O'$ will take the direction of $B\,O$ and coincide with it.

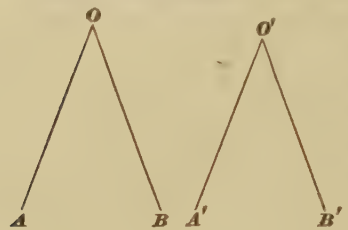

FIG. 10

17. Any angle may be thought of as being formed, or **generated,** by a line turning about the vertex as a pivot, from the position of one side to the position of the other. Thus, the angle $A\,O\,B$, Fig. 8, may be conceived as generated by a line turning about O from the position $O\,A$ to the position $O\,B$. The size of the angle does not depend on the length of the sides, which are supposed to be of indefinite length, but on the opening between the sides; or, what is the same thing, on the amount of turning necessary to bring one side to the position of the other.

18. If a straight line, as $A\,B$, Fig. 11, meets another straight line, as $C\,D$, so as to make with it two equal adjacent angles, each of these angles is a **right angle,** and the first line is said to be **perpendicular** to the second. The point where the first line meets the second is called the **foot** of the perpendicular. It is evident that all right angles are equal.

FIG. 11

FIG. 12

19. A **horizontal line** is a line parallel to the horizon, or to the surface of still water.

20. A **vertical line** is a line perpendicular to a horizontal line, and having, therefore, the direction of a plumb-line. See Fig. 12.

21. An **oblique angle** is any angle that is not a right angle. An **acute angle** is an oblique angle that is less than a right angle. An **obtuse angle** is an oblique angle that is greater than a right angle. In Fig. 13, BOC and AOC are oblique angles, BOC being an acute angle, and AOC an obtuse angle.

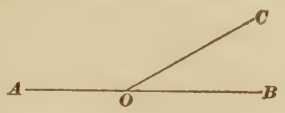

Fig. 13

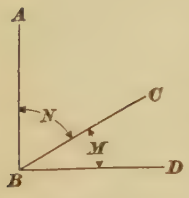

Fig. 14

22. Two angles are said to be **complementary** when their sum is equal to one right angle. Each of two complementary angles is called the **complement** of the other. Thus, in Fig. 14, in which AB is perpendicular to BD, the angles M and N are complementary, their sum being equal to the right angle ABD.

23. Two angles are said to be **supplementary** when their sum is equal to two right angles. Each of two supplementary angles is called the **supplement** of the other. In Fig. 15, AOD and DOB are supplementary angles, their sum being evidently equal to the sum of the two right angles POB and POA.

Fig. 15

It will be seen from this illustration that two adjacent angles whose non-common sides are in the same straight line are always supplementary. Conversely, if two adjacent angles are supplementary, their non-common sides are in the same straight line.

24. At a given point in a straight line, one perpendicular to the line and only one can be drawn.

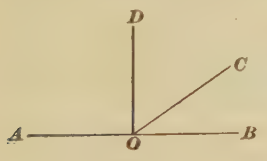

Fig. 16

Let O, Fig. 16, be the given point in the line OB. Suppose that with the point O fixed, the line OC starts from the position OB and revolves about O. In any position, as OC, it makes two angles with the line AB; one AOC, the other BOC. As OC revolves from the position OB to the position OA, the angle BOC will continually increase, and the

angle $A\,O\,C$ will continually decrease. There will therefore be one position, as $O\,D$, where the two angles are equal, and there can evidently be but one such position.

25. The sum of all the angles formed on the same side of a straight line about the same point in the line is equal to two right angles.

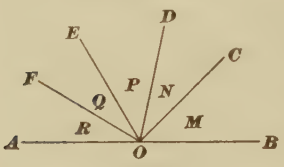

Fig. 17

In Fig. 17, the sum of the three angles M, N, and P is evidently equal to the angle $B\,O\,E$, and the sum of the angles Q and R is equal to the angle $E\,O\,A$. But, by Art. **23**, $B\,O\,E + E\,O\,A$ is equal to two right angles. Hence, $M + N + P + Q + R =$ two right angles.

26. The sum of all the angles formed in the same plane about one point is equal to four right angles. Thus, in Fig. 18, $M + N + P + Q + R + S + T + U =$ four right angles.

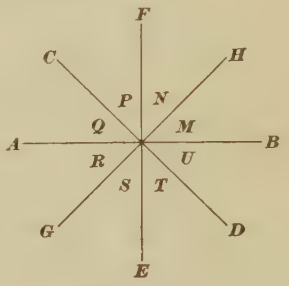

Fig. 18

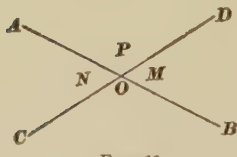

Fig. 19

27. When two lines, as $A\,B$ and $C\,D$, Fig. 19, cut or cross each other, they are said to **intersect.** Their common point O is called their **point of intersection,** or simply their **intersection.**

28. Two intersecting straight lines determine four angles having a common vertex. Any one of these angles and the angle on the opposite side of both lines, as the angles M and N, Fig. 19, are called **vertical angles** with respect to each other. Vertical angles may also be defined as those having a common vertex and in which the sides of the one are the prolongations of the sides of the other.

Since M and N are each the supplement of P, they are equal to each other. Any angle is equal to its vertical angle.

29. If two straight lines intersect and one of the angles is a right angle, the other three angles are right angles, and the lines are perpendicular to each other.

30. Two oblique lines drawn from the same point in a perpendicular to a line, and cutting off on that line equal distances from the foot of the perpendicular, are equal.

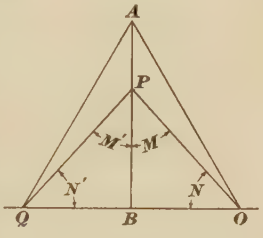

FIG. 20

Let PO and PQ, Fig. 20, be two oblique lines drawn from the point P in the perpendicular AB, and let BO and BQ be equal. Then, by turning the right side of the figure about AB, it will coincide with the left side; O will fall on Q, and PO will coincide with PQ. Hence, PO is equal to PQ.

31. Every point in the perpendicular at the middle point of a straight line is equally distant from the ends of the line. Thus, in Fig. 20, P, which may be any point in the perpendicular AB at the middle point B of OQ is equally distant from Q and O.

32. Two equal oblique lines drawn from the same point in the perpendicular to a straight line make equal angles with the straight line and with the perpendicular.

Since when PBO, Fig. 20, is brought to coincide with PBQ, PO coincides with PQ and BO with BQ, the angle M = angle M', and angle N = angle N'.

33. A line that divides an angle into two equal angles is called the **bisector** of that angle. In Fig. 20, PB is the bisector of OPQ, since $M = M'$.

34. Two points, each of which is equally distant from the two extremities of a line, determine a perpendicular bisecting the line. Thus, in Fig. 20, A and P are two points equally distant from Q and O and determine the perpendicular bisecting the line OQ.

EXAMPLES FOR PRACTICE

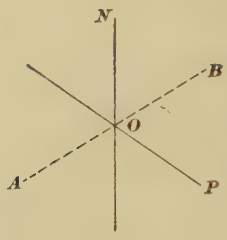

Fig. 21

1. Show that the bisectors of two vertical angles are in the same straight line.

Suggestion.—In Fig. 21, show that the sum of the angles on one side of the bisector $A\,B$ of the angle $N\,O\,P$ is equal to the sum of the angles on the other side.

2. Show that the bisectors of two supplementary adjacent angles are perpendicular to each other.

Suggestion.—In Fig. 22, show that the angle $E\,O\,F$ is one-half of two right angles.

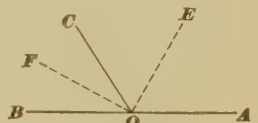

Fig. 22

PARALLELS

35. Parallel lines, Fig. 23, are straight lines that lie in the same plane and never meet, however far they are produced. Any two parallel lines have the same direction and are everywhere equally distant from each other.

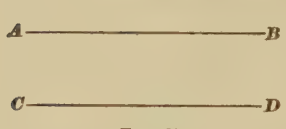

Fig. 23

36. When two parallel lines, as $P\,Q$ and $R\,S$, Fig. 24, are cut by a third line, as $X\,Y$, the cutting line $X\,Y$ is called a **secant line** or a **transversal.**

The eight angles thus formed are named as follows: The angles a, A, d, and D are **exterior angles.** The angles b, B, c, and C are **interior angles.** The pairs of angles a and d or A and D are **alternate-exterior angles.** The pairs of angles b and c or B and C are **alternate-interior angles.** The pairs of angles a and c, A and C, b and d, or B and D are **exterior-interior** or **corresponding angles.**

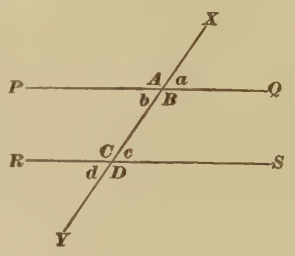

Fig. 24

37. When two parallel lines are cut by a transversal, the alternate-interior angles are equal.

Let CD and EF, Fig. 25, be the parallel lines and AB the transversal. The angles M and M' have their sides GD and HF parallel and AG and GH in the same line; hence, the turning in changing from the direction HF to the direction HG is equal to the turning in changing from the direction GD to the direction GA. That is, angle AGD, or M, is equal to the angle GHF, or M', Art. **17.** But angle M is equal to angle N, Art. **28**; therefore, angle N is equal to angle M'. In like manner, it can be shown that the angle DGH is equal to the angle GHE.

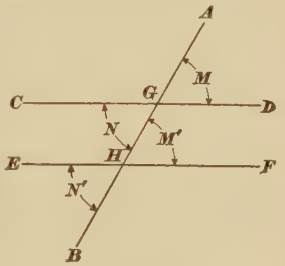

Fig. 25

38. It follows from the preceding article that the alternate-exterior angles are equal; also, the exterior-interior angles. Thus, in Fig. 24, we have $a = d$, $A = D$; $B = D$, $b = d$; $B = C$, $b = c$.

39. In Fig. 24, the angle a and the angle A are supplementary adjacent angles, and their sum is, therefore, equal to two right angles. From this, and from the principle stated in the preceding article, it follows that any angle in Fig. 24 marked by a capital letter and any angle marked by a small letter are together equal to two right angles.

The principles stated in this and in the two preceding articles may be summed up as follows: When two parallel lines are cut by an oblique transversal, the four obtuse angles are equal to one another; the four acute angles are equal to one another; and any of the obtuse angles is the supplement of any of the acute angles.

40. If a straight line is perpendicular to one of two parallel lines, it is perpendicular to the other also.

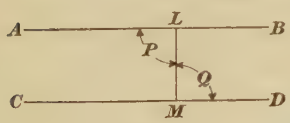

Fig. 26

In Fig. 26, AB and CD are parallel, and LM is drawn perpendicular to AB. Then, since the alternate-interior angles P and Q are equal, and since P is a right angle, Q must be a right angle also; that is, LM is perpendicular to CD.

41. The distance between two parallel lines is the length intercepted by the two parallels on any line perpendicular to them. Thus, $L\,M$, Fig. 26, is the distance between $A\,B$ and $C\,D$.

42. If two straight lines $A\,B$ and $C\,D$, Fig. 27, are cut by a third straight line $E\,F$ so that the exterior-interior angles M and N are equal, the two straight lines are parallel.

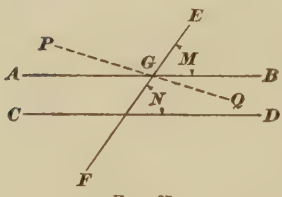

FIG. 27

If $A\,B$ were not parallel to $C\,D$, we might draw through G a line $P\,Q$ that was parallel to $C\,D$. But then the exterior-interior angles N and $E\,G\,Q$ would be equal (Art. **38**), which is obviously inconsistent with the supposition that N is equal to M.

43. If two lines, as $A\,B$ and $C\,D$, Fig. 28, are parallel to a third line, as $E\,F$, they are parallel to each other.

Draw a transversal $G\,H$. Then, since $A\,B$ is parallel to $E\,F$, the alternate-interior angles M and N are equal; and, since $C\,D$ is parallel to $E\,F$, the alternate-interior angles P and N are equal. We have, therefore, $N = M$, $N = P$, and, consequently, $M = P$. As M and P are exterior-interior angles, it follows, from Art. **42**, that $A\,B$ and $C\,D$ are parallel.

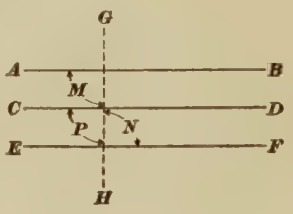

FIG. 28

44. Two angles whose sides are respectively parallel and lie in the same or opposite directions from their vertexes are equal.

In Fig. 29 (*a*), $B\,A$ and $E\,D$ are parallel and extend in the same direction; also, $B\,C$ and $E\,F$ are parallel and extend in the same direction from the vertexes. Let O be the point of intersection of the sides $B\,C$ and $E\,D$ produced. Then, since $B\,Q$ and $E\,F$ are parallel, the exterior-interior angles E and M are equal; and, since $B\,A$ and $E\,G$ are parallel, the exterior-interior angles B and M are equal. Therefore, the angles B and E, being each equal to M, are equal to each other.

In Fig. 29 (*b*), $B\,A$ and $E\,F$ are parallel and extend in opposite directions; also, $B\,C$ and $E\,D$ are parallel and extend in opposite directions from the vertexes. Producing $F\,E$ and $D\,E$, we have, by the preceding case, $B = D'\,E\,F'$. As $D\,E\,F$ and $D'\,E\,F'$ ar vertical

angles, they are equal, and, therefore, B, which is equal to $D'E'F'$, is also equal to DEF.

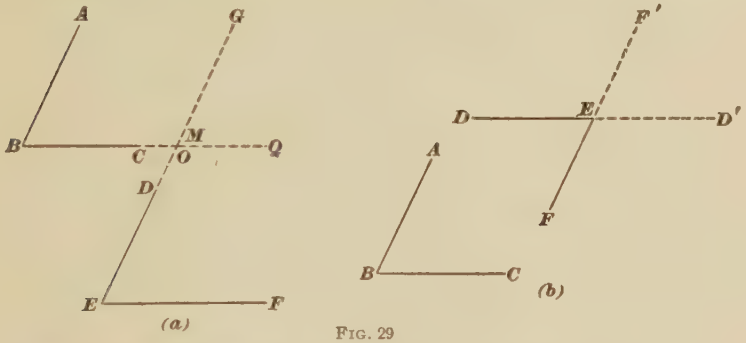

FIG. 29

45. If one side of an angle is parallel to one side of another angle, the two extending in the same direction from the vertexes, and if the other sides of the two angles are also parallel, but extend in opposite directions from the vertexes, the two angles are supplementary.

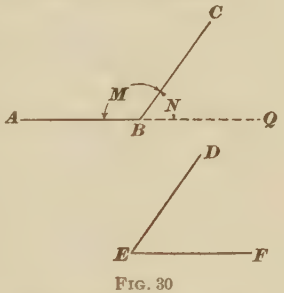

FIG. 30

In Fig. 30, BC and ED are parallel and extend in the same direction, while BA and EF are parallel and extend in opposite directions from the vertexes. Producing AB, we have, by Art. **44**, $N = E$. Now, $M + N =$ two right angles; therefore, $M + E =$ two right angles.

46. Two angles that have their sides perpendicular, each to each, are either equal or supplementary; they are equal if both are acute or both obtuse; and supplementary if one is acute and the other obtuse.

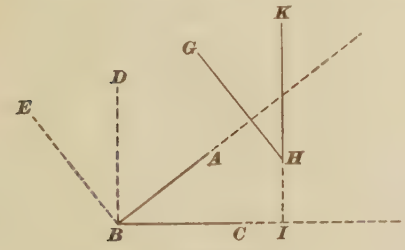

FIG. 31

In Fig. 31, let GH be perpendicular to AB, and KH perpendicular to BC. Draw BD parallel to KH, and BE parallel to GH. Then, by Art. **44**, DBE is equal to KHG. Since EBA and DBC are right angles, by taking DBA

from each of them EBD is seen to be equal to ABC. Hence, the acute angle ABC is equal to the acute angle KHG. Also, when one angle is the acute angle ABC and the other is the obtuse angle GHI, since GHI is the supplement of KHG, it must be the supplement of ABC.

POLYGONS

DEFINITIONS

47. A **polygon** is a portion of a plane bounded by straight lines. The boundary lines are the **sides** of the polygon. The angles formed by the sides are the **angles** of the polygon. The vertexes of the angles of the polygon are the **vertexes** of the polygon. The broken line that bounds it, or the whole distance around it, is the **perimeter** of the polygon. Thus, $ABCDE$, Fig. 32, is a polygon; the sides of this polygon are AB, BC, CD, DE, and EA; its angles are ABC, BCD, CDE, DEA, and EAB; and its vertexes are A, B, C, D, and E.

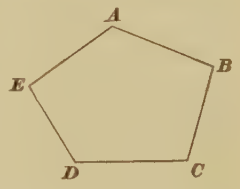

FIG. 32

48. The number of vertexes of a polygon is the same as the number of sides.

49. The least number of sides that a polygon can have is three, since two straight lines cannot enclose space.

50. Polygons are classified in various manners. One of these classifications is based on the number of sides. A polygon of three sides is a **triangle**; a polygon of four sides, a **quadrilateral**; a polygon of five sides, a **pentagon**; a polygon of six sides, a **hexagon**; a polygon of seven sides, a **heptagon**; a polygon of eight sides, an **octagon**; a polygon of nine sides, a **nonagon**; a polygon of ten sides, a **decagon**; a polygon of twelve sides, a **dodecagon**.

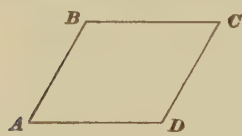

FIG. 33

51. An **equilateral polygon** is a polygon whose sides are all equal. Thus, in Fig. 33, $AB = BC = CD = DA$; hence, $ABCD$ is an equilateral polygon.

52. An **equiangular polygon** is a polygon whose angles are all equal. Thus, in Fig. 34, angle A = angle B = angle D = angle C; hence, $A\,B\,D\,C$ is an equiangular polygon.

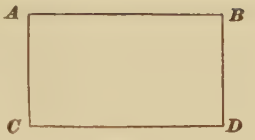

FIG. 34

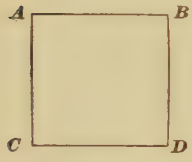

FIG. 35

53. A **regular polygon** is a polygon in which all the sides and all the angles are equal. Thus, in Fig. 35, $A\,B = B\,D = D\,C = C\,A$; and angle A = angle B = angle D = angle C; hence, $A\,B\,D\,C$ is a regular polygon. Some regular polygons are shown in Fig. 36.

FIG. 36

54. A **reentrant angle** of a polygon is an angle whose sides if produced through the vertex will enter the surface bounded by the perimeter of the polygon. Thus, $B\,C\,D$, Fig. 37, is a reentrant angle.

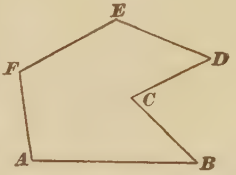

FIG. 37

TRIANGLES

55. Triangles are classified with regard to their sides into *scalene*, *isosceles*, and *equilateral* triangles.

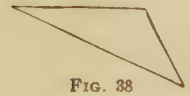
FIG. 38

56. A **scalene triangle**, Fig. 38, is a triangle that has no two of its sides equal.

57. An **isosceles triangle**, Fig. 39, is a triangle that has two of its sides equal.

FIG. 39

58. An **equilateral triangle**, Fig. 40, is a triangle that has its three sides equal. An equilateral triangle is a particular kind of isosceles triangle. Thus, the triangle ABC, Fig. 40, may be regarded as an isosceles triangle whose equal sides are AB and AC, as an isosceles triangle whose equal sides are BA and BC, or as an isosceles triangle whose equal sides are CA and CB. All the statements made with regard to isosceles triangles are, therefore, true of equilateral triangles.

FIG. 40

59. Triangles are classified with regard to their angles into *right-angled*, *obtuse-angled*, and *acute-angled triangles*. See Fig. 41.

FIG. 41

60. A **right-angled triangle**, or a **right triangle**, is a triangle having a right angle. The **hypotenuse** of a right triangle is the side opposite the right angle. The **legs** of a right triangle are the sides that include the right angle.

61. An **obtuse-angled triangle** is a triangle having an obtuse angle.

62. An **acute-angled triangle** is a triangle all the angles of which are acute.

63. An **oblique triangle** is a triangle that has no right angle. The class oblique triangles includes all obtuse-angled and acute-angled triangles.

64. An **equiangular triangle** is a triangle whose three angles are equal.

65. The **base** of a triangle is the side on which the triangle is supposed to stand. In a scalene triangle, any side

may be considered as the base. In an isosceles triangle, the unequal side is usually, though not necessarily, taken as the base.

The angle opposite the base of a triangle is sometimes called the **vertical angle** of the triangle. In Figs. 42 and 43, $A\,C$ is the base.

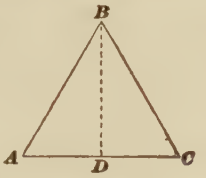

FIG. 42

66. The **altitude** of a triangle is the length of a line drawn from the vertex of the angle opposite the base perpendicular to the base. Thus, in Figs. 42 and 43, the length of $B\,D$ is the altitude.

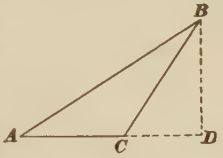

FIG. 43

67. An **exterior angle** of a triangle is an angle formed by a side and the prolongation of another side. Thus, in Figs. 43 and 44, the angle $B\,C\,D$, formed by the side $B\,C$ and the prolongation of the side $A\,C$, is an exterior angle of the triangle $A\,B\,C$. The angle $B\,C\,A$ is adjacent to the exterior angle $B\,C\,D$. The angles A and B are **opposite-interior angles** to the angle $B\,C\,D$.

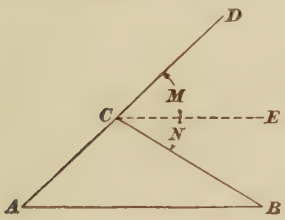

FIG. 44

68. In any triangle, an exterior angle is equal to the sum of the opposite-interior angles.

Let $D\,C\,B$, Fig. 44, be an exterior angle of the triangle $A\,B\,C$. Draw $C\,E$ through C parallel to $A\,B$. Then, the angles M and A, being exterior-interior angles, are equal. Also, N and B, being alternate-interior angles, are equal. Hence, angle M plus angle N, that is, the exterior angle $D\,C\,B$, is equal to angle A plus angle B, or the sum of the opposite-interior angles.

69. The sum of the interior angles of a triangle is equal to two right angles.

In Fig. 44, the angles $B\,C\,D$ and $B\,C\,A$, being supplementary adjacent angles, are together equal to two right angles. But, by the preceding article, the angle $B\,C\,D$ is equal to the sum of the angles A and B. Hence, the sum of the three interior angles A, B, and $B\,C\,A$ is equal to two right angles.

70. The following important propositions are immediate consequences of that stated in Art. **69**:

1. If two angles of a triangle are known, or if their sum is known, the third angle can be found by subtracting their sum from two right angles.

2. If two angles of a triangle are equal, respectively, to two angles of another triangle, the third angle of the first-mentioned triangle is equal to the third angle of the other triangle.

3. A triangle can have but one right angle, or one obtuse angle.

4. In any right triangle, the two acute angles are complementary.

5. Each angle of an equiangular triangle is equal to one-third of two right angles, or two-thirds of one right angle.

6. From a point without a line, only one perpendicular to the line can be drawn.

EXAMPLES FOR PRACTICE

1. If one acute angle of a right triangle is one-third of a right angle, what is the value of the other? Ans. Two-thirds of a right angle

2. If one angle of a triangle is one-half of a right angle, and another is five-sixths of a right angle, what is the third angle?

Ans. Two-thirds of a right angle

3. The exterior angle of a triangle is $1\frac{2}{5}$ right angles, and one of the opposite-interior angles is one-fourth of a right angle; what are the other angles of the triangle?

Ans. { Other opposite-interior angle $= \frac{2}{20} = 1.15$ right angles
Angle adjacent to exterior angle = three-fifths of a right angle

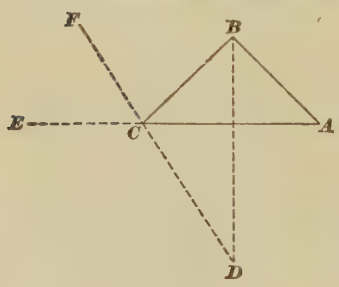

FIG. 45

4. Show that in the triangle ABC, Fig. 45, the bisector of the right angle ABC forms with the bisector of the exterior angle at C an angle that is equal to one-half of the angle A.

SUGGESTION.—Let BD be the bisector of ABC and FD the bisector of BCE. Then BCF is equal to CBD plus CDB, or CDB is equal to BCF minus CBD. Also, ECB is equal to CBA plus A, or A is equal to ECB minus CBA. Furthermore, ECB is equal to twice BCF and CBA is equal to twice CBD.

5. One angle of a triangle is one-half of a right angle: (*a*) What are the remaining two angles, if one is twice as large as the other? (*b*) What kind of triangle is this?

Ans. { (*a*) One-half of a right angle and one right angle
(*b*) An isosceles right triangle

---

71. Two plane figures are **equal** when one can be placed on the other so that they will coincide in all their parts.

Thus, the triangles $A\,B\,C$ and $A'\,B'\,C'$, Fig. 46, are equal, because if $A'\,B'\,C'$ is imagined to be lifted off the paper, moved over and placed on $A\,B\,C$, the sides $A'\,B'$, $B'\,C'$, and $C'\,A'$ can be made to coincide

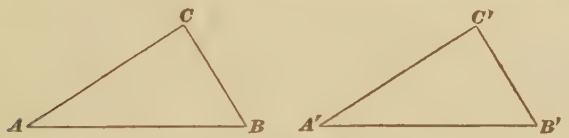

FIG. 46

with $A\,B$, $B\,C$, and $C\,A$, respectively, and the angles A', B', and C' to coincide with the angles A, B, and C. It is evident, from the figure, that if the vertexes of the two triangles coincide, the triangles will coincide throughout, and are, therefore, equal.

The polygons $A\,B\,C\,D\,E$ and $A'\,B'\,C'\,D'\,E'$, Fig. 47, are equal, because $A'\,B'\,C'\,D'\,E'$ can be imagined to be lifted, turned over, and placed on $A\,B\,C\,D\,E$ so as to make the two polygons coincide in all their parts.

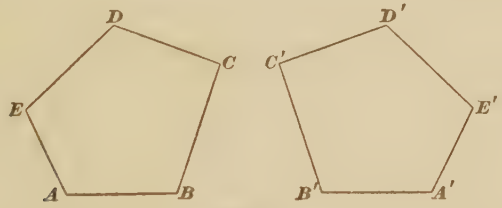

FIG. 47

72. Two triangles are equal when a side and two adjacent angles of one are equal to a side and two adjacent angles of the other.

Let $A'\,B'$, Fig. 48, equal $A\,B$, the angle A' equal the angle A, and the angle B' equal the angle B. Now, if $A'\,B'\,C$ is placed on $A\,B\,C$ so that $A'\,B'$ coincides with its equal $A\,B$, with A' on A and B' on B, $A'\,C'$ will take the direction $A\,C$; since the angle A' is equal to the angle A, and as B' is equal to B, $B'\,C'$ will take the direction $B\,C$.

Now, the point C' will fall somewhere on the line AC, and also somewhere on the line BC, and since two lines can intersect in only one point, C' must fall at the intersection of AC and BC, or at C. Hence, the vertexes of the triangles coincide and the triangles are equal.

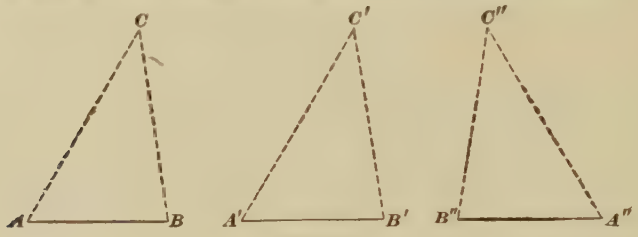

FIG. 48

The same reasoning applies to the triangles ABC and $A''B''C''$, in which $AB = A''B''$, and $A = A''$, $B = B''$; but the triangle $A''B''C''$ must be imagined to be lifted and turned over before it can be placed on ABC.

73. The following important principles are consequences of the preceding proposition:

1. Two triangles are equal when one side and any two angles of one are equal, respectively, to one side and the two similarly situated angles of the other.

2. Two right triangles are equal when one side and one acute angle of the one are equal, respectively, to one side and the similarly situated acute angle of the other.

74. Two triangles are equal when two sides and the included angle of one are equal to two sides and the included angle of the other.

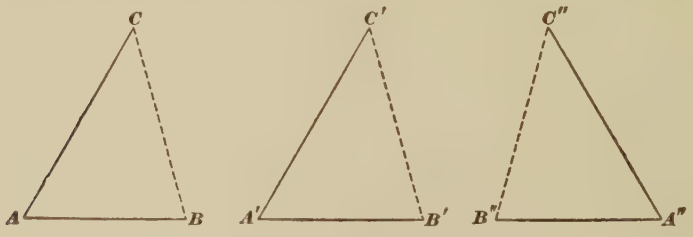

FIG. 49

In Fig. 49, $AB = A'B' = A''B''$, $AC = A'C' = A''C''$, and $A = A' = A''$. If $A'B'C'$ is placed on ABC, so that A' will coincide with A, and $A'B'$ with AB, the rest of the triangles will evidently

coincide; for since $A' = A$, $A'C'$ will take the direction AC, and since $A'C' = AC$, C' will coincide with C. The same reasoning applies to $A''B''C''$, after the latter triangle has been turned over.

75. Two triangles are equal when the three sides of the one are equal, respectively, to the three sides of the other.

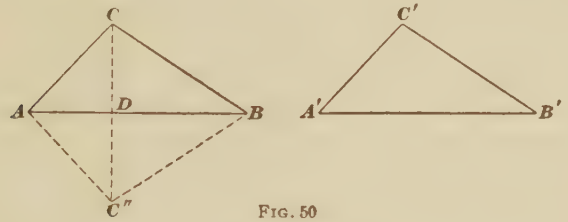

FIG. 50

In Fig. 50, let $A'B'$, $B'C'$, and $C'A'$ be equal, respectively, to AB, BC, and CA. Place $A'B'C'$ in the position ABC'', with its longest side $A'B'$ coinciding with AB, and C'' on the opposite side of AB from C; then join C and C''. Now, AC is equal to AC'', and BC is equal to BC''; hence, A and B determine a perpendicular to CC'' at its mid-point (Art. **34**). Then, by Art. **32**, the angle CAD is equal to the angle $C''AD$, or to $C'A'B'$, and by Art. **74** the triangles CAB and $C'A'B'$ are equal.

76. In an isosceles triangle, the angles opposite the equal sides are equal.

Let ABC, Fig. 51, be an isosceles triangle in which $AB = BC$. Draw the bisector BD of the angle B. Then, by Art. **74**, the triangles ABD and CBD are equal. Therefore, $A = C$.

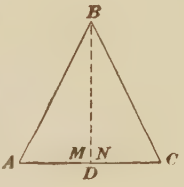

FIG. 51

77. The equality of the triangles ABD and CBD, Fig. 51, gives $AD = DC$, and angle M = angle N = one right angle (since $M + N$ = two right angles). Hence,

1. The bisector of the vertical angle of an isosceles triangle bisects the base and is perpendicular to it.

2. Conversely, the perpendicular bisecting the base of an isosceles triangle passes through the vertex of the opposite angle and bisects that angle.

3. Also, the perpendicular drawn from the vertical angle of an isosceles triangle to the base, bisects both the base and the vertical angle.

78. If two angles of a triangle are equal, the sides opposite these two angles are equal, and the triangle is therefore isosceles.

In Fig. 51, let $A = C$. Draw BD perpendicular to AC. The right triangles BDC and BDA have the common side BD, and acute angle $A = C$. Therefore (Art. **73**), they are equal, and their hypotenuses BA and BC are equal.

79. It follows from Art. **76** that an equilateral triangle is also equiangular, and from the preceding article that an equiangular triangle is also equilateral.

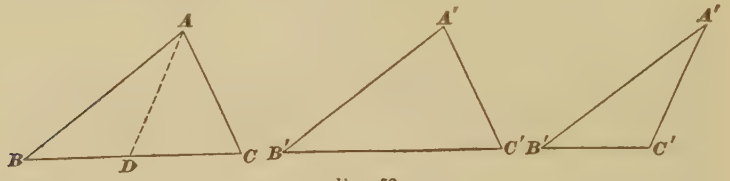

FIG. 52

80. If two sides of a triangle are equal, respectively, to two sides of another triangle, and the angle opposite one of these two sides in the first triangle is equal to the corresponding angle in the second triangle, the angles opposite the other two equal sides are either equal or supplementary.

In Fig. 52, let $A'C' = AC$, $A'B' = AB$, and the angle $B' = B$. Place $A'B'C'$ on ABC so that $A'B'$ coincides with AB. Then since $B' = B$, $B'C'$ will take the direction BC, and since $A'C'$ joins $B'C'$, C' must fall on BC, at either C or D. If C' falls at C, the triangles are equal and the angle $C' = C$; but if C' falls at D, ADB is the angle C', and ADB, the supplement of ADC, is the supplement of C, since, by Art. **76**, $ADC = C$.

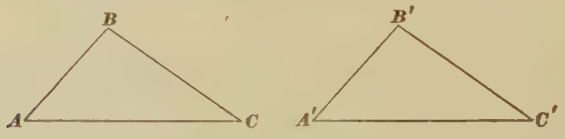

FIG 53

81. If two triangles have two sides of the one equal to two sides of the other, and the angles opposite one pair of the equal sides are right angles or equal obtuse angles, the triangles are equal.

Since a triangle can have but one right or one obtuse angle, when the angles B and B', Fig. 53, are obtuse, the angle C' cannot be the supplement of C, hence C' must equal C.

82. Of two sides of a triangle, that is greater which is opposite the greater angle.

In the triangle ABC, Fig. 54, let the angle C be greater than the angle B. Draw CD, making with CB an angle BCD equal to the angle B. Then BCD is an isosceles triangle, and $CD = DB$. Therefore, $AD + DB$, or AB, is the same as $AD + DC$, which is evidently greater than AC.

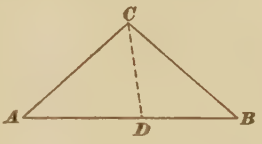

FIG. 54

83. Of two angles of a triangle, that is greater which is opposite the greater side.

Let A and B be two angles of a triangle, a the side opposite A, and b the side opposite B. Suppose that a is greater than b. If A were equal to B, the triangle would be isosceles, and $a = b$. If B were greater than A, then by the preceding article, b would be greater than a. Therefore, since B cannot be equal to or greater than A, it must be less, or A must be greater than B.

84. If from a point O, Fig. 55, without a line AB, a perpendicular OP to the line is drawn, and also two oblique lines OL and OL', the oblique line OL, whose foot L is farther from the foot P of the perpendicular, is the greater of the two oblique lines.

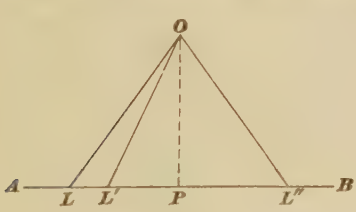

FIG. 55

Suppose the two oblique lines OL and OL' to be on the same side of the perpendicular. Since $OL'P$ is a right triangle and OPL' the right angle, the angle $OL'P$ is acute; also, the angle $OL'L$ is obtuse, since it is the supplement of $OL'P$. As the triangle OLL' can have but one obtuse angle, $OL'L$ is greater than OLP, and, therefore (Art. **82**), OL is greater than OL'. If OL lies on the opposite side of the perpendicular from OL', as in the position OL'', and if $PL'' = PL$, which is greater than PL', then, by Art. **30**, $OL'' = OL$, which is greater than OL'.

85. If the hypotenuse, as AB, Fig. 56, and one leg, as BC, of a right triangle are equal, respectively, to the

hypotenuse and one leg of another right triangle, as $A'B'C'$, the two triangles are equal.

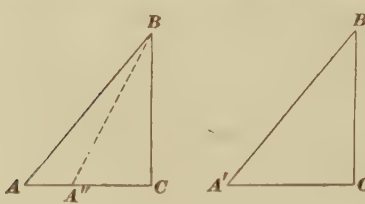

FIG. 56

Place $A'B'C'$ on ABC, so that $B'C'$ will coincide with its equal BC. Since $B'C'A'$ is a right angle, $C'A'$ will take the direction CA; and, since $B'A' = BA$, A' must fall on A; for if it fell to the right of A, as at A'', the hypotenuse BA'', or $B'A'$, would be less than BA (Art. 84); and, if A' fell on the left of A, the hypotenuse $B'A'$ would be greater than BA.

EXAMPLES FOR PRACTICE

1. Show that, if two intersecting lines, as AB and DC, Fig. 57, bisect each other, the lines AC and DB are parallel.

FIG. 57

2. If the value of the unequal or vertical angle of an isosceles triangle is two-fifths of a right angle, what is the value of each of the base angles? Ans. Four-fifths of a right angle

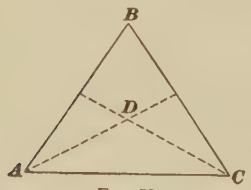

FIG. 58

3. Show that the bisectors of the base angles of an isosceles triangle form with the base an isosceles triangle; or that, ADC, Fig. 58, is an isosceles triangle.

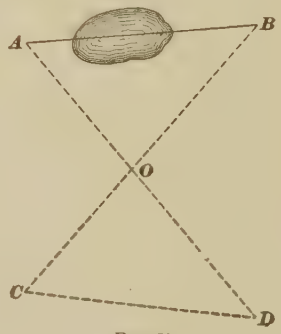

FIG. 59

4. Show that the length of the inaccessible line AB, Fig. 59, can be found by measuring AO and BO, then making $OD = OB$ and $OC = OA$, and finally measuring CD.

QUADRILATERALS

86. There are three kinds of quadrilaterals: the *parallelogram*, the *trapezoid*, and the *trapezium*.

87. A **parallelogram** is a quadrilateral whose opposite sides are parallel. There are four kinds of parallelograms: the *rectangle*, the *square*, the *rhomboid*, and the *rhombus*.

88. A **rectangle**, Fig. 60, is a parallelogram whose angles are all right angles.

FIG. 60

FIG. 61

89. A **square**, Fig. 61, is a rectangle whose sides are equal.

90. A **rhomboid**, Fig. 62, is a quadrilateral whose opposite sides are parallel, and whose angles are not right angles.

FIG. 62

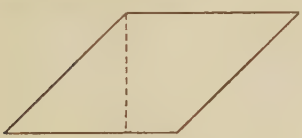

FIG. 63

91. A **rhombus**, Fig. 63, is a rhomboid having equal sides.

92. A **trapezoid**, Fig. 64, is a quadrilateral that has only two of its sides parallel.

FIG. 64

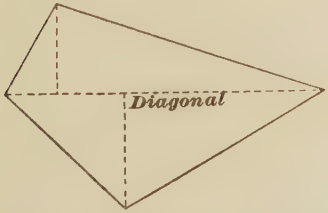

FIG. 65

93. A **trapezium**, Fig. 65, is a quadrilateral having no two sides parallel.

94. The **altitude** of a parallelogram, or of a trapezoid, is the length of the perpendicular distance between the parallel sides. See dotted line in Figs. 62, 63, and 64.

95. A **diagonal** of a quadrilateral is a straight line drawn from the vertex of any angle of the quadrilateral to the vertex of the angle opposite. A diagonal divides a quadrilateral into two triangles. See Figs. 60 and 65.

96. In a parallelogram, as $ABCD$, Fig. 66, the opposite sides and opposite angles are equal; that is, $AB = DC$, $AD = BC$, angle A = angle C, angle B = angle D.

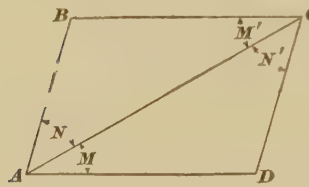

FIG. 66

Draw the diagonal AC. Then, angle M = angle M', and $N = N'$ (Art. **37**). The triangles ADC and ABC, having the common side AC and the adjacent angles M and N' equal, respectively, to M' and N, are equal (Art. **72**). Therefore, $AD = BC$, $AB = DC$, and angle B = angle D. Also, since $M = M'$ and $N = N'$, it follows that $M + N$, or BAD, is equal to $M' + N'$, or BCD.

97. The diagonal of a parallelogram divides the parallelogram into two equal triangles.

98. Parallel lines intercepted between parallel lines are equal. Thus, if the parallels AB and CD, Fig. 67, are cut by the parallels EF, GH, IJ, KL, we have, from Art. **96**, $MN = OP = QR = ST$.

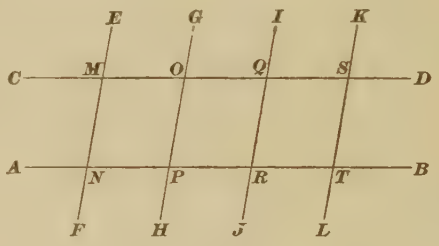

FIG. 67

99. The diagonals of a parallelogram, as AC and BD, Fig. 68, bisect each other; that is, denoting by O the point of intersection of the diagonals, $OA = OC$ and $OB = OD$.

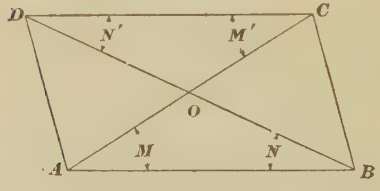

FIG. 68

In the triangles AOB and DOC, $AB = DC$ (Art. **96**), $M = M'$ and $N = N'$ (Art. **37**). Therefore, the triangles are equal (Art. **72**) and $OA = OC$. $OB = OD$.

EXAMPLES FOR PRACTICE

1. Show that if the diagonals of a quadrilateral bisect each other the figure is a parallelogram.

SUGGESTION.—In Fig. 68, assume that $OA = OC$, $OB = OD$. Then show that triangle BOC = triangle AOD, and triangle AOB = triangle DOC.

2. Show that the diagonals of a rectangle are equal.

SUGGESTION.—Show that in any rectangle $ABCD$ the triangle ABC = triangle ABD.

3. Show that if the opposite sides of a quadrilateral are equal, the figure is a parallelogram.

SUGGESTION.—Draw the diagonal. Then, by Art. **75**, the triangles formed are equal.

4. Show that if two sides of a quadrilateral are equal and parallel, the figure is a parallelogram.

5. Show that if one angle of a parallelogram is a right angle, the parallelogram is a rectangle.

ADDITIONAL PROPERTIES OF TRIANGLES

100. The bisectors of the three angles of a triangle meet in a point.

In the triangle ABC, Fig. 69, draw the bisectors of the angles A and B and let them meet at O. Join C and O, and draw the perpendiculars from O to the sides of the triangle. Then, in the right triangles BOF and BOE, BO is common and the angle OBF = angle OBE. Hence, by Art. **73**, these triangles are equal. Therefore, $OF = OE$. In a similar manner it can be shown that $OD = OE$. Therefore, $OD = OF$. The right triangles OFC and ODC, having $OD = OF$ and OC common, are equal (Art. **85**). Hence, angle OCF = angle OCD; that is, OC, which meets the bisectors AO and BO in O, is the bisector of the angle C.

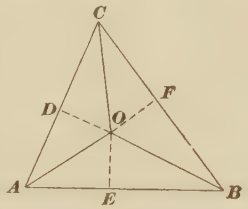

FIG. 69

101. Any point in the bisector of an angle is equally distant from the sides of the angle. For it has just been shown that, in Fig. 69, $OF = OE$.

102. The perpendiculars erected at the middle points of the three sides of a triangle meet in a point equally distant from the vertexes of the triangle.

In Fig. 70, draw the perpendiculars to CB and AC at their midpoints D and E, and let O be the point in which these perpendiculars meet. Now, O, being in OD, is equally distant from C and B (Art. **31**), that is, $OB = OC$: and being in OE, is equally distant from A and C; that is, $OA = OC$. From these two equalities it follows that $OB = OA$. Therefore, the perpendicular to AB at its middle point F passes through O. (Art. **77**).

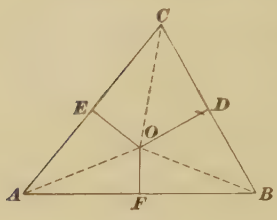

FIG. 70

103. If several parallel lines intercept equal distances on one transversal, they intercept equal distances on any other transversal.

In Fig. 71, let the parallels AE, BG, CI, DK intercept the equal distances AB, BC, and CD on the transversal PQ, and let RS be any other transversal. Draw EF, GH, IJ, parallel to AB. Then, by Art. **98**, $EF = AB$, $GH = BC$, $IJ = CD$. Hence, $EF = GH = IJ$. In the triangles EFG, GHI, and IJK, angle E = angle G = angle I (Art. **38**), and angle F = angle H = angle J (Art. **44**). Hence, by Art. **72**, these triangles are equal, and, therefore, $EG = GI = IK$.

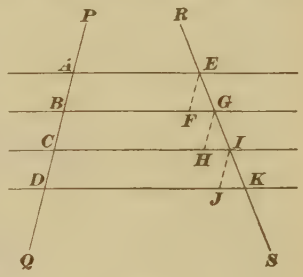

FIG. 71

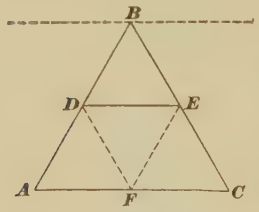

FIG. 72

104. A line parallel to one of the sides of a triangle and bisecting one of the other sides, bisects the third side also.

In Fig. 72, let DE bisect AB and be parallel to AC. Draw a line through B parallel to AC. Then since the three parallels intercept equal parts on AB, they intercept equal parts on BC; that is, $BE = EC$.

105. A line joining the middle points of two sides of a triangle is parallel to the third side and equal to one-half of that third side.

In Fig. 72, let DE join D and E, the middle points of AB and BC. The first part of this proposition follows at once from the preceding

article. Let F be the middle point of AC, and draw FE. This line is parallel to AB, and, therefore, $ADEF$ is a parallelogram. Consequently (Art. **96**), $DE = AF = \frac{1}{2} AC$.

106. The lines joining the middle points of the three sides of a triangle divide it into four equal triangles.

The diagonal DF, Fig. 72, divides the parallelogram $ADEF$ into two equal triangles AFD and DFE. Likewise, the diagonal EF divides $DECF$ into two equal triangles DFE and EFC; and the diagonal DE divides the parallelogram $BDFE$ into the two equal triangles DFE and BDE. Hence, triangle AFD = triangle DFE = triangle EFC = triangle BDE.

107. Any of the parallelograms $ADEF$, $FCED$, $DBEF$, Fig. 72, is equal to one-half the given triangle, since it contains two of the four equal triangles into which the given triangle is divided.

108. A line, as EF, Fig. 73, parallel to the bases AB and DC of a trapezoid and passing through the middle point E of one of the non-parallel sides, passes through the middle point of the other non-parallel side and is equal to one-half the sum of the parallel sides or bases.

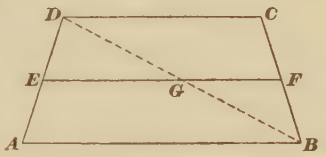

FIG. 73

Since the parallels AB, EF, and DC intercept equal parts on AD, they intercept equal parts on BC (Art. **103**); that is, $BF = FC$.

Draw BD, meeting EF in G. Then, by Art. **105**, in the triangle DCB, FG is one-half CD. Also, in the triangle ADB, GE is one-half BA. Hence, $FG + GE$, or FE, $= \frac{1}{2}(CD + BA)$.

109. The **medians** of a triangle are the lines drawn from the vertexes to the middle points of the opposite sides.

110. The medians of a triangle meet in a point whose distance from any vertex is two-thirds the length of the median from that vertex.

In Fig. 74, AD, BE, CF are the median lines of the triangle ABC; they meet at O, and $AO = \frac{2}{3} AD$, $BO = \frac{2}{3} BE$ and $CO = \frac{2}{3} CF$.

Let AD and CF meet at O. Join I and H, the mid-points of CO and AO, respectively; also join D and F. Then, in the triangle AOC, IH is parallel to AC and equal to one-half AC (Art. **105**). Also, in triangle ABC, DF is parallel to AC and equal to one-half AC. Hence, IH and DF are equal and parallel. It follows that the triangles DOF and HOI are equal, and that, therefore, $HO = OD$. But, by construction, $AH = HO$. Hence, $AH = HO = OD$, whence, $AO = \frac{2}{3}AD$. Similarly $CO = \frac{2}{3}CF$. That is, one median cuts off on the other median two-thirds of the distance from the vertex to the opposite side.

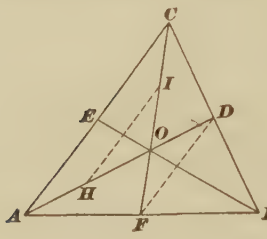

FIG. 74

POLYGONS IN GENERAL

111. Two polygons are equal when they can be divided into the same number of triangles equal each to each and

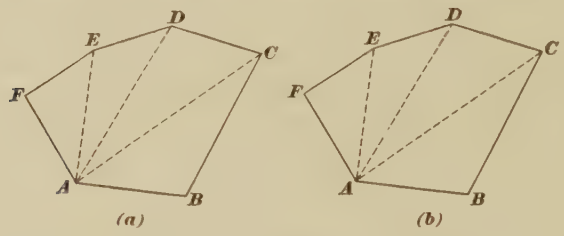

FIG. 75

similarly placed. Thus, the polygons shown at (a) and (b) in Fig. 75 are composed of the same number of triangles equal each to each and similarly placed, and it is evident that one polygon can be placed on the other so that they will coincide throughout; hence, they are equal.

112. An **exterior angle** of a polygon is an angle formed by any side and the prolongation of an adjacent side. In Fig. 76, the angles M and N are exterior angles of the polygon $ABCDE$.

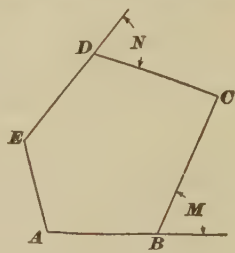

FIG. 76

113. A **diagonal** of a polygon is any line joining two vertexes not adjacent to the same side of the polygon. Thus, in Fig. 75, AC, AD, and AE are diagonals of the polygon $ABCDEF$.

114. The sum of the interior angles of any polygon is equal to two right angles multiplied by a number that is two less than the number of sides of the polygon.

Let (*a*), Fig. 75, be any polygon. Draw the diagonals from one vertex and thus divide the polygon into triangles. It is seen that the first triangle ABC and the last triangle AFE, each contains two sides of the polygon, while each of the other triangles contains but one side of the polygon. Thus, the number of triangles formed is two less than the number of the sides of the polygon. Hence (Art. **69**), the sum of the angles of the triangles, or of the polygon, is two right angles multiplied by a number that is two less than the number of sides of the polygon.

115. Let n = number of sides of a polygon;
S = sum of interior angles of the polygon, expressed in right angles.

Then, $$S = 2\,(n - 2) = 2\,n - 4$$

If $n = 4$, then $S = 2 \times 4 - 4 = 4$ right angles; that is, the sum of the angles of a quadrilateral is equal to four right angles.

Example 1.—What is the value of one of the interior angles of an equiangular hexagon?

Solution.—The number of sides of a hexagon is six; hence, applying the formula, $S = 2 \times (6 - 2) = 8$ right angles, that is, the sum of the interior angles of a hexagon is equal to eight right angles. Since the hexagon is equiangular, one of the angles is equal to one-sixth of eight right angles, or $1\frac{1}{3}$ right angles. Ans.

Example 2.—If one of the interior angles of an equiangular polygon is equal to $1\frac{3}{7}$ right angles, what is the name of the polygon?

Solution.—If one of the interior angles is equal to $1\frac{3}{7}$ or $\frac{10}{7}$ right angles, their sum S is equal to $\frac{10}{7} \times n = \frac{10\,n}{7}$. But from the formula, $S = 2\,n - 4$. Therefore, $\frac{10\,n}{7} = 2\,n - 4$; whence, $n = 7$. A polygon of seven sides is a heptagon; therefore, the polygon is a heptagon. Ans.

EXAMPLES FOR PRACTICE

1. Show that if two angles of a quadrilateral are supplementary the other two angles are supplementary.

2. In a triangle $A\,B\,C$, the angle C is twice the angle B. Show that the line that bisects the angle C meets the line $A\,B$ at a point D so that $C\,D = B\,D$.

SUGGESTION.—Half the angle C = angle B. Then in the triangle $C\,D\,B$, angle $B\,C\,D$ = angle $C\,B\,D$.

3. What is the value of one of the interior angles of an equiangular octagon? Ans. $1\frac{1}{2}$ right angles

4. (*a*) What is the value of one of the interior angles of an equiangular quadrilateral? (*b*) What kind of quadrilateral is it?

Ans. { (*a*) One right angle
(*b*) Rectangle

5. If one of the interior angles of an equiangular polygon is equal to $1\frac{5}{9}$ right angles, what is the name of the polygon? Ans. Nonagon

THE CIRCLE

DEFINITIONS AND GENERAL PROPERTIES

116. A **circle**, Fig. 77, is a plane figure bounded by a curved line every point of which is equally distant from a point within called the **center.**

FIG. 77

117. The **circumference** of a circle is the line that bounds the circle. The term *circle* is often used in the sense of *circumference.*

118. The **diameter** of a circle is a straight line drawn through the center and terminated at both ends by the circumference. Thus, $A\,E$, Fig. 78, is a diameter of the circle whose center is O.

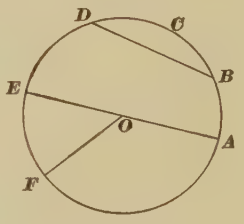

FIG. 78

119. The **radius** of a circle is any straight line drawn from the center to the circumference. The plural of *radius* is *radii*. Thus, OA, OE, and OF, Fig. 78, are radii of the circle whose center is O.

120. The distance from the center to the circumference is, by the definition of a circle, the same for all points in the same circle; hence, all radii are equal.

121. When any two radii, as OA and OE, Fig. 78, are in the same straight line, they form a diameter. Hence, the length of the diameter is twice the length of the radius.

122. An **arc** of a circle is any part of its circumference, as DCB, Fig. 78.

123. An arc equal to one-half the circumference is a **semi-circumference**; and an arc equal to one-fourth the circumference is a **quadrant.**

124. A **chord** is a straight line, as BD, Fig. 78, joining any two points in a circumference, or it is a line joining the extremities of an arc.

125. The longest chord that can be drawn in a circle is a chord that passes through the center and is, therefore, a diameter.

126. An arc of a circle is said to be **subtended** by its chord. Thus, the arc BCD, Fig. 78, is subtended by the chord BD.

Every chord in a circle subtends two arcs. Thus, BD subtends both the arcs BCD and $BAFED$.

When an arc and its chord are spoken of, the arc less than a semi-circumference is meant, unless the contrary is stated. The shorter arc is usually referred to by naming the letters at its extremities; thus, the arc BCD is called the arc BD

127. A **segment** of a circle is a part of the circle enclosed by an arc and its chord. In Fig. 78, the part of the circle between the chord BD and the arc BD is a segment.

A segment equal to one-half the circle is a **semicircle.**

128. A **sector** of a circle is the space included between an arc and the two radii drawn to the extremities of the arc. In Fig. 78, the space included between the arc FE and the radii OF and OE is a sector.

129. Two circles are equal when the radius or diameter of one is equal to the radius or diameter of the other.

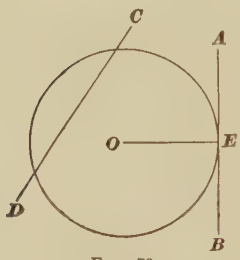

Fig. 79

130. A **tangent** to a circle is a line that touches the circumference in only one point. In Fig. 79, AB is tangent to the circle whose center is O. The point E at which the tangent touches the circumference is the **point of contact,** or **point of tangency.**

131. Two circles are tangent when they touch each other in one point only, as in Fig. 80. When two circles are tangent, they are tangent to the same straight line at the point of tangency.

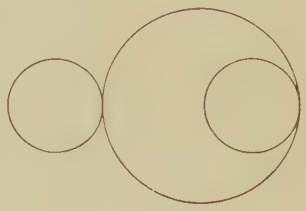
Fig. 80

132. A **secant,** as the term is used in geometry, is a line that intersects the circumference of a circle in two points. In Fig. 79, CD is a secant to the circle whose center is O.

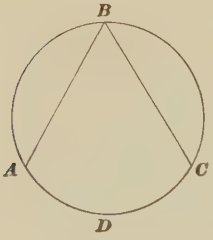

Fig. 81

133. An **inscribed angle** is an angle whose vertex lies on the circumference of a circle, and whose sides are chords. In Fig. 81, ABC is an inscribed angle.

134. A **central angle,** or an **angle at the center,** is an angle whose vertex is at the center of a circle and whose sides are radii. Thus, in Fig. 82, AOB is a central angle.

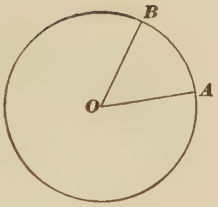

Fig. 82

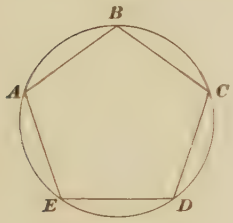

Fig. 83

135. An **inscribed polygon** is a polygon each of whose vertexes lies on the circumference of a circle, as in Fig. 83. The circle is said to be **circumscribed** about the polygon.

136. An **inscribed circle** is a circle whose circumference touches but does not intersect each of the sides of a polygon, as in Fig. 84. The polygon is said to be **circumscribed** about the circle.

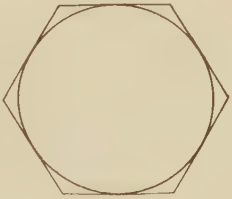
Fig. 84

Fig. 85

137. **Concentric circles** are circles having the same center. See Fig. 85.

138. Every diameter of a circle bisects the circle and its circumference. Thus, in Fig. 86, both the arc and the portion of the circle on one side of the diameter AB are equal, respectively, to the arc and the portion of the circle on the other side.

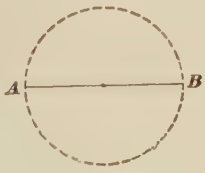

Fig. 86

139. In the same circle, or equal circles, equal angles at the center intercept equal arcs on the circumference.

Let O and O', Fig. 87, be equal circles, and AOB and $A'O'B'$ equal angles. Place the circle O' on O so that the point O' coincides with O and the line $O'B'$ takes the direction OB. Then, since OB and $O'B'$ are equal, being radii of equal circles, B' will fall on B, and, since the angle O' is equal to the angle O, the line $O'A'$ will take the direction of OA, and, being equal to OA, its extremity A' will fall on A. Hence, the arcs AB and $A'B'$ will coincide and are equal.

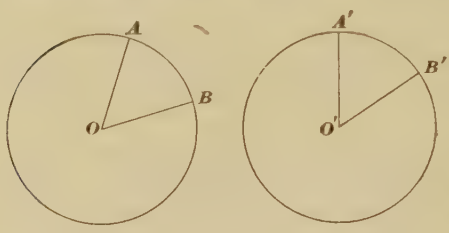

Fig. 87

140. In the same circle, or equal circles, equal arcs are intercepted by equal angles at the center.

Let O and O', Fig. 87, be equal circles, and AB and $A'B'$ equal arcs. Place the circle O' on the circle O, with the points O' and A' on O and A, respectively. Then, since the arc $A'B'$ is equal to the arc AB, B' will fall on B. Then the angle O' is equal to the angle O, as the vertex and the sides of the angles coincide.

141. In the same circle or equal circles, equal chords subtend equal arcs.

Let AB and CD, Fig. 88, be equal chords. Draw the radii AO, BO, CO, and DO, joining A, B, C and D to O. Then the triangles AOB and COD, having three sides of one equal to three sides of the other, are equal. Hence, the angle AOB is equal to the angle COD, and, therefore (Art. **139**), the arc AB is equal to the arc CD.

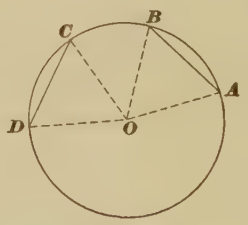

Fig. 88

142. In the same circle, or equal circles, equal arcs are subtended by equal chords.

143. A perpendicular from the center of a circle to a chord bisects the chord and the arc subtended by it.

Let OM, Fig. 89, be drawn from O perpendicular to the chord AB. Join O to A and B. The triangle AOB is isosceles, since the two sides OA and OB are radii of the same circle. Therefore (Art. **77**), $AM = MB$. Also, $\angle AOM = \angle MOB$ (Art. **77**); therefore (Art. **139**), arc AC = arc CB.

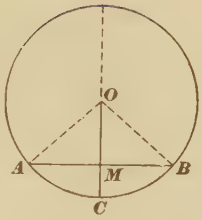

FIG. 89

144. The perpendicular erected at the middle of a chord passes through the center of the circle and bisects the arc subtended by the chord.

145. Through any three points not in a straight line a circumference can be passed.

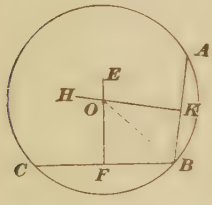

FIG. 90

Let A, B, and C, Fig. 90, be any three points. Draw AB and BC. At the middle point of AB draw KH perpendicular to AB; at the middle point of CB draw FE perpendicular to BC and meeting KH at O. As O is a point in the perpendiculars at the middle points of AB and BC, it is equally distant from A, B, and C. Therefore, a circle with O as center and OB as radius will pass through A, B, and C.

146. A straight line perpendicular to a radius at its extremity is tangent to the circle.

Let AB, Fig. 91, be perpendicular to OH at its extremity H. As OH is perpendicular to AB it is shorter than any other line, as OM, drawn from O to AB. Hence, M is without the circle, and any point in AB other than H is without the circle. Therefore, AB touches the circle in only the point H, and is, consequently, tangent to the circle.

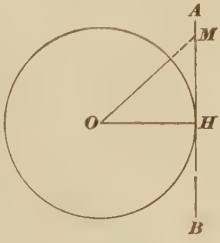

FIG. 91

147. A perpendicular to a tangent at the point of tangency passes through the center of the circle.

148. A tangent to a circle is perpendicular to the radius drawn to the point of tangency.

149. If two circles intersect, the line joining their centers bisects at right angles the line joining the points where the circles intersect.

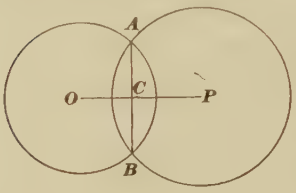

Fig. 92

Let the two circles whose centers are O and P, Fig. 92, intersect at A and B. The point P, being the center of a circle, is equally distant from A and B, points on the circumference. Similarly, O is equally distant from A and B. Hence, by Art. **34**, O and P determine the perpendicular bisecting AB.

150. The two tangents from a point to a circle are equal.

Let PA and PB, Fig. 93, be tangents from P to the circle whose center is O. Draw OA, OP, OB. Then the triangles POB and POA are right triangles (Art. **148**). In these triangles, PO is common and OA is equal to OB. Hence, the triangles are equal, and $PA = PB$.

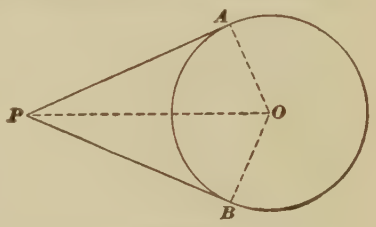

Fig. 93

151. The line joining an external point to the center of a circle bisects the angle made by the two tangents drawn from the point to the circle. Thus, the angle OPA, Fig. 93, is equal to the angle OPB.

EXAMPLES FOR PRACTICE

1. Show that the line joining the intersection of two tangents to the center of the circle bisects the chord joining the points of tangency.

2. Show that the bisector of the angle between two tangents passes through the center of the circle.

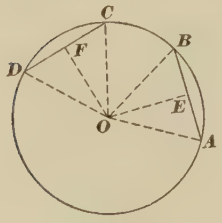

Fig. 94

3. Show that in the same circle, or equal circles, equal chords are equally distant from the center.

Suggestion.—Draw OE and OF, Fig. 94, perpendicular to the equal chords AB and CD. Then what is true of the triangles AEO and DOF?

4. Show that the tangents to a circle at the extremities of a diameter are parallel.

5. Show that in any circle a chord parallel to a tangent is bisected by the diameter drawn to the point of contact.

MEASUREMENT OF ANGLES

152. The **ratio** of one quantity to another of the same kind is the number of times that the first contains the second. When both quantities are represented by numbers, their ratio is the same as the quotient obtained by dividing one of the numbers by the other.

153. In the same circle, or equal circles, two central angles have the same ratio as their intercepted arcs; that is, in Fig. 95, angle AOB : angle COD = arc AB : arc CD.

Suppose the arc AB to be three-fifths of the arc CD. Divide AB into three equal parts, and CD into five equal parts, as shown, and join the points of division with the center.

Since $AB : CD = 3 : 5$, or $\frac{AB}{CD} = \frac{3}{5}$, it follows that one-third of AB is one-fifth of CD; that is, arc AE = arc DF, and, therefore, angle AOE = angle DOF. We have, therefore, angle $AOB = 3 \times$ angle AOE, angle $COD = 5 \times$ angle $DOF = 5 \times$ angle AOE; whence, $\frac{\text{angle } AOB}{\text{angle } COD} = \frac{3 \times \text{angle } AOE}{5 \times \text{angle } AOE} = \frac{3}{5} = \frac{\text{arc } AB}{\text{arc } CD}$.

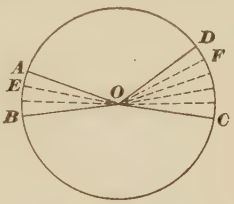

FIG. 95

154. Since the angle at the center and its intercepted arc increase and decrease in the same ratio, it is said that *an angle at the center is measured by its intercepted arc.*

155. The whole circumference of a circle is divided into 360 equal parts, called **degrees.** A degree is divided into 60 equal parts, called **minutes;** and a minute is divided into 60 equal parts, called **seconds.** Degrees, minutes, and seconds of arc are used as units for measuring circular arcs. Since the circumference of every circle contains 360 degrees, the length of a degree differs in different circles. Thus, if AOB, Fig. 96, is an angle of 1°, AB is an arc of 1° in the larger circle and CD is also and arc of 1° in the smaller concentric circle. A degree of the earth's equator is a little more than 69 miles

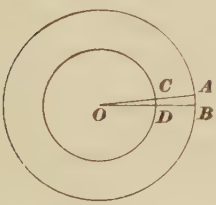

FIG. 96

long; and a degree of the circumference of a circle whose diameter is 360 inches is 3.1416 inches long.

Degrees, minutes, and seconds are indicated by °, ′, ″. Thus, 25° 3′ 10″ means 25 degrees, 3 minutes, and 10 seconds.

Since a right angle intercepts one quarter of a circumference, the number of degrees measuring it is $360 \div 4 = 90°$. The number of degrees measuring an angle equal to one-half of a right angle is $90° \div 2 = 45°$.

Usually, the magnitude of an angle is expressed by stating the number of degrees that it subtends. Thus, a right angle is referred to as an angle of 90°; one-third of a right angle, as an angle of 30°, etc.

156. An inscribed angle is measured by one-half the intercepted arc. Thus, in Fig. 97, the angle ABC is measured by one-half the arc ADC.

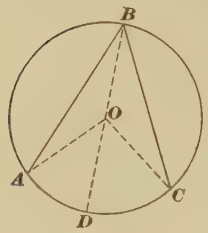

Fig. 97

Draw the diameter BOD and the radii OC and OA. The angle COD, the exterior angle of the triangle OBC, is equal to the angle OBC plus the angle OCB. But the angle OCB is equal to the angle OBC, as they are opposite the equal sides of an isosceles triangle. Hence, the angle COD, which is measured by the arc CD, is equal to $2 \times OBC$. Therefore, OBC is measured by one-half the arc CD. Similarly, the angle OBA is measured by one-half the arc AD. Therefore, the angle ABC is measured by one-half the arc AD plus one-half the arc DC; that is, by one-half the arc AC.

157. In the same circle, or equal circles, equal arcs are intercepted by equal inscribed angles.

158. All angles inscribed in the same segment are equal.

159. Any angle inscribed in a semicircle is a right angle.

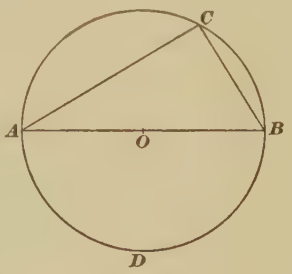

Fig. 98

The angle ACB, Fig. 98, is measured by one-half the arc ADB, which is a semi-circumference. As a semi-circumference contains 180°, the angle ACB is measured by one-half of 180°, or 90°, and is, therefore, a right angle.

160. The vertexes of all the angles of a given magnitude whose sides pass through two fixed points, lie on a circle that passes through the two fixed points and any one of the vertexes.

In Fig. 99, let APB be an angle of the given magnitude and A and B the fixed points. Through A, B, and P, pass a circle. Now, any angle, as AP_1B or AP_2B, whose sides pass through A and B and whose vertex lies on the arc APB is (Art. **158**) equal to the given angle APB.

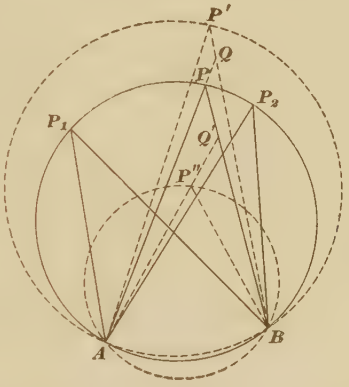

FIG. 99

Again, any angle, as $AP'B$, whose sides pass through A and B and whose vertex lies without the arc APB is less than the angle APB. For if AP is produced to meet BP' at Q, the angle APB being an exterior angle of the triangle BPQ, is equal to $PQB + PBQ$ and is therefore greater than PQB, and, as PQB is greater than $AP'B$ (since $PQB = AP'B + QAP'$), it follows that APB is greater than $AP'B$.

In like manner it can be shown that any angle, as $AP''B$, whose sides pass through A and B and whose vertex lies within the arc APB, is greater than the given angle APB.

161. An angle formed by a tangent, as TM, Fig. 100, and a chord, as TP, is measured by one-half the intercepted arc TEP.

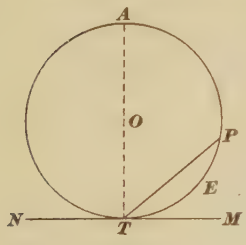

FIG. 100

Draw the diameter TOA. Then MTA is a right angle and is, therefore, measured by one-half the semi-circumference $TEPA$. The angle PTA is measured by one-half the arc PA. Hence, the angle MTP, equal to MTA minus PTA, is measured by one-half the difference between the semi-circumference and PA; that is, by one-half the arc TEP.

EXAMPLES FOR PRACTICE

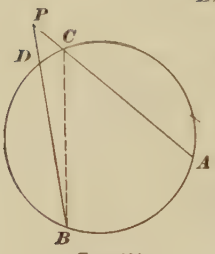

FIG. 101

1. Prove that the angle BPA, Fig. 101, formed by two secants intersecting without the circumference is measured by one-half the difference of the intercepted arcs AB and CD; that is, by $\frac{1}{2}(AB - CD)$.

SUGGESTION.—Join C and B. Then angle BCA is an exterior angle of the triangle BCP, and angle BPC is equal to angle ACB minus angle DBC.

2. Show that the angle APB, Fig. 102, formed by two tangents PT and PT' is measured by one-half the difference of the intercepted arcs TQT' and TRT'.

SUGGESTION.—Join T and T'. Then ATT' is an exterior angle of triangle $TT'P$, while ATT' and $PT'T$ are angles formed by a tangent and a chord.

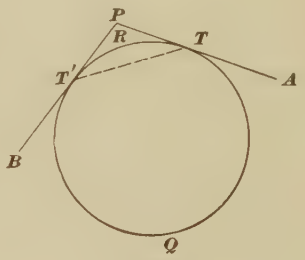

FIG. 102

162. The **angle of intersection** of two tangents is the angle formed by one tangent with the prolongation of the other tangent. Thus, the angle APT', Fig. 103, is the angle of intersection of the two tangents TP and PT'.

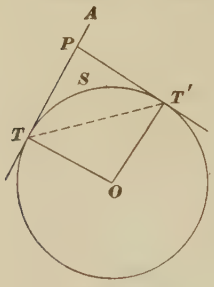

FIG. 103

163. The angle of intersection of two tangents is equal to the central angle whose sides pass through the points of tangency.

In Fig. 103, join TT'. Then, the angle APT' is equal to the sum of the equal angles PTT' and $PT'T$. But each of these angles is made by a tangent and a chord and is, therefore, measured by one-half of the arc TST'. Hence, the angle APT' is measured by the arc TST'. The central angle O is also measured by this arc; therefore, the angle O is equal to the angle APT'.

164. The opposite angles of an inscribed quadrilateral are supplementary; that is, their sum is equal to two right angles or 180°.

In Fig. 104, the angle B is measured by one-half the arc ADC, and the opposite angle D is measured by one-half the arc ABC. The sum of the arcs ADC and ABC is a circumference, or 360°. Hence, the sum of the angles ADC and ABC is measured by one-half of 360°, or 180°.

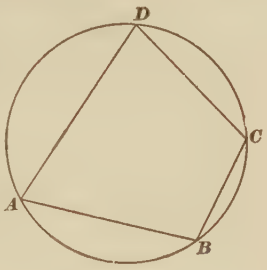

FIG. 104

165. If the opposite angles of a quadrilateral are supplementary, the quadrilateral can be inscribed in a circle.

EXAMPLE 1.—What is the number of degrees in each angle of an equilateral triangle?

SOLUTION.—The sum of the three angles of the triangle is two right angles, or 180°. Since the three angles are equal, each angle is one-third of 180°, or $\frac{180°}{3} = 60°$. Ans.

EXAMPLE 2.—The unequal angle of an isosceles triangle is 75° 32′ 10″; what is the magnitude of each of the equal angles?

SOLUTION.—Since the sum of the three angles is 180°, the sum of the two equal angles is 180° minus the other angle, or 180° − 75° 32′ 10″ = 104° 27′ 50″, and each of them is one-half of this sum, or (104° 27′ 50″) ÷ 2 = 52° 13′ 55″. Ans.

EXAMPLE 3.—The exterior angle of a triangle is 124° 3′ 40″, and one of the opposite-interior angles is 60°; find the other two angles of the triangle.

SOLUTION.—Let the given exterior angle be denoted by A, the given interior angle by B, the other opposite-interior angle by C, and the third angle of the triangle by A'. (Let the student draw the triangle and mark these angles.) Then, $A = B + C$; whence, $C = A - B$ = 124° 3′ 40″ − 60° = 64° 3′ 40″. Ans. Also, $A + A' = 180°$; whence, $A' = 180° - A = 180° - 124° 3' 40'' = 55° 56' 20''$. Ans.

EXAMPLES FOR PRACTICE

1. Show that the only parallelogram that can be inscribed in a circle is a rectangle.

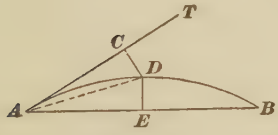

FIG. 105

2. Show that if from a point A, Fig. 105, on the arc of a circle a chord AB and a tangent AT are drawn, the perpendiculars DC and DE drawn to them from the middle point D of the subtended arc are equal.

3. The angle of intersection of two tangents is 100°; find the number of degrees in each angle formed by the tangents and the chord through the points of contact. Ans. 50°

4. One of the acute angles of a right triangle is 50°; what is the magnitude of the other acute angle? Ans. 40°

5. Each of the equal angles of an isosceles triangle is 45°; show that the triangle is right-angled.

6. Two angles of a triangle are 37° 41′ 36″ and 86° 51′ 2″; what is the value of the other angle? Ans. 55° 27′ 22″

GEOMETRY

(PART 2)

PROPORTION

DEFINITIONS AND GENERAL PRINCIPLES

1. A **proportion** is an equality of ratios or of fractions. Thus, the fractions $\frac{4}{5}$ and $\frac{8}{10}$, being equal, form a proportion. In general, if $\frac{a}{b}$ is equal to $\frac{c}{d}$, these two ratios or fractions form a proportion, which may be written in any of the following forms: $\frac{a}{b} = \frac{c}{d}$, $a : b = c : d$, $a : b :: c : d$. When written in either of the last two forms, the proportion is read a is to b as c is to d.

2. Properties of Proportions.—The first and the fourth term of a proportion are called the **extremes**; the second and the third, the **means**. Thus, in the proportion $a : b = c : d$, the extremes are a and d, and the means, b and c.

3. If any four quantities are in proportion, the product of the extremes is equal to the product of the means. This principle follows at once from the definition of a proportion, as will be explained presently. If a, b, c, and d, are in proportion, then, by the definition,

$$\frac{a}{b} = \frac{c}{d} \qquad (1)$$

This equation may be treated the same as any other algebraic equation. Both members of the equation may be

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multiplied or divided by the same quantity, or the same quantity may be added to or subtracted from both members, and the proportion may thus be changed to a great number of forms without destroying the equality of the ratios. Different names are applied to these changes, some of the most common of which are given in the following articles.

In order to show that the product of the means is equal to the product of the extremes, multiply both members of equation (1) by $b\,d$ to clear of fractions; the equation then becomes

$$a\,d = b\,c \qquad (2)$$

4. It is evident that if two fractions are equal, their reciprocals are also equal. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{b}{a} = \frac{d}{c}$; that is, if $a : b = c : d$, we have also $b : a = d : c$.

Taking the reciprocal of a fraction is called **inverting** the fraction. The operation of inverting the two fractions of a proportion is called **inversion.**

5. If both members of equation (2), Art. **3,** are divided by $c\,d$, there results

$$\frac{a}{c} = \frac{b}{d}, \text{ or } a : c = b : d$$

Or, if both members of equation (2) are divided by $b\,a$, the result is

$$\frac{d}{b} = \frac{c}{a}, \text{ or } d : b = c : a$$

Therefore, either the means or the extremes of a proportion can be interchanged. This operation is called **alternation.**

6. If 1 is added to each member of equation (1), Art. **3,** the equation becomes

$$\frac{a}{b} + 1 = \frac{c}{d} + 1$$

Reducing each member to an improper fraction,

$$\frac{a+b}{b} = \frac{c+d}{d}, \text{ or } a + b : b = c + d : d \qquad (1)$$

In a similar manner it can be shown that

$$a + b : a = c + d : c \qquad (2)$$

The proportions (1) and (2) are said to be derived from the original proportion by **composition.**

7. If 1 is subtracted from each member of equation (1), Art. **3,** the equation becomes

$$\frac{a}{b} - 1 = \frac{c}{d} - 1$$

Reducing each member to an improper fraction,

$$\frac{a-b}{b} = \frac{c-d}{d}, \text{ or } a-b : b = c-d : d \qquad (1)$$

In a similar manner it can be shown that

$$a-b : a = c-d : c \qquad (2)$$

The proportions (1) and (2) are said to be derived from the original proportion by **division.**

LINES DIVIDED PROPORTIONALLY

8. Two straight lines are **divided proportionally** when the corresponding segments or parts are in proportion; or when the ratio of the two segments of one is the same as the ratio of the two segments of the other. Thus, the lines AB and CD, Fig. 1, are divided proportionally in the points E and F if $AE : EB = CF : FD$.

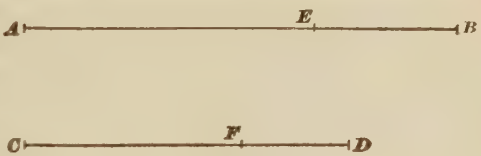

Fig. 1

9. A line parallel to one of the sides of a triangle divides the other two sides proportionally. Thus, in Fig. 2, where DE is parallel to BC, $AD : DB = AE : EC$.

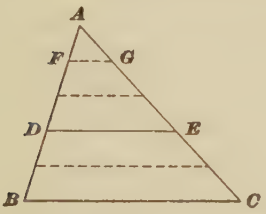

Fig. 2

Suppose that the ratio of AD to DB is as 3 to 2; that is, let $\frac{AD}{DB} = \frac{3}{2}$. Divide AB into five equal parts, and through the points of division draw lines parallel to BC. These lines will intercept equal distances on AC (see *Geometry*, Part 1).

As the ratio of AD to DB is that of 3 to 2, AD will contain three, and DB will contain two, of the equal parts into which AB is divided

Also, AE will contain three and EC two of the equal parts into which AC is divided; so that $AE = 3 \times AG$, and $EC = 2 \times AG$; whence

$$\frac{AE}{EC} = \frac{3 \times AG}{2 \times AG} = \frac{3}{2} = \frac{AD}{DB}$$

or,
$$AE : EC = AD : DB$$

10. Any two sides of a triangle are to each other as the segments into which they are divided by any line parallel to the third side. Thus, in Fig. 2, $AB : AC = AD : AE = DB : EC$.

From the preceding article, we have

$$AD : DB = AE : EC$$

whence (Art. 6),

$$AD + DB : DB = AE + EC : EC$$

that is,
$$AB : DB = AC : EC$$

and, interchanging the means (Art. 5),

$$AB : AC = DB : EC$$

In the same manner it may be shown that

$$AB : AC = AD : AE$$

11. If a line divides two sides of a triangle proportionally, it is parallel to the third side. Thus, if DE, Fig. 3, divides AB and AC so that $AD : DB = AE : EC$, then DE is parallel to BC.

If DE were not parallel to BC, a line DE' could be drawn through D parallel to BC. Then, by Art. 9, we should have $\frac{AD}{DB} = \frac{AE'}{E'C}$; whence, since we have assumed that $\frac{AD}{DB} = \frac{AE}{EC}$,

$$\frac{AE}{EC} = \frac{AE'}{E'C}$$

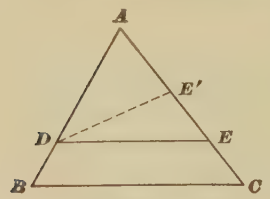

FIG. 3

By interchanging the means of this proportion, we obtain

$$\frac{AE}{AE'} = \frac{EC}{E'C}$$

This equality is evidently absurd, since AE is greater than AE', whereas EC is less than $E'C$. Therefore, no other line than DE can pass through D and be parallel to BC.

EXAMPLE 1.—Find the length of the line AB, Fig. 4, of which the end B is inaccessible.

SOLUTION.—There are several ways of solving this problem in practice. The one illustrated in the figure is as follows: Any convenient distance AC is measured and the angle C observed with a transit or compass. From C, a distance CE is measured, and at E an angle AED equal to C is turned off. The point D where the line of sight ED meets AB is marked, and the distances AD and AE are measured. Then, since AED equals C, the lines ED and CB are parallel (see *Geometry*, Part 1) and, therefore (Art. 9),

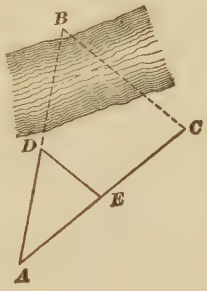

FIG. 4

$$AB : AC = AD : AE$$

whence (Art. 3),

$$AB \times AE = AC \times AD$$

and, dividing by AE,

$$AB = \frac{AC \times AD}{AE}. \quad \text{Ans.}$$

EXAMPLE 2.—Divide a line AB, Fig. 1, of given length into two parts AE and EB whose ratio shall be the same as that of two given numbers m and n; that is, so that $AE : EB = m : n$.

SOLUTION.—Since $AE : EB = m : n$, we must have (Art. 6),

$$\frac{AE + EB}{EB} = \frac{m+n}{n}, \text{ or, } \frac{AB}{EB} = \frac{m+n}{n},$$

whence, solving for EB,

$$EB = \frac{n \times AB}{m+n}. \quad \text{Ans.}$$

AE can be found in a similar manner, or by subtracting the value of EB from AB.

EXAMPLES FOR PRACTICE

1. If the measured distances in Fig. 4 are $AC = 100$ feet, $AE = 45.2$ feet, $AD = 48.36$ feet, what are the distances AB and DB?

Ans. $\begin{cases} AB = 106.99 \text{ ft.} \\ DB = 58.63 \text{ ft.} \end{cases}$

2. If, in Fig. 3, $AD = 75$ feet, $DB = 16.25$ feet, and $AC = 80$ feet, find AE and EC.

Ans. $\begin{cases} AE = 65.75 \text{ ft.} \\ EC = 14.25 \text{ ft.} \end{cases}$

3. If AB, Fig. 1, is equal to 125 feet, find the distances AE and EB so that the line will be divided at E in the ratio of 5 to 2.

Ans. $\begin{cases} AE = 89.286 \text{ ft.} \\ EB = 35.714 \text{ ft.} \end{cases}$

12. If two lines, as AB and CD, Fig. 5, are cut by any number of parallel lines, as EM, GN, IO, etc., the corresponding intercepts are proportional; that is, $EG : GI = MN : NO$; $GI : IK = NO : OP$, or, by interchanging the means, $EG : MN = GI : NO = IK : OP$, etc.

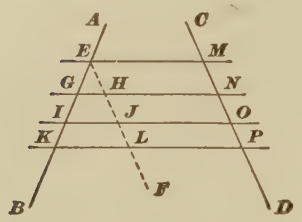

FIG. 5

Through E, draw EF parallel to CD. Then, $EH = MN$, $HJ = NO$, $JL = OP$. (See *Geometry*, Part 1.) Also, by Arts. **9** and **10**,

$$\frac{EK}{EL} = \frac{EG}{EH} = \frac{GI}{HJ} = \frac{IK}{JL}$$

that is,

$$\frac{EK}{MP} = \frac{EG}{MN} = \frac{GI}{NO} = \frac{IK}{OP}$$

or, $$EK : MP = EG : MN = GI : NO = IK : OP$$

13. In any triangle ABC, Fig. 6, the bisector BD of an angle divides the side opposite proportionally to the including sides; that is, $AB : BC = AD : DC$.

Draw CE parallel to BD and meeting AB produced in E. Then, in the triangle AEC, by Art. **9**,

$$AB : BE = AD : DC \qquad (1)$$

The angles DBC and M, being alternate-interior angles, are equal; that is, $M = \frac{1}{2}B$. The angles DBA and E, being exterior-interior angles, are equal; that is, $E = \frac{1}{2}B$. Therefore, $E = M$, and $BE = BC$. (See *Geometry*, Part 1.)

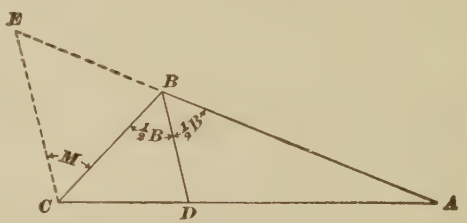

FIG. 6

Substituting, in equation (1), BC for its equal BE,

$$AB : BC = AD : DC$$

POLYGONS

SIMILAR POLYGONS

SIMILAR TRIANGLES

14. **Similar** polygons are those whose corresponding angles are equal and whose corresponding sides are proportional.

In order that two polygons may be similar, it is manifestly necessary that each angle of the one shall be equal to the corresponding angle of the other. But this is not sufficient; the corresponding sides must be proportional. For example,

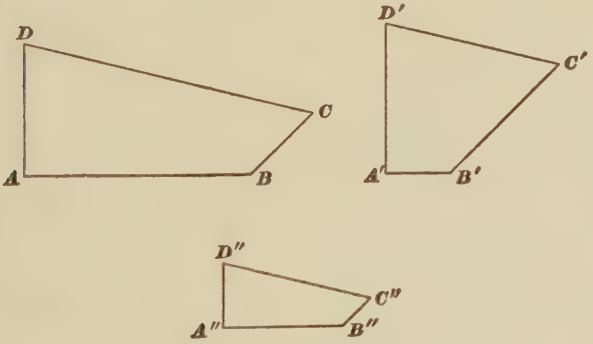

FIG. 7

the quadrilaterals $A\,B\,C\,D$ and $A'\,B'\,C'\,D'$, Fig. 7, have their corresponding angles equal, but they are not similar, because their corresponding sides are not proportional. The quadrilaterals $A\,B\,C\,D$ and $A''\,B''\,C''\,D''$ have their corresponding angles equal and their corresponding sides proportional, and are, therefore, similar.

The corresponding sides of similar polygons are called **homologous sides.**

15. Two triangles are similar when the angles of one are equal to the angles of the other.

In Fig. 8, let the angles of the triangle $A\,B\,C$ be equal, respectively, to those of the triangle $A'\,B'\,C'$. Place the triangle $A'\,B'\,C'$ upon $A\,B\,C$, so that the angle A' will coincide with its equal A. Then B' will fall along $A\,B$ and C' along $A\,C$, as at B'' and C'', respectively, and $B'\,C'$ will take the position $B''\,C''$. The angle B'', which is equal to B', is equal to B, and the angle C'', which is equal to C', is equal to C; hence, $B''\,C''$ is parallel to $B\,C$ (see *Geometry*, Part 1). Then, by Art. **10,**

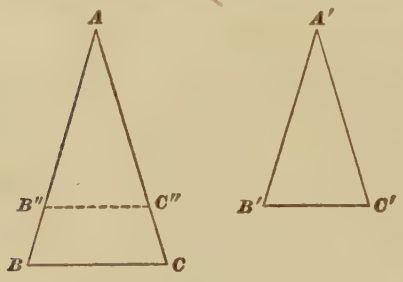

FIG. 8

$$A\,B : A\,B'' = A\,C : A\,C''$$

Substituting $A'\,B'$ and $A'\,C'$ for their respective equals $A\,B''$ and $A\,C''$,

$$A\,B : A'\,B' = A\,C : A'\,C'$$

In like manner, it can be proved that

$$A\,B : A'\,B' = B\,C : B'\,C'$$

Therefore, the triangles, having their angles equal and their corresponding sides proportional, are similar.

16. Two triangles are similar when two angles of the one are equal respectively to two angles of the other.

17. Two right triangles are similar when an acute angle of one is equal to an acute angle of the other.

18. A triangle is similar to any triangle formed by a line parallel to one of its sides and the segments it intercepts on the other two sides or the other two sides prolonged.

19. Two triangles are similar when the three sides of one are either parallel or perpendicular to the three sides of the other.

20. Two triangles are similar when their corresponding sides are proportional.

In Fig. 9, $A\,B : A'\,B' = A\,C : A'\,C' = B\,C : B'\,C'$

On AB, lay off AD equal to $A'B'$; on AC, lay off AE equal to $A'C'$, and join DE. Then, since $AB : AD = AC : AE$, DE is parallel to BC. Hence, by Art. **18**, triangles ABC and ADE are similar, and, consequently, triangles ABC and $A'B'C'$ are similar if it can be shown that $DE = B'C'$. Now,

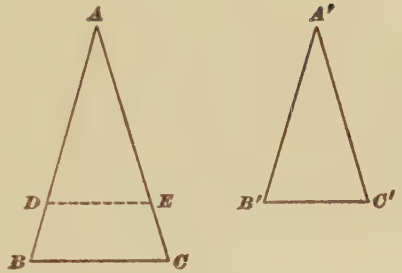

FIG. 9

$$AB : AD = BC : DE, \text{ or } AB : A'B' = BC : DE$$

But,
$$AB : A'B' = BC : B'C'$$

The last two proportions are the same, term for term, excepting the last term; hence, DE is equal to $B'C'$, and the triangles ADE and $A'B'C'$ are equal. Therefore, the triangles ABC and $A'B'C'$ are similar.

21. Two triangles are similar when an angle of the one is equal to an angle of the other and the including sides are proportional.

22. In two similar triangles, corresponding altitudes have the same ratio as any two corresponding sides.

Let CD and $C'D'$, Fig. 10, be the corresponding altitudes of the two similar triangles ABC and $A'B'C'$. The right triangles ACD and $A'C'D'$, having angle A equal to the angle A', are similar; hence,

$$CD : C'D' = AC : A'C'$$

But, since the triangles ABC and $A'B'C'$ are similar,

$$AC : A'C' = AB : A'B' = BC : B'C'$$

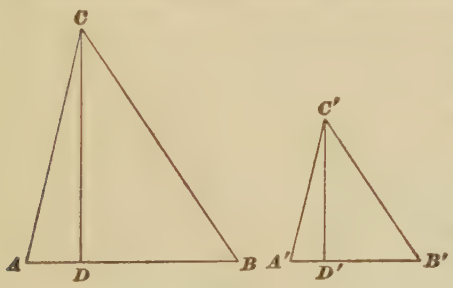

FIG. 10

Therefore,

$$CD : C'D' = AC : A'C' = AB : A'B' = BC : B'C'$$

23. As stated in Art. **14**, two polygons are similar when their corresponding angles are equal and their corresponding sides are proportional. It has now been shown that, in triangles, either of these conditions includes the other. This could have been expected from the fact that either the three

angles or the three sides of a triangle fix its shape. This is not true of a polygon of more than three sides, as the angles can be changed without altering the sides, or the proportions of the sides can be changed without altering the angles.

EXAMPLE 1.—In the triangles $A\,B\,C$ and $A'\,B'\,C'$, Fig. 11, angle A = angle A', angle B = angle B', and angle C = angle C', and the

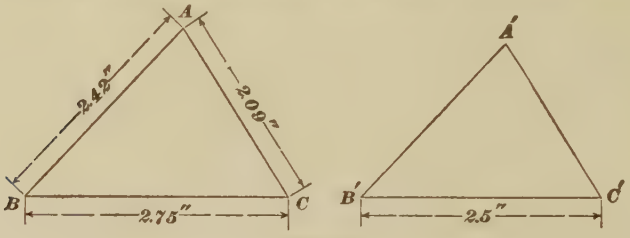

FIG. 11

sides $B\,C$, $C\,A$, $A\,B$, and $B'\,C'$ have the dimensions that are marked on them; find the lengths of the sides $C'\,A'$ and $A'\,B'$.

SOLUTION.—Since the two triangles are equiangular, they are similar, and hence the value of $A'\,B'$ and that of $A'\,C'$ are conveniently found as follows:

$$2.75 : 2.42 = 2.5 : A'\,B'$$

whence $$A'\,B' = \frac{2.42 \times 2.5}{2.75} = 2.2 \text{ in.} \quad \text{Ans.}$$

$$2.75 : 2.09 = 2.5 : A'\,C'$$

whence $$A'\,C' = \frac{2.09 \times 2.5}{2.75} = 1.9 \text{ in.} \quad \text{Ans.}$$

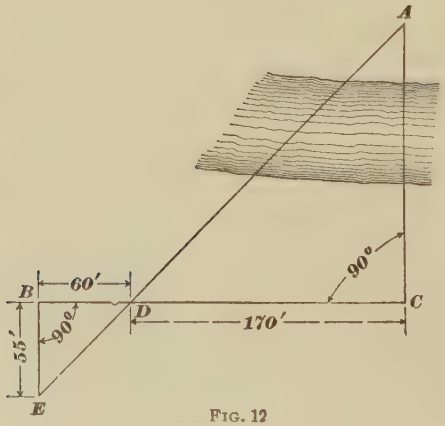

FIG. 12

EXAMPLE 2.—In Fig. 12, $C\,D$ is perpendicular to $A\,C$ and is 170 feet long; $D\,B$ is 60 feet; and $B\,E$, perpendicular to $B\,D$, is 55 feet long; find the distance $A\,C$.

SOLUTION.—The right triangles $A\,C\,D$ and $D\,B\,E$ have the angles $A\,D\,C$ and $B\,D\,E$ equal; hence, they are similar, and

$A\,C : C\,D = B\,E : B\,D$,

or $A\,C : 170 = 55 : 60$

whence,

$$AC = \frac{170 \times 55}{60} = 155.83 \text{ ft.} \quad \text{Ans.}$$

EXAMPLE 3.—It is required to cut from a triangular plate $A\ B\ C$, Fig. 13, having the dimensions shown, a trapezoidal plate $B\ D\ E\ C$ whose upper base $D\ E$ shall be 6 inches; find the distances $A\ D$ and $A\ E$ that must be cut off.

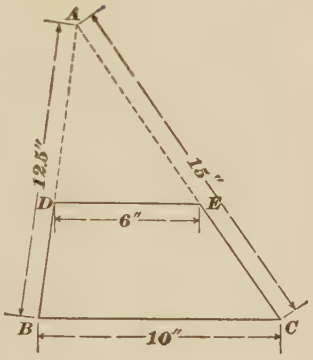

FIG. 13

SOLUTION.—The similar triangles $A\ D\ E$ and $A\ B\ C$ give,

$$\frac{AD}{AB} = \frac{DE}{BC}; \quad AD = \frac{AB \times DE}{BC} = \frac{12.5 \times 6}{10} = 7.5 \text{ in.} \quad \text{Ans.}$$

$$\frac{AE}{AC} = \frac{DE}{BC}; \quad AE = \frac{AC \times DE}{BC} = \frac{15 \times 6}{10} = 9 \text{ in.} \quad \text{Ans.}$$

EXAMPLE 4.—In order to measure the height $R\ H$ of a pier $A_1\ P\ Q\ R$, Fig. 14, whose base and top are, respectively, 22 feet and 12 feet square and whose sides all have the same inclination, a transit was set at a point O distant 250 feet from the side A_1 of the pier; that is, so that $O\ M_1 = 250$ feet. $A_1\ B_1$ was a rod on which the horizontal line of sight $O\ M_1$ intercepted a distance $A_1\ M_1 = 4.5$ feet. The same rod was

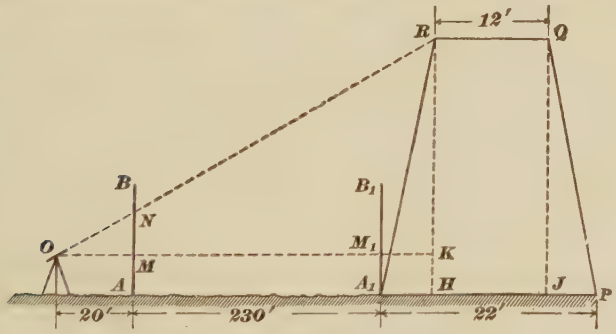

FIG. 14

held at a distance $O\ M = 20$ feet from the instrument, and the height $A\ M$ above the ground noted. Then the telescope of the transit was directed to the top R of the pier, and, with the rod still held at A, the height $A\ N$ was read on the rod. By subtracting $A\ M$ from $A\ N$, the distance $M\ N$ intercepted between the lines $O\ M_1$ and $O\ R$ was found to be 6 feet. What was the height $K\ H$?

SOLUTION.—The inclination of $A_1 R$ and that of $P Q$ being equal, we have, $A_1 H = J P$, and $H J = R Q = 12$ ft. Now,

$$A_1 P = A_1 H + H J + J P = 2 A_1 H + 12$$

whence,
$$A_1 H = \frac{A_1 P - 12}{2} = \frac{22 - 12}{2} = 5 \text{ ft.}$$

The similar triangles $O M N$ and $O K R$ give

$$\frac{O M}{M N} = \frac{O K}{K R}; \quad K R = \frac{O K \times M N}{O M}$$

$$= \frac{(O M_1 + M_1 K) \times M N}{O M} = \frac{(O M_1 + A_1 H) \times M N}{O M}$$

$$= \frac{(250 + 5) \times 6}{20} = \frac{255 \times 6}{20} = 76.5 \text{ ft.}$$

Finally,

$R H = R K + K H = R K + M_1 A_1 = 76.5 + 4.5 = 81$ ft. **Ans.**

EXAMPLES FOR PRACTICE

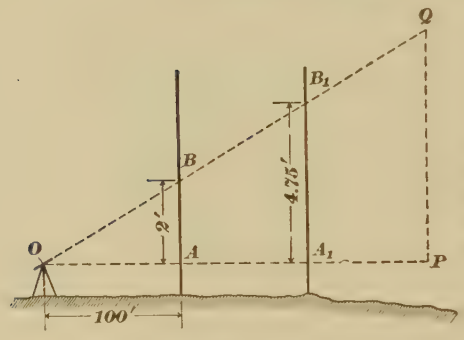

FIG. 15

1. In Fig. 15, the lines of sight $O P$ and $O Q$ of a transit intercept on a rod distances, $A B = 2$ feet and $A_1 B_1 = 4.75$ feet; if the distance $O A$ is 100 feet, what is the distance $O A_1$? Ans. 237.5 ft.

2. In order to find the stress in the member $B D$, Fig. 16, by the method of moments, it is necessary to find the distance $D O$ from D to

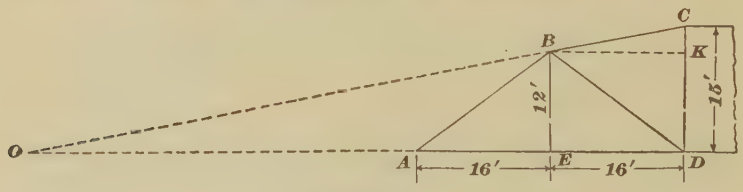

FIG. 16

the point of intersection O of $D A$ and $C B$, both produced; the dimensions being as shown, what is that distance? Ans. $D O = 80$ ft.

3. $A\,B\,C\,D$, Fig. 17, is a trapezoid whose non-parallel sides produced meet at O; the line $M\,N$ is parallel to the bases $A\,D$ and $B\,C$; the dimensions of $A\,D$, $B\,C$, $B\,M$, and $M\,A$ being as shown, find $O\,B$ and $M\,N$.

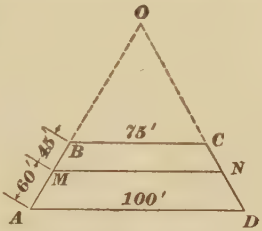

FIG. 17

Ans. $\begin{cases} O\,B = 315 \text{ ft.} \\ M\,N = 85.714 \text{ ft.} \end{cases}$

4. In a triangle $A\,B\,C$, side $A\,B = 32$ feet, $B\,C = 34$ feet, and $A\,C = 48$ feet; if side $A'\,B'$ of a similar triangle $A'\,B'\,C'$ is 72 feet long, what are the lengths of the other two sides?

Ans. $\begin{cases} A'\,C' = 108 \text{ ft.} \\ B'\,C' = 76.5 \text{ ft.} \end{cases}$

5. The base of a right triangle is 24 inches, and its altitude 72 inches; at what distance from the top is the triangle 16 inches wide?

Ans. 48 in.

IMPORTANT CONSEQUENCES OF THE THEORY OF SIMILAR TRIANGLES

24. When the first of three quantities is to the second as the second is to the third, the three quantities are in **continued proportion**; the second is a **mean proportional** between the first and third; and the third is a **third proportional** to the first and second. Thus, if $a : b = b : c$, the three quantities a, b, and c are in continued proportion; b is a mean proportional between a and c; and c is a third proportional to a and b.

25. In a right triangle, as $A\,B\,C$, Fig. 18, the perpendicular $C\,D$ drawn from the vertex of the right angle to the hypotenuse, divides the triangle into two triangles $A\,C\,D$ and $C\,D\,B$ that are similar to the whole triangle and to each other.

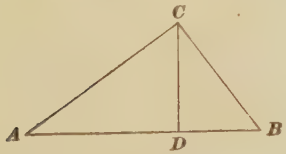

FIG. 18

The right triangles $A\,B\,C$ and $A\,C\,D$ are similar, by Art. **17**, as the angle A is common. Also, the triangles $A\,B\,C$ and $C\,B\,D$, having angle B in common, are similar. Again, the triangles $A\,C\,D$ and $C\,B\,D$, being each similar to $A\,B\,C$, are similar to each other.

26. In a right triangle, the perpendicular to the hypotenuse from the vertex of the right angle is a mean

proportional between the two parts or segments into which it divides the hypotenuse; that is, Fig. 18, $AD:CD = CD:DB$.

As the triangles BCD and ABC are similar, and the angle B is common, the angle BCD must equal the angle A, and similarly the angle ACD must equal the angle B. The triangles ACD and BCD are similar, hence the sides opposite equal angles are in proportion; that is,

$$\frac{AD \text{ (side opposite } ACD)}{CD \text{ (side opposite } B)} = \frac{CD \text{ (side opposite } A)}{DB \text{ (side opposite } BCD)}$$

Or, $$AD : CD = CD : DB$$

27. The side AC, Fig. 18, is a mean proportional between the whole hypotenuse and the segment AD on the same side of CD as the side AC; that is, $AB : AC = AC : AD$. Similarly, $AB : BC = BC : BD$.

The triangles ABC and ACD are similar, hence the sides opposite equal angles are proportional; that is,

$$\frac{AB \text{ (opposite right angle)}}{AC \text{ (opposite right angle)}} = \frac{AC \text{ (opposite } B)}{AD \text{ (opposite } ACD)}$$

Or, $$AB : AC = AC : AD$$

EXAMPLE 1.—In the right triangle ABC, Fig. 19, find the length of the perpendicular CD.

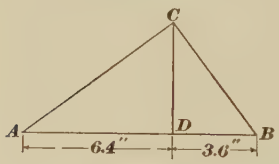

FIG. 19

SOLUTION.—The perpendicular is a mean proportional between the parts AD and DB into which it divides the hypotenuse; therefore,

$$6.4 : CD = CD : 3.6$$

whence, $$\overline{CD}^2 = 6.4 \times 3.6$$

and $$CD = \sqrt{6.4 \times 3.6} = 4.8 \text{ in.} \quad \text{Ans.}$$

EXAMPLE 2.—Find the length of the sides of the right triangle ABC, Fig. 20, in which CD is the perpendicular from the vertex of the right angle to the hypotenuse.

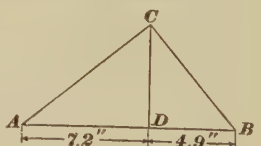

FIG. 20

SOLUTION.—The hypotenuse is 7.2 in. + 4.9 in. = 12.1 in. The side CB is a mean proportional between the hypotenuse AB and the part DB; therefore,

$$12.1 : CB = CB : 4.9$$

$$\overline{CB}^2 = 12.1 \times 4.9$$

$$CB = \sqrt{12.1 \times 4.9} = 7.7 \text{ in.} \quad \text{Ans.}$$

The leg AC is a mean proportional between AB and AD; that is,

$$AB : AC = AC : AD$$

$$AC = \sqrt{AB \times AD}$$

$$= \sqrt{12.1 \times 7.2} = 9.34 \text{ in.} \quad \text{Ans.}$$

28. Since an angle inscribed in a semicircle is a right angle, it follows from Arts. **26** and **27**, that:

(a) A perpendicular CD, Fig. 21, drawn from any point on the circumference of a circle to a diameter AB, is a mean proportional between the segments into which it divides the diameter; that is,

$$AD : CD = CD : DB$$

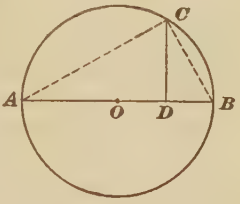

FIG. 21

(b) A chord CA drawn from a point in a circumference to the end of a diameter is a mean proportional between the whole diameter and the adjacent segment AD; that is,

$$AB : AC = AC : AD$$

29. If from a point without a circle, a tangent and a secant are drawn, the tangent is a mean proportional between the whole secant and the exterior segment; that is, in Fig. 22, $PB : PT = PT : PA$.

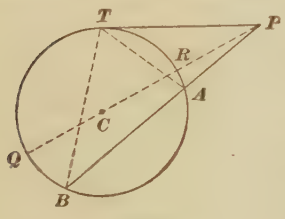

FIG. 22

In the triangles BPT and APT, the angle P is common. The angle B, an inscribed angle, and the angle PTA, an angle formed by a tangent and a chord, are equal, since each is measured by one-half the same arc AT. Hence, the triangles are similar by Art. **16**, and

$$\frac{PB \text{ (opposite angle } PTB)}{PT \text{ (opposite angle } PAT)} = \frac{PT \text{ (opposite angle } B)}{PA \text{ (opposite angle } PTA)}$$

$$PT^2 = PB \times PA$$

$$PT = \sqrt{PB \times PA}$$

30. If from a point without a circle any two secants are drawn, the product of one secant and its external segment is equal to the product of the other secant and its external segment.

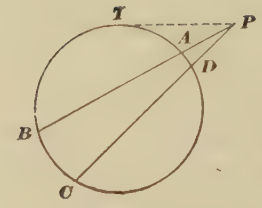

FIG. 23

In Fig. 23, PB and PC are secants. Draw the tangent PT. Then, from Art. **29**,

$$\overline{PT}^2 = PA \times PB$$

and $$\overline{PT}^2 = PC \times PD$$

hence, $$PA \times PB = PC \times PD$$

31. If any two chords be drawn through a point within a circle, the product of the segments of one is equal to the product of the segments of the other.

In Fig. 24, the angles D and B, being measured by one-half the arc AC, are equal. The angles BPC and DPA, being vertical angles, are equal. Hence, by Art. **16**, the triangles CBP and ADP are similar. Therefore,

$$\frac{AP}{CP} = \frac{PD}{PB}$$

and $$AP \times PB = CP \times PD$$

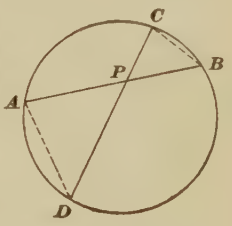

FIG. 24

EXAMPLES FOR PRACTICE

1. The perpendicular from the vertex of the right angle of a right triangle divides the hypotenuse into parts of 23.04 inches and 1.96 inches. Find: (*a*) the length of the perpendicular; (*b*) the length of the two sides of the triangle.

Ans. { (*a*) 6.72 in.
(*b*) 24 in. and 7 in.

2. If, in Fig. 22, the distance CP of the point P from the center of the circle is 65 feet, and the radius CR is 25 feet, what is the length of the tangent PT? Ans. 60 ft.

3. The chord of the arc of a segment is 14 inches long and the height of the segment is 2 inches; what is the radius? Ans. $13\frac{1}{4}$ in.

OTHER SIMILAR POLYGONS

32. Two polygons are similar when they are composed of the same number of triangles similar each to each and similarly placed.

Thus, in Fig. 25, the polygons $ABCDE$ and $A'B'C'D'E'$ are composed of the same number of similar triangles similarly placed

Since the triangle $A\,E\,D$ is similar to the triangle $A'\,E'\,D'$, angle E = angle E' and angle $A\,D\,E$ = angle $A'\,D'\,E'$. Also, in the similar triangles $A\,D\,C$ and $A'\,D'\,C'$, angle $A\,D\,C$ = angle $A'\,D'\,C'$. Hence, the sum of the angles $A\,D\,E$ and $A\,D\,C$, or the angle $E\,D\,C$, is equal to the sum of the angles $A'\,D'\,E'$ and $A'\,D'\,C'$, or the angle $E'\,D'\,C'$. In like manner, angle $D\,C\,B$ = angle $D'\,C'\,B'$,

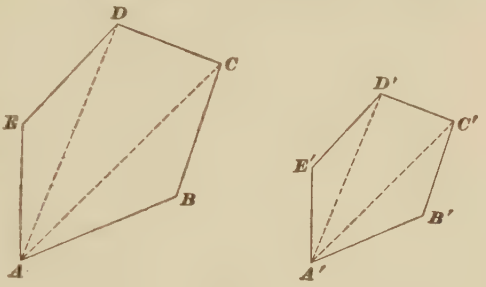

FIG. 25

angle B = angle B', and angle $B\,A\,E$ = angle $B'\,A'\,E'$. Since the triangles are similar,

$$E\,D : E'\,D' = A\,D : A'\,D' \text{ and } A\,D : A'\,D' = D\,C : D'\,C'$$

hence,
$$E\,D : E'\,D' = D\,C : D'\,C'$$

In like manner,

$$D\,C : D'\,C' = C\,B : C'\,B' = B\,A : B'\,A' = A\,E : A'\,E'$$

Therefore, as the angles of the one polygon are equal to the corresponding angles of the other and the sides of the one polygon are proportional to the sides of the other, the polygons are similar.

33. Two similar polygons can be divided into the same number of similar triangles similarly placed.

34. The perimeters of two similar polygons are in the same ratio as any two homologous sides.

In Fig. 25, let P be the perimeter of the polygon $A\,B\,C\,D\,E$, and P' the perimeter of the polygon $A'\,B'\,C'\,D'\,E'$. Since the polygons are similar

$$\frac{A\,E}{A'\,E'} = \frac{E\,D}{E'\,D'} = \frac{D\,C}{D'\,C'} = \frac{C\,B}{C'\,B'} = \frac{B\,A}{B'\,A'} \qquad (1)$$

Let each of these equal ratios be denoted by R; that is, let

$$\frac{A\,E}{A'\,E'} = R,\ \frac{E\,D}{E'\,D'} = R,\ \frac{D\,C}{D'\,C'} = R,\ \frac{C\,B}{C'\,B'} = R,\ \frac{B\,A}{B'\,A'} = R.$$

From these equations we obtain,

$$A\,E = R \times A'\,E',\ E\,D = R \times E'\,D',\ D\,C = R \times D'\,C',$$
$$C\,B = R \times C'\,B',\ B\,A = R \times B'\,A'$$

Adding the sides of these equalities,

$$A\,E + E\,D + D\,C + C\,B + B\,A$$
$$= R \times A'\,E' + R \times E'\,D' + R \times D'\,C' + R \times C'\,B' + R \times B'\,A'$$
$$= R\,(A'\,E' + E'\,D' + D'\,C' + C'\,B' + B'\,A')$$

whence $$\frac{AE + ED + DC + CB + BA}{A'E' + E'D' + D'C' + C'B' + B'A'} = R$$

But $$R = \frac{AE}{A'E'} = \frac{ED}{E'D'} = \frac{DC}{D'C'}, \text{ etc.;}$$

therefore, $$\frac{P}{P'} = \frac{AE}{A'E'} = \frac{ED}{E'D'} = \frac{DC}{D'C'}, \text{ etc.}$$

35. Equation (1) of the preceding article is a series of equal ratios, of which the numerators are the antecedents and the denominators the consequents. The general truth was shown in that article, that in a series of equal ratios the sum of the antecedents is to the sum of the consequents as any antecedent is to its consequent.

AREAS OF POLYGONS

36. Definitions.—The **area** of a surface is the superficial space included within its boundary lines. Area is expressed by the ratio of the surface to a surface of fixed value chosen as a unit and called the **unit of area.**

37. A square whose side is equal in length to the unit of length is usually taken as the unit of area, and its area is called the **square unit.** For example, if the unit of length is 1 inch, the unit of area, or square inch, is the square whose sides measure 1 inch, and the area of any surface is expressed by the number of square inches that the surface contains. If the unit of length were 1 foot, the unit of area would measure 1 foot on each side, and the area of the surface would be expressed in square feet. Square inch and square foot are abbreviated to sq. in. and sq. ft., respectively, and are often indicated by the symbols $\square''$ and $\square'$.

38. Two surfaces are **equivalent** when their areas are equal.

39. Comparison of the Areas of Two Rectangles. The areas of two rectangles $AB\,CD$ and $A'B'\,C'D'$, Fig. 26, having equal altitudes are to each other as their bases; that is, area $AB\,CD$: area $A'B'\,C'D' = AB : A'B'$.

Suppose that $A'B'$ is four-fifths of AB, or that $AB : A'B' = 5 : 4$. Divide AB into five equal parts AE, EF, etc., and $A'B'$ into four

equal parts $A'E'$, $E'F'$, etc. It is evident that $A'E' = AE$, for, since AB is to $A'B'$ in the ratio of 5 to 4, any quantity, as AE, that is contained five times in AB must be contained four times in $A'B'$. Through the points of division E, F, E', F', etc., draw perpendiculars

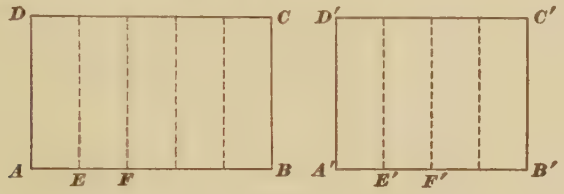

FIG. 26

to AB and $A'B'$. Each large rectangle is thus divided into small rectangles, all of which are equal. As $ABCD$ contains five, and $A'B'C'D'$ contains four, of the small rectangles, the ratio of the two large rectangles is that of 5 to 4, which is also the ratio of their bases.

40. Since any of the sides of a rectangle can be considered as the base, it follows that the area of two rectangles having equal bases are to each other as their altitudes.

41. The areas of any two rectangles are to each other as the products of their bases by their altitudes.

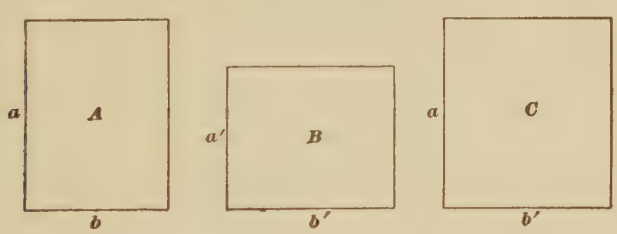

FIG. 27

Let A and B, Fig. 27, be two rectangles whose altitudes are a and a' and whose bases are b and b', respectively. Construct a rectangle C with an altitude a and a base b'. Then, by Arts. **39** and **40**,

$$A : C = b : b' \qquad (1)$$

and

$$C : B = a : a' \qquad (2)$$

Multiplying equation (1) by equation (2),

$$AC : BC = ab : a'b' \qquad (3)$$

Dividing the terms of the first member of equation (3) by C,

$$A : B = ab : a'b'$$

42. Area of a Rectangle.—The area of a rectangle is equal to its base multiplied by its altitude; that is, in Fig. 28,

$$A = b\,h.$$

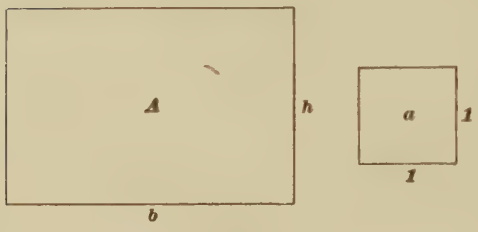

Fig. 28

Construct a unit square a. Then (Art. 41),

$$A : a = h \times b : 1 \times 1;$$

or,
$$\frac{A}{a} = \frac{h \times b}{1 \times 1}$$

But a is a unit square, and its area is therefore equal to 1; hence,

$$A = b\,h$$

43. Area of a Triangle.—The area of a right triangle is equal to one-half the product of the two legs of the triangle; that is, in Fig. 29, area $A\,B\,C = \frac{1}{2}\,a\,b$.

For the triangle $A\,B\,C$ is one-half the rectangle $A\,B\,C\,D$ and the area of the latter is $a\,b$.

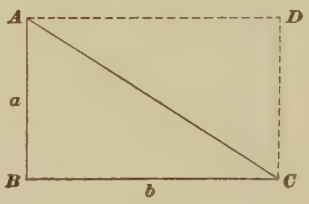

Fig. 29

44. The area of any triangle is equal to one-half the product of its base and altitude.

In Fig. 30 (a), let $A\,C$ be the base and $B\,H$ the altitude of the triangle $A\,B\,C$. The area $A\,B\,C$ is equal to the sum of the right triangles $A\,H\,B$ and $C\,H\,B$, which, by the last article, is

$$\tfrac{1}{2}\,B\,H \times A\,H + \tfrac{1}{2}\,B\,H \times H\,C = \tfrac{1}{2}\,B\,H \times (A\,H + H\,C) = \tfrac{1}{2}\,B\,H \times A\,C$$

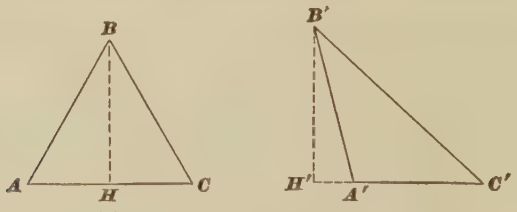

Fig. 30

In Fig. 30 (b), the area $A'\,B'\,C'$ is the difference between the areas of the right triangles $B'\,H'\,C'$ and $B'\,H'\,A'$; that is,

$$\tfrac{1}{2}\,B'\,H' \times H'\,C' - \tfrac{1}{2}\,B'\,H' \times H'\,A' = \tfrac{1}{2}\,B'\,H' \times (H'\,C' - H'\,A')$$
$$= \tfrac{1}{2}\,B'\,H' \times A'\,C'$$

Let b be the base, h the altitude, and A the area of any triangle; then,

$$A = \tfrac{1}{2}\, b\, h$$

45. Two triangles having the same base are to each other as their altitudes, and two triangles having the same altitude are to each other as their bases.

46. Two triangles having the same base and the same altitude are equivalent.

It should be borne in mind that any side of a triangle can be taken as the base, the altitude being the perpendicular to that side from the opposite vertex.

47. To find the area of a triangle from the lengths of its three sides, apply the following:

Rule.—*From half the sum of the three sides subtract each side separately; multiply together the half sum and the three remainders and extract the square root of the product.*

Let a, b, and c be the three sides of a triangle, and A the area; let

$$s = \tfrac{1}{2}(a + b + c)$$

Then $$A = \sqrt{s(s-a)(s-b)(s-c)}$$

The geometrical proof of this rule is very laborious, and will not be given here. A proof will be found in *Trigonometry*.

EXAMPLE.—What is the area of a triangle having two sides 19.8 feet long, and one side 28 feet long?

SOLUTION.—It is immaterial which side is called a, b, or c. $s = \frac{a+b+c}{2} = \frac{28 + 19.8 + 19.8}{2} = 33.8$; taking b and c as the short sides, $s - a = 33.8 - 28 = 5.8$, and $s - b$ and $s - c$ are each $33.8 - 19.8 = 14$. Then, applying the formula

$A = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{33.8 \times 5.8 \times 14 \times 14} = 196$ sq. ft., nearly. Ans.

48. A triangle equivalent to any given polygon may be constructed as follows:

Let $A\,B\,C\,D\,E\,F$, Fig. 31, be the given polygon. Produce any of the sides, as $A\,F$, in both directions, as indicated by $X\,Y$. This line

will be referred to as the base. Starting from one of the ends of AF as A, draw a diagonal AC forming a triangle with AB and BC. Draw BB_1 parallel to CA, meeting the base at B_1, and join C to B_1.

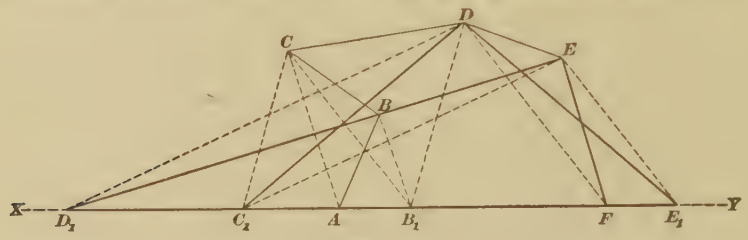

FIG. 31

The polygon B_1CDEF has one side less than the given polygon, and is equivalent to it. For

$$ABCDEF = B_1BCDEF + \text{triangle } B_1BA$$
$$B_1CDEF = B_1BCDEF + \text{triangle } B_1BC$$

The two triangles B_1BA and B_1BC are equivalent, for they have the common base B_1B, and their altitudes, being each equal to the distance between the parallels AC and B_1B, are equal. Proceeding with the polygon B_1CDEF as with the original polygon, draw the diagonal B_1D, forming a triangle with B_1C and CD. Draw CC_1 parallel to DB_1, and join D and C_1. It can be shown as before that the polygon C_1DEF is equivalent to B_1CDEF, and, therefore, to the original polygon. Finally, draw the diagonal C_1E, and DD_1 parallel to it, meeting the base at D_1. Then will the triangle D_1EF be the required triangle equivalent to the given polygon.

In practice, it is more convenient, as well as more accurate, to reduce about one-half of the polygon on one side of A and the rest on the other side of F. Thus, having reduced the polygon to the quadrilateral C_1DEF, the diagonal FD is drawn from F; EE_1 is drawn through E parallel to DF, and E_1 joined to D. This gives C_1DE_1 as the required triangle.

49. Area of a Parallelogram.—The area of a parallelogram is equal to its base multiplied by its altitude; that is, in Fig. 32, area $ABCD = AD \times MN$.

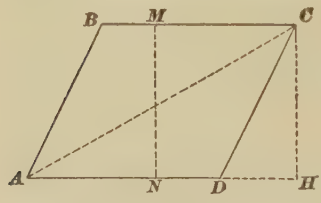

FIG. 32

For $ABCD$ is equal to the sum of the equal triangles ABC and ADC, or to twice either of them, as ADC; that is, $ABCD = 2 \times \frac{1}{2} AD \times CH = AD \times CH = AD \times MN$.

EXAMPLE 1.—What is the cost of paving a street 1,800 feet long and 36 feet wide with asphalt, the price being $2 per square yard?

SOLUTION.—The surface to be covered is a rectangle whose sides are 36 ft. and 1,800 ft., or 12 yd. and 600 yd., and whose area is, therefore, $12 \times 600 = 7{,}200$ sq. yd. The cost of paving is, then, $2 \times 7{,}200 = \$14{,}400$. Ans.

EXAMPLE 2.—One side of a triangular plot of land is 125 feet long and the perpendicular distance from the opposite vertex to this side is 174.24 feet; it is desired to find a side of a rectangle that has the same area as the triangle and one side 75 feet long.

SOLUTION.—The area of the triangle is $\frac{1}{2} \times 125 \times 174.24 = 10{,}890$ sq. ft. Then the other side of the rectangle is $10{,}890 \div 75 = 145.2$ ft. Ans.

EXAMPLE 3.—Divide a triangular plot of land into any number of equal parts by lines from a vertex to the opposite side.

SOLUTION.—Divide the side opposite the vertex through which the lines are to be run into the required number of equal parts and run lines from the vertex of the triangle to the points of division. Then, since the triangles thus formed have equal bases and their vertexes in the same point, they are equivalent. Ans.

EXAMPLE 4.—Divide a given triangle into parts proportional to any given numbers by lines run through a vertex.

SOLUTION.—Let the given triangle be ABC, Fig. 33, and let it be required to divide it into parts proportional to 3, 4, and 5, by lines drawn from the vertex A.

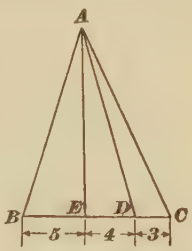

FIG. 33

The base BC is divided into parts proportional to the numbers 3, 4, and 5, by dividing it into $3 + 4 + 5 = 12$ equal parts, and then marking the third and the seventh points of division. From the points thus marked, lines are run to the vertex A. Then, by Art. **45**,

$$CAD : ADE = 3 : 4$$

and,

$$ADE : ABE = 4 : 5 \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Find the area of a square whose side is 5 feet 9 inches. Ans. 33.062 sq. ft.

2. Find the area of a rhombus whose length is 12.5 feet, and whose height is 9.25 feet. Ans. 115.62 sq. ft.

3. One side of a room is 16 feet long; if the floor contains 240 square feet, what is the length of the other side? Ans. 15 ft.

4. In a trapezium two not adjacent sides are 16 and 14 inches, respectively. A diagonal divides the trapezium into two triangles whose altitudes from their vertexes to the given sides as bases are 17 inches and 3 inches, respectively; what is the area of the trapezium? Ans. 157 sq. in.

5. The base BC of a triangle is 150 chains and the perpendicular from the opposite vertex A to BC is 45 chains; it is desired to divide the triangle into two parts equal in area by a line from A to BC; how far from B is D, the intersection of this line with BC?
Ans. 75 ch.

6. From the mid-point E of the side AB of a parallelogram $ABCD$, lines are drawn to the vertexes D and C and to the mid-point of the side CD; show that these lines divide the parallelogram into four triangles that are equal in area.

7. Find the area of a triangle whose three sides are 13, 14, and 15 feet. Ans. 84 sq. ft.

8. Find the area of a right triangle whose hypotenuse is 50 feet and one of whose legs is 40 feet. Ans. 600 sq. ft.

---

50. Area of a Trapezoid.—The area of a trapezoid is equal to one-half the sum of the parallel sides multiplied by the altitude; that is, in Fig. 34, area of trapezoid $ABCD = \frac{1}{2}(AB + DC) \times MN$.

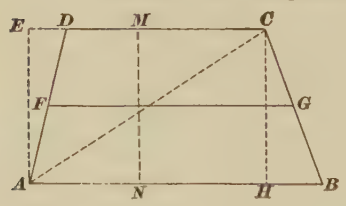

FIG. 34

The area of the trapezoid is equal to the sum of the areas of the two triangles ABC and ADC; hence,

$$\begin{aligned} ABCD &= \tfrac{1}{2}AB \times CH + \tfrac{1}{2}DC \times AE \\ &= \tfrac{1}{2}AB \times MN + \tfrac{1}{2}DC \times MN \\ &= \tfrac{1}{2}(AB + DC) \times MN \end{aligned}$$

Let b_1 = length of lower base;
b_2 = length of upper base;
h = altitude.

Then, the area A of the trapezoid $A\,B\,C\,D$ is

$$A = \tfrac{1}{2}(b_1 + b_2)h$$

51. Since the median line $F\,G$, Fig. 34, joining the mid-points of the non-parallel sides is equal to $\tfrac{1}{2}(A\,B + D\,C)$, the area of a trapezoid is equal to the product of the median line by the altitude.

EXAMPLE.—Divide a plot of ground in the form of a trapezoid into any number of equal parts by lines intersecting the two bases.

SOLUTION.—Divide each of the bases into the same number of equal parts into which the trapezoid is to be divided and run lines through the corresponding points of division. The trapezoids thus formed have equal bases and the same altitude and are, therefore, equal in area. Ans.

EXAMPLES FOR PRACTICE

1. The parallel sides of a trapezoid are 321.51 and 214.24 feet, and the perpendicular distance between them is 171.16 feet; what is the area of the trapezoid? Ans. 45,849 sq. ft.

2. Find the area of a trapezoid whose parallel sides are 20.5 and 12.25 chains, the perpendicular distance between them being 10.75 chains. Ans. 17.603 A.

3. The parallel sides of a trapezoidal plot of ground are 400 feet and 360 feet long; the distance between the parallel sides is 100 feet. It is desired to divide this plot into five lots by lines intersecting the parallel sides; what will be the length of the front and the rear of one of the lots? Ans. 80 ft. and 72 ft.

4. How many square feet are there in a board 12 feet long, 18 inches wide at one end, and 12 inches wide at the other end? Ans. 15 sq. ft.

---

52. Area of Any Polygon.—The area of any polygon can be found by dividing the polygon into triangles, determining the area of each triangle, and adding the results.

53. Comparison of the Areas of Similar Polygons. The areas of two similar triangles are to each other as the squares of their homologous sides.

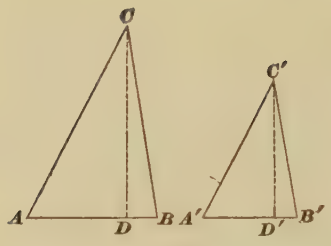

FIG. 35

In Fig. 35,

Area $ABC = \frac{1}{2} AB \times CD$ (1)

Area $A'B'C' = \frac{1}{2} A'B' \times C'D'$ (2)

Dividing equation (1) by equation (2), $\frac{ABC}{A'B'C'} = \frac{A'B}{A'B'} \times \frac{CD}{C'D'}$ (3)

but, by Art. **22**, $\frac{CD}{C'D'} = \frac{AB}{A'B'}$;

hence, substituting in (3)

$$\frac{AB}{A'B'} \text{ for } \frac{CD}{C'D'}, \quad \frac{ABC}{A'B'C'} = \frac{AB}{A'B'} \times \frac{AB}{A'B'} = \frac{\overline{AB}^2}{\overline{A'B'}^2}$$

that is, $$ABC : A'B'C' = \overline{AB}^2 : \overline{A'B'}^2$$

54. The areas of two similar triangles are to each other as the squares of any two homologous lines.

55. The areas of two similar polygons are to each other as the squares of their homologous lines.

By Art. **33**, two similar polygons can be divided into the same number of similar triangles. The sums of these triangles will, by Art. **35**, be to each other as any triangle of one polygon is to the corresponding triangle of the other. But these triangles are to each other as the squares of any two homologous lines. Hence, the sum of the triangles, or the polygons, are to each other as the squares of any two homologous lines.

EXAMPLE 1.—Divide a given triangle by a line parallel to the base into parts such that the given triangle shall be to the triangle cut off as $m : n$.

SOLUTION.—Let ABC, Fig. 36, be the given triangle, and ADE be the triangle cut off so that $ABC : ADE = m : n$. By Art. **18**, ADE and ABC are similar; hence, by Art. **53**,

$$ABC : ADE = \overline{AB}^2 : \overline{AD}^2$$

But by the conditions of the problem,

$$ABC : ADE = m : n$$

Therefore, $$\overline{AB}^2 : \overline{AD}^2 = m : n;$$

whence, $$AD = AB\sqrt{\frac{n}{m}}.$$ Ans.

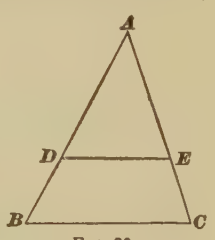

FIG. 36

When the triangle ABC is to be divided into two equal parts,

$$AD = AB\sqrt{\frac{1}{2}} = .70711\, AB$$

EXAMPLE 2.—Let the length AB of example 1 be 32 chains and the area of ABC be 25.6 acres; what is the length of AD, if it is desired to make the triangle ADE contain 15 acres?

SOLUTION.—The area ABC is to be to the area of ADE as 25.6 : 15; hence, $m : n = 25.6 : 15$.

Then, $$AD = 32\sqrt{\frac{15}{25.6}} = 24.495 \text{ ch.} \quad \text{Ans.}$$

EXAMPLE 3.—Divide a given triangle ABC, by lines parallel to the base, into n equal parts.

SOLUTION.—Let ABC, Fig. 37, be the triangle, and DE, FG, HI, etc., divide it into n equal parts. Then ADE is one part, AFG is two parts, and so on. Hence,

$$ABC : ADE = n : 1$$
$$ABC : AFG = n : 2; \text{ etc.}$$

Then, by example 1,

$$AD = AB\sqrt{\frac{1}{n}};\ AF = AB\sqrt{\frac{2}{n}};$$

$$AH = AB\sqrt{\frac{3}{n}};\ \text{etc.} \quad \text{Ans.}$$

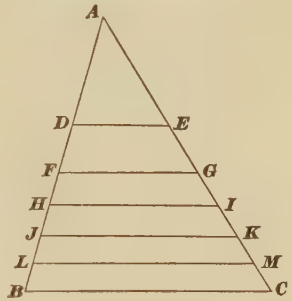

FIG. 37

EXAMPLE 4.—Two triangles ABC and $A'B'C'$ are similar. The sides of the triangle ABC are: $AB = 10$ inches, $BC = 21$ inches, $AC = 17$ inches, and in the triangle $A'B'C'$ the side $B'C' = 42$ inches; what is the area of the triangle $A'B'C'$?

SOLUTION.—In the triangle ABC, $s = \frac{10 + 17 + 21}{2} = 24$. Then $s - a = 3$, $s - b = 7$, $s - c = 14$, and the area is $\sqrt{24 \times 3 \times 7 \times 14} = 84$ sq. in. By the principle of Art. **53**,

$$\text{area of } A'B'C' : \text{area of } ABC = \overline{B'C'}^2 : \overline{BC}^2;$$

that is, $$\text{area of } A'B'C' : 84 = 42^2 : 21^2$$

But $$42^2 : 21^2 = 4 : 1$$

hence, $$\text{area of } A'B'C' : 84 = 4 : 1$$

whence, $$\text{area of } A'B'C' = 4 \times 84 = 336 \text{ sq. in.} \quad \textbf{Ans.}$$

EXAMPLES FOR PRACTICE

1. Suppose that the sides of the triangle $A'B'C'$ in example 4 of Art. **55** are $A'B' = 20$ inches, $B'C' = 42$ inches, and $C'A' = 34$ inches; show that the answer that is given to the example is correct.

2. The triangles ABC and $A'B'C'$ are similar; being given $BC = 13$ inches, $CA = 14$ inches, $AB = 15$ inches, and $B'C' = 19.5$ inches; find the area of the triangle $A'B'C'$. Ans. 189 sq. in.

3. Let AB, one side of a triangle ABC, be 60 chains long, and let it be required to divide, by lines parallel to BC, the triangle ABC into five equal parts. (a) What are the lengths of the lines AD, AF, AH, and AT? (b) Let the area of ABC be 120 acres; by means of Art. **53**, prove your results.

Ans. $\begin{cases} AD = 26 \text{ ch. } 83.3 \text{ l.} \\ AF = 37 \text{ ch. } 94.7 \text{ l.} \\ AH = 46 \text{ ch. } 47.6 \text{ l.} \\ AT = 53 \text{ ch. } 66.6 \text{ l.} \end{cases}$

4. Find the lengths of AD and AF when the triangle of example 3 is divided into three parts, whose areas shall be proportional to the numbers 3, 4, and 5.

Ans. $\begin{cases} AD = 30 \text{ ch.} \\ AF = 45 \text{ ch. } 82.6 \text{ l.} \end{cases}$

HINT.—This is the same as if the triangle were divided into $3 + 4 + 5$ equal parts and ADE contained three, and AFG, seven of these equal parts.

---

56. The Theorem of Pythagoras.—In any right triangle, the square described on the hypotenuse is equivalent to the sum of the squares described on the other two sides.

Let ABC, Fig. 38, be a right triangle. Draw an equal triangle in the position $CB'C'$, so that CB' will be in the prolongation of BC. Construct the squares $ABDE$ and $B'C'FD$ on AB and $B'C'$, respectively. Since $M + N_1$ ($= M + N$) is a right angle, ACC' is also a right angle. Produce EF to A', making $FA' = BA = DE$.

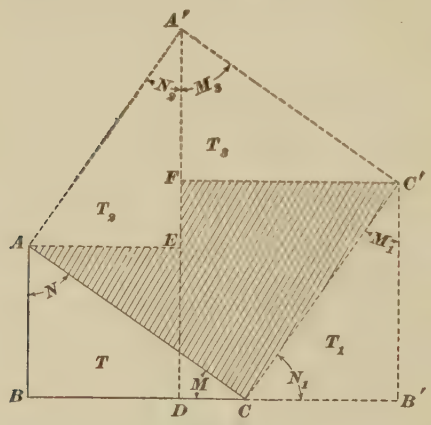

FIG. 38

Then, since EF is the difference between DF and DE, or BC and AB, $EA' = BC$. Draw AA' and $C'A'$. Each of the right triangles T_2 and T_3 is equal to T, since their legs are respectively equal. The quadrilateral $ACC'A'$, having all its sides equal and a right angle C, is a square—the square on the hypotenuse AC. This square is equal to the shaded figure plus the sum of the triangles T_2 and T_3; or to the shaded figure plus twice the triangle T. The sum of the squares $ABDE$ and $B'C'FD$ is equal to the shaded figure plus the sum of the triangles T and T_1, or to the shaded figure plus twice the triangle T. Therefore, square $ACC'A'$ = square $ABDE$ + square $B'C'FD$.

A particular case of the proposition just proved is shown in Fig. 39.

Let c be the hypotenuse, and a and b the other two sides of any right triangle.

Then,

$$c^2 = a^2 + b^2 \qquad (1)$$

$$c = \sqrt{a^2 + b^2} \qquad (2)$$

$$a = \sqrt{c^2 - b^2} \qquad (3)$$

Formula **3** may be written

$$a = \sqrt{(c - b)(c + b)} \qquad (4)$$

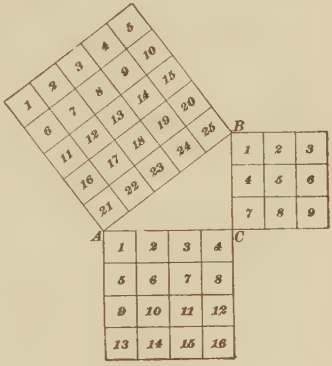

FIG. 39

EXAMPLE 1.—If $AB = 3$ inches and $BC = 4$ inches, what is the length of the hypotenuse AC, Fig. 38?

SOLUTION.—

$$AC = \sqrt{\overline{AB}^2 + \overline{BC}^2}$$
$$= \sqrt{3^2 + 4^2} = \sqrt{25} = 5 \text{ in.} \quad \text{Ans.}$$

EXAMPLE 2.—The side given is 3 inches ($= b$, say), the hypotenuse is 5 inches ($= c$); what is the length of the other side?

SOLUTION.—Applying formula **4**, Art. **56**,

$$a = \sqrt{(5 - 3)(5 + 3)} = \sqrt{16} = 4 \text{ in.} \quad \text{Ans.}$$

Also, $$a = \sqrt{c^2 - b^2} = \sqrt{5^2 - 3^2} = 4 \text{ in.} \quad \text{Ans.}$$

EXAMPLE 3.—If, from a church steeple that is 150 feet high, a rope is to be attached at the top and to a stake in the ground 85 feet from its foot (the ground being supposed to be level), what must be the length of the rope?

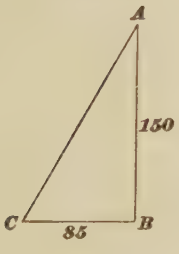

FIG. 40

SOLUTION.—In Fig. 40, AB represents the steeple 150 ft. high; C, a stake 85 ft. from the foot of the steeple; and AC, the rope. Here we have a triangle right-angled at B, of which AC is the hypotenuse. The square of $AC = 85^2 + 150^2 = 7{,}225 + 22{,}500 = 29{,}725$. Therefore,

$$AC = \sqrt{29{,}725} = 172.4 \text{ ft., nearly.} \quad \text{Ans.}$$

EXAMPLE 4.—Referring to Fig. 16, it is required to find the length of the post AB and that of the member BC.

SOLUTION.—Draw BK parallel to ED. Then, $BK = ED = 16$ ft. and $CK = CD - DK = CD - EB = 15 - 12 = 3$ ft. The right triangles AEB and BCK give

$$AB = \sqrt{AE^2 + EB^2} = \sqrt{16^2 + 12^2} = \sqrt{400} = 20 \text{ ft.} \quad \text{Ans.}$$

$$BC = \sqrt{BK^2 + CK^2} = \sqrt{16^2 + 3^2} = \sqrt{265} = 16.279 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. If the two sides about the right angle in a right triangle are 52 and 39 feet long, how long is the hypotenuse? Ans. 65 ft.

2. A ladder 65 feet long reaches to the top of a house when its foot is 25 feet from the house; how high is the house, supposing the ground to be level? Ans. 60 ft.

3. The shortest distance from a point to a line is 25 inches; the distances from this point to the extremities of the line are 54 inches and 40 inches, respectively; what is the length of the line? Ans. 79.08 in.

4. Show that the diagonal of a square is equal to the side multiplied by $\sqrt{2}$.

REGULAR POLYGONS

57. A **regular** polygon is a polygon that has equal sides and equal angles, that is, it is equilateral and equiangular.

58. A circle can be circumscribed about any regular polygon.

Take any three vertexes of the regular polygon $ABCDE$, Fig. 41, as the vertexes A, B, C, and pass a circle through them. Let O be the center of this circle. Join O to A, B, C, D, and E. The polygon being equiangular, the angle ABC = angle BCD. The angles OCB and OBC, being opposite equal sides OC and OB of the triangle OBC, are equal. Hence,

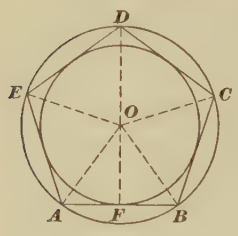

FIG. 41

$$ABC - OBC = BCD - OCB$$

or,

$$ABO = OCD$$

The polygon being equilateral, the sides AB and CD are equal. Hence, the triangles AOB and OCD, having two sides and included angle of one equal to two sides and included angle of the other equal, are equal. Therefore, $OD = OA$, and a circle passing through A,

B, and C must pass through D. In like manner, it can be shown that the circle passes through E.

59. A circle can be inscribed in any regular polygon.

In Fig. 41, OA, OB, OC, OD, and OE, being radii of the circumscribed circle, are equal and divide the polygon into equal isosceles triangles that have a common vertex O. The altitudes of these equal triangles are equal, hence the perpendicular distances, as OF, from O to each of the sides are the same. Therefore, a circle drawn with O as center and a radius equal to OF will be inscribed in the regular polygon.

60. The **center** of a regular polygon is the common center of the circumscribed and the inscribed circle.

61. The **radius** of a regular polygon is the radius of the circumscribed circle, as OA, Fig. 41.

62. The **apothem** of a regular polygon is the radius of the inscribed circle, as OF, Fig. 41.

63. The **angle at the center** of a regular polygon is the angle included by the radii drawn to the extremities of any side.

64. The angle at the center of any regular polygon is equal to four right angles, or 360°, divided by the number of the sides.

65. If n is the number of sides of a regular polygon, the sum of its interior angles is $2(n-2)$ right angles (see *Geometry*, Part 1), or, $90° \times 2(n-2) = 180° \times (n-2)$, and, since all the angles are equal, each angle is equal to $\frac{180° \times (n-2)}{n} = 180° - \frac{360°}{n}$. Since this value depends only on the number of sides, all regular polygons of the same number of sides have the same angles.

66. Regular polygons of the same number of sides are similar; their perimeters are to each other as any two homologous lines, and their areas are to each other as the squares of any two homologous lines.

67. The area of a regular polygon is equal to one-half the product of the perimeter and the apothem.

Let l be the side MN of a regular polygon, Fig. 42, n the number of sides, $p(=nl)$ the perimeter, $a(=OF)$ the apothem, and A the area. As A is equal to the sum of n triangles, each equal to MON, we have,

$$A = (\tfrac{1}{2} MN \times OF) \times n = \tfrac{1}{2} l a \times n = \tfrac{1}{2} nl \times a,$$

or,
$$A = \tfrac{1}{2} pa$$

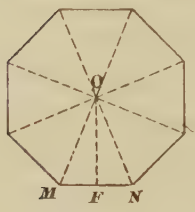

FIG. 42

EXAMPLE.—Find the area of a regular pentagon whose side is 25 feet and apothem is 17.2 feet.

SOLUTION.—The figure is a pentagon, hence it has five sides. The perimeter is 5×25 and the area is $\dfrac{5 \times 25 \times 17.2}{2}$ = 1,075 sq. ft. Ans.

68. The areas of regular polygons each of whose sides is equal to 1 are given in the following table:

TABLE I

AREAS OF REGULAR POLYGONS

Name	Number of Sides	Area When Side = 1	Name	Number of Sides	Area When Side = 1
Triangle .	3	.4330	Octagon .	8	4.8284
Square . .	4	1.0000	Nonagon .	9	6.1818
Pentagon .	5	1.7205	Decagon .	10	7.6942
Hexagon .	6	2.5981	Undecagon	11	9.3656
Heptagon .	7	3.6339	Dodecagon	12	11.1960

From the principle of Art. **55**, the following rule is derived:

Rule.—*To find the area of any regular polygon, square the length of a side and multiply by the area of the similar polygon whose side is equal to the unit of length.*

Let A = area; l = length of side of required polygon; a = area of similar polygon whose side is 1; then, by Art. **55**,

$$A : a = l^2 : 1^2$$

whence,
$$A = a l^2$$

EXAMPLE.—The side of a regular octagon is 3 inches, find its area.

SOLUTION.—From the table, the area of a regular octagon whose side is 1 in. is 4.8284 sq. in. Hence, the area of the octagon whose side is 3 in. is $4.8284 \times 3^2 = 43.456$ sq. in. Ans.

69. If the vertexes of a regular inscribed polygon are joined to the middle points of the arcs subtended by the sides of the polygon, the joining lines form a regular inscribed polygon of double the number of sides. Thus, the octagon $AFBG$, etc., Fig. 43, is formed by joining the middle points of the arcs subtended by the sides of the square $ABCD$.

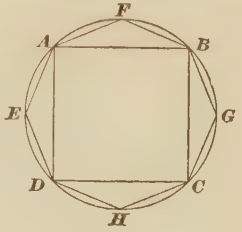

Fig. 43

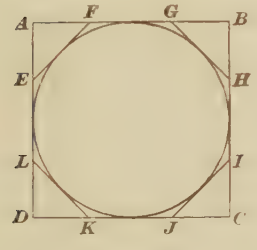

Fig. 44

70. If tangents are drawn at the middle points of the arcs between adjacent points of contact of the sides of a regular circumscribed polygon, a regular circumscribed polygon of double the number of sides is formed. Thus, in Fig. 44, the octagon $EFGH$, etc., is formed by drawing tangents at the middle points of the arcs between adjacent points of contact of the sides of the circumscribed square $ABCD$.

CIRCULAR MEASUREMENTS

THE CIRCLE

LENGTH OF ANY ARC

71. If any two circles are taken, and two regular polygons of the same number of sides are inscribed in them, the perimeters of these polygons are to each other as the radii of the circles (Art. **66**). This relation holds whatever the number of sides of the polygon. Now, it is evident that, as this number increases, the perimeters of the two polygons approach the circumferences of their respective circles. We may, therefore, consider these circumferences as extreme cases of the perimeters of regular polygons, in which the number of sides is increased indefinitely; whence we conclude that the circumferences, also, are to each other as their radii.

If c and c' are the circumferences of any two circles, and r and r' their respective radii, we may write,

$$c : c' = r : r'$$

whence,
$$c : r = c' : r'$$

or,
$$\frac{c}{r} = \frac{c'}{r'}$$

Dividing both numbers by 2, and denoting the diameters by d and d',

$$\frac{c}{2r} = \frac{c'}{2r'}$$

that is,
$$\frac{c}{d} = \frac{c'}{d'}$$

As c and c' are any two circumferences, it is seen that the ratio obtained by dividing any circumference by its diameter is the same for all circumferences. This ratio is usually

denoted by the Greek letter π (pronounced *pi*). We have, therefore, for any circle,

$$\frac{c}{d} = \pi$$

whence, $$c = \pi d = 2\pi r$$

72. The quantity π can be determined by elementary geometrical methods, which may be found in treatises on geometry; but these methods are very laborious. A much better method is afforded by the theory of series, which is treated in works on trigonometry and the differential calculus. It is found that π cannot be expressed as an exact fraction, either decimal or vulgar. Its value can, however, be calculated to any desired degree of approximation. The following value is approximate to fifteen decimal places:

$$\pi = 3.141592653589793+$$

For nearly all practical purposes, 3.1416 is a sufficiently close value. This value is used very generally, and will be used in this Course, unless otherwise stated. The student should commit it to memory. A value that is often used in rough calculations is $\frac{22}{7}$; it can be used when no more than three significant figures are required in the result.

73. The length of an arc, when the number of degrees in the arc and the radius of the circle are given, may be found as follows:

The length of the arc is evidently the same part of the length of the circumference ($2\pi r$) as the number of degrees in the arc is of the number of degrees in the whole circumference, or 360°. Thus, if n is the number of degrees in the arc, and l is its length, we shall have,

$$\frac{2\pi r}{l} = \frac{360}{n}$$

whence, $$l = \frac{\pi r n}{180}$$

In applying this formula, minutes and seconds should be expressed as fractions of a degree.

EXAMPLE 1.—Find the length of a rope that will go around a wheel or drum 7.5 feet in diameter.

SOLUTION.—The required length is equal to the length c of the circumference of the wheel or drum. Here $d = 7.5$ ft., and, taking $\pi = 3.1416$, we have, by formula of Art. **71**,

$$c = 3.1416 \times 7.5 = 23.562 \text{ ft.} \quad \text{Ans.}$$

Using $\frac{22}{7}$ for π, the result, to three significant figures, is

$$c = \frac{22}{7} \times 7.5 = 23.6 \text{ ft.} \quad \text{Ans.}$$

EXAMPLE 2.—Find the diameter of a circular race track 1 mile in length.

SOLUTION.—Here c is given (= 1 mi. = 5,280 ft.) and the quantity required is d. From the formula $c = \pi d$, we get

$$d = \frac{c}{\pi} = \frac{5{,}280}{3.1416} = 1{,}680.7 \text{ ft.} \quad \text{Ans.}$$

EXAMPLE 3.—What is the length of a railroad circular curve having a radius of 1,540 feet and subtending an angle at the center equal to 26° 35′?

SOLUTION.—To apply formula of Art. **73**, we have $r = 1{,}540$ ft., $n = 26\frac{35}{60}° = 26.583°$, nearly. Therefore,

$$l = \frac{3.1416 \times 1{,}540 \times 26.583}{180} = 714.50 \text{ ft.} \quad \text{Ans.}$$

74. When only the chord AB, Fig. 45, of an arc and the height, or "rise," CD of the segment are known, the following approximate method gives good results. AC, the chord of half the arc, has the value

$$AC = \sqrt{\overline{AD}^2 + \overline{CD}^2} = \sqrt{\left(\frac{AB}{2}\right)^2 + \overline{CD}^2}$$

Then, to find the length of the arc:

Rule.—*From eight times the chord of half the arc, subtract the chord of the whole arc and divide the remainder by 3.*

That is,

$$\text{arc } ACB = \frac{8 \times AC - AB}{3}$$

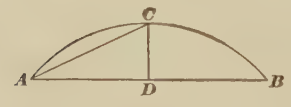

FIG. 45

Let c = chord of whole arc;
h = height of segment;
l = length of arc.

Then,
$$AC = \sqrt{\frac{c^2}{4} + h^2} = \frac{1}{2}\sqrt{c^2 + 4h^2}$$

and
$$l = \frac{4\sqrt{c^2 + 4h^2} - c}{3}$$

This formula gives the length of an arc less than one-sixth of the circumference correct to four figures, and it gives the length of an arc less than one-third of the circumference correct to three figures.

EXAMPLE.—Find the length of the arc ACB, Fig. 46.

SOLUTION.—In this example, $c = 72$, $h = 8$. Therefore,

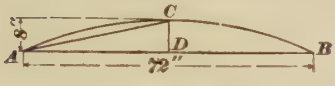

FIG. 46

$$l = \frac{4\sqrt{72^2 + 4 \times 8^2} - 72}{3} = 74.34 \text{ in.}$$

Ans.

75. For very flat arcs, that is, when $\frac{h}{c}$ is very small (say not greater than .1), the following approximate formula may be used, the notation being the same as in the preceding article:

$$l = c + \frac{8\,h^2}{3\,c}$$

EXAMPLE 1.—Find the length of the arc AB, Fig. 46.

SOLUTION.—

$$l = 72 + \frac{8 \times 8^2}{3 \times 72} = 72 + 2.37 = 74.37. \quad \text{Ans.}$$

This is not a very close approximation, because the ratio $\frac{h}{c}\left(= \frac{8}{72} = \frac{1}{9}\right)$ is not very small; however, the approximate value thus found would be close enough for most practical purposes.

EXAMPLE 2.—The chord of a railroad curve is 675 feet long, and the rise (or, "middle ordinate," as the rise is called in railroad work) is 40 feet; what is the length of the curve?

SOLUTION.—Here $c = 675$, $h = 40$, and therefore

$$l = 675 + \frac{8 \times 40^2}{3 \times 675} = 675 + 6.32 = 681.32 \text{ ft.} \quad \text{Ans.}$$

76. Circular Measure of an Angle.—The following equation follows from the formula of Art. **73:**

$$\frac{l}{r} = \frac{\pi n}{180} = \frac{\pi}{180} \times n$$

If we assume the radius to be 1, then

$$l = \frac{\pi}{180} \times n \qquad (1)$$

This equation gives the length of the arc that the angle subtends on a circle whose radius is equal to unity. The length of such arc is called the **circular measure** of the angle, and the angle is often referred to by stating that measure. Thus, an angle of 1.34, circular measure, means an angle that subtends an arc of length 1.34 on a circle whose radius is 1. An angle expressed in circular measure is also said to be expressed **in radians.**

If in equation 1 we make $n = 180°$, we obtain, for the circular measure of 180°, $l = \pi$, that is, 180° is equivalent to π radians. Likewise, 90° is equivalent to $\frac{\pi}{2}$ radians, etc.

EXAMPLES FOR PRACTICE

1. Find the distance around the outside of a waterwheel whose outside diameter is 22 feet 8 inches. Ans. 71.21 ft.

2. The wheel of a carriage is observed to turn 375 times in going from a certain place to another; the diameter of the wheel is 3.5 feet; what is the distance between the two places? Ans. 4,123.4 ft.

3. A circular column measures 45.5 inches around the outside; what is its diameter? Ans. 14.483 in.

4. A belt covers an arc of 50° on a pulley whose diameter is 5 feet; what length of the belt is in contact with the pulley? Ans. 2.1817 ft.

5. How long will it take a train to move over a curve subtending an angle of 100°, the radius of the curve being 1,800 feet, and the train going at the rate of 20 miles an hour? Ans. 1.79 min.

6. The length of arc of a circle is equal to the radius; find the number of degrees in the arc. Ans. 57.3° = 57° 18′, nearly

7. The chord of a railroad curve is 600 feet long and the middle ordinate is 80 feet; what is the length of the curve? Ans. 628 ft.

AREAS BOUNDED BY CIRCULAR ARCS

77. The area of a circle is equal to one-half the product of its circumference and radius (Art. **67**). This at once follows by considering the circle as an extreme case of a regular polygon.

Let A = area of circle;
c = circumference of circle;
r = radius of circle.

Then, $$A = \tfrac{1}{2} c r$$

or, since $c = 2 \pi r$,

$$A = \tfrac{1}{2} 2 \pi r \times r$$

or, simplifying,

$$A = \pi r^2 = 3.1416 r^2 \qquad (1)$$

Writing $\frac{d}{2}$ for r, we obtain for the area in terms of the diameter,

$$A = \frac{\pi d^2}{4} = .7854 d^2 \qquad (2)$$

These formulas serve likewise to find r or d when A is given. Since $2 \pi r = c$, we have

$$r = \frac{c}{2 \pi}, \text{ and } \pi r^2 = \pi \left(\frac{c}{2 \pi}\right)^2 = \frac{c^2}{4 \pi}$$

that is, $$A = \frac{c^2}{4 \pi} \qquad (3)$$

This formula gives the area of a circle when its circumference is known.

EXAMPLE 1.—The steam pressure on a piston is 75 pounds per square inch, and the diameter of the piston is 15 inches; what is the pressure on the whole surface of the piston?

SOLUTION.—The required pressure is evidently seventy-five times the number of square inches in the surface of the piston, or seventy-five times the area A of the piston. Here $d = 15$ in., and formula **2** gives

$$A = .7854 \times 15^2$$

whence the total pressure is

$$75 \times .7854 \times 15^2 = 13{,}254 \text{ lb.} \quad \text{Ans.}$$

EXAMPLE 2.—The distance around a circular park is 2.75 miles; what is the area of the park, in acres?

SOLUTION.—Here c is given equal to 2.75 mi. $= (2.75 \times 80)$ ch. Therefore, the area of the park, in square chains, is (formula **3**)

$$\frac{(2.75 \times 80)^2}{4 \times 3.1416}$$

The area, in acres, is one-tenth of this, or

$$\frac{1}{10} \times \frac{(2.75 \times 80)^2}{4 \times 3.1416} = \frac{220^2}{125.664} = 385.15 \text{ A.} \quad \text{Ans.}$$

EXAMPLE 3.—What must be the diameter of a circular sewer pipe that its cross-section may be 12.75 square feet?

SOLUTION.—Solving formula **2** for d,

$$d = \sqrt{\frac{A}{.7854}} = \sqrt{\frac{12.75}{.7854}} = 4.03 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. The cable of a suspension bridge measures 40 inches around its circumference; find: (*a*) the diameter d of the cable; (*b*) the area A of the cross-section.

Ans. $\begin{cases} (a)\ d = 12.732 \text{ in.} \\ (b)\ A = 127.32 \text{ sq. in.} \end{cases}$

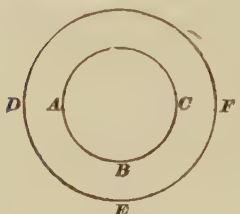

FIG. 47

2. Find a formula for the area A of the space enclosed between two circles $A\,B\,C$ and $D\,E\,F$, Fig. 47, the diameter of the outer circle being D, and that of the inner circle d.

Ans. $\begin{cases} A = \frac{\pi}{4}(D^2 - d^2) \\ A = \frac{\pi}{4}(D + d)(D - d) \end{cases}$

3. What must be the inner diameter of a circular chimney, that its inner cross-section may be 14 square feet? Ans. 4.2221 ft.

4. The diameter of a circular airway of a mine is 10 feet; find: (*a*) the circumference c; (*b*) the area A of the cross-section.

Ans. $\begin{cases} (a)\ c = 31.416 \text{ ft.} \\ (b)\ A = 78.54 \text{ sq. ft.} \end{cases}$

---

78. A sector is the same part of a circle as its arc is of the circumference.

Let A = area of circle;
A' = area of sector;
n = number of degrees in arc of sector.

Then, $$A' : A = n : 360$$

whence, $$A' = \frac{n\,A}{360} = \frac{\pi\,r^2\,n}{360}$$

EXAMPLE.—The angle of a sector of a circle is 75°; the diameter of the circle is 12 inches; what is the area of the sector?

SOLUTION.—The area A of the circle is $12^2 \times .7854$ sq. in. Then the area of the sector is

$$\frac{n\,A}{360} = \frac{75 \times 12^2 \times .7854}{360} = 23.562 \text{ sq. in.} \quad \text{Ans.}$$

79. The area of a sector is equal to one-half the product of its base by the radius of the circle.

$$A' = \tfrac{1}{2}\,r\,l$$

If l is the length of the arc, or **base**, of a sector, we have (Art. **73**),

$$l = \frac{\pi r n}{180}$$

whence,

$$n = \frac{180\, l}{\pi r}$$

This value of n substituted in formula of Art. **78** gives

$$A' = \frac{\pi r^2}{360} \times \frac{180\, l}{\pi r}$$

or, reducing,

$$A' = \tfrac{1}{2} r l$$

EXAMPLE.—If the radius of an arc is 5 feet and the length of the arc is 4 feet, what is the area of the sector?

SOLUTION.—By formula of Art. **79**,

$$A' = \frac{l r}{2} = \frac{4 \times 5}{2} = 10 \text{ sq. ft.} \quad \text{Ans.}$$

80. The area of a segment, as ADB, Fig. 48, is evidently equal to the area of the sector $AOBD$ minus the area of the triangle AOB.

EXAMPLE 1.—The diameter of a circle is 10 inches, and the chord of the arc of a segment is 7 inches; what is the area of the segment?

SOLUTION.—In Fig. 48, let $AB = 7$ in. and the diameter $= 10$ in. Then, $OB = 5$ in., and $CB = 3.5$ in. Hence, $OC = \sqrt{5^2 - 3.5^2} = 3.57$ in., and $CD = 5 - 3.57 = 1.43$ in. Then, by formula of Art. **74**,

arc $ADB = \dfrac{4\sqrt{7^2 + 4 \times 1.43^2} - 7}{3} = 7.75$ in.

Hence, area of sector $AOBD = \frac{1}{2} \times 5 \times 7.75 = 19.38$ sq. in. The area of the triangle $AOB = \frac{1}{2} \times 3.57 \times 7 = 12.50$ sq. in. Therefore, the area of the segment is $19.38 - 12.50 = 6.88$ sq. in. Ans.

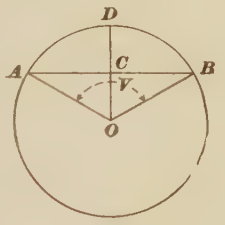

FIG. 48

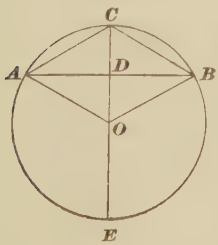

FIG. 49

EXAMPLE 2.—The chord of the arc of a segment is 79 inches and the height of the segment is 20 inches; find the area of the segment.

SOLUTION.—Let $ACBE$, Fig. 49, be the circle; let $AB = 79$ in. and $CD = 20$ in. Then, $AD = \frac{1}{2} \times 79$ in. $= 39.5$ in. By Art. **28**,

$$CD : AD = AD : DE$$

or,

$$20 : 39.5 = 39.5 : DE$$

whence,

$$DE = 78.01$$

Hence, the diameter = 20 + 78.01 = 98.01 in., and the radius = 49. Then the arc $ACB = \frac{4\sqrt{79^2 + 4 \times 20^2} - 79}{3} = 91.7$ in. Hence, the area of sector $AOBC = 91.7 \times \frac{1}{2} \times 49 = 2{,}246.65$ sq. in. The area of the triangle $AOB = \frac{1}{2} \times 79 \times 29 = 1{,}145.5$ sq. in. Therefore, the area of the segment = 2,246.65 − 1,145.50 = 1,101.15 sq. in. Ans.

THE ELLIPSE

81. An **ellipse** is a plane figure bounded by a curved line such that the sum of the distances of any point on that line from two fixed points within is always equal to the length of the line passing through the fixed points and terminating at both ends in the curved line.

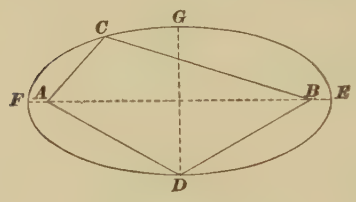

FIG. 50

In Fig. 50, the fixed points are A and B, and if C and D are any two points on the curve, $AC + CB = AD + DB = FE$. The two fixed points are the **foci.** The line FE through the foci is the **transverse,** or **major, axis.** The line GD, which is the perpendicular bisector of FE, is the **conjugate,** or **minor, axis.** The foci may be located from G or D as a center by striking arcs with a radius equal to one-half FE.

82. There is no simple and exact method of finding the periphery (perimeter) of an ellipse. The following formula gives values very nearly exact:

Let C = periphery;
a = half the major axis;
b = half the minor axis;

$$D = \frac{a - b}{a + b}.$$

Then,
$$C = \pi(a + b)\frac{64 - 3D^4}{64 - 16D^2}$$

EXAMPLE.—What is the periphery of an ellipse whose axes are 10 inches and 4 inches long?

SOLUTION.—In this example, $a = 5$, $b = 2$, $D = \frac{5-2}{5+2} = \frac{3}{7}$.

Then, $$C = 3.1416(5+2)\frac{64 - 3(\frac{3}{7})^4}{64 - 16(\frac{3}{7})^2} = 23.013$$

Therefore, the periphery is 23.013 in. Ans.

83. The area of an ellipse is equal to the product of its two semiaxes multiplied by π.

Let a = half the major axis;
b = half the minor axis;
A = area.

Then, $$A = \pi a b = 3.1416\, a b$$

EXAMPLE.—What is the area of an ellipse whose axes are 10 inches and 6 inches?

SOLUTION.—Here, $a = \frac{1}{2} \times 10 = 5$, $b = \frac{1}{2} \times 6 = 3$.

Then, $$A = 3.1416 \times 5 \times 3 = 47.124$$

Therefore, the area is 47.12 sq. in. Ans.

EXAMPLES FOR PRACTICE

1. The number of degrees in the angle formed by drawing radii from the center of a circle to the extremities of an arc of the circle is 84; the diameter of the circle is 17 inches; what is the area of the sector? Ans. 52.96 sq. in.

2. Given the chord of the arc of a segment equal to 24 inches, and the height of the segment equal to 6.5 inches, find: (*a*) the diameter of the circle; (*b*) the area of the segment. Ans. { (*a*) 28.7 in. (*b*) 109.5 sq. in.

3. (*a*) What is the perimeter of an ellipse whose axes are 15 inches and 9 inches? (*b*) What is the area? Ans. { (*a*) 38.29 in. (*b*) 106.03 sq. in.

4. The base of a sector is 24 inches and the diameter of the circle is 54 inches; what is the area of the sector? Ans. 324 sq. in.

THE MENSURATION OF SOLIDS

84. A **solid**, or body, has three dimensions: length, breadth, and thickness.

85. The **entire area** of a solid is the area of the whole outside of the solid.

The **convex area** of a solid having one or two flat ends is the same as the entire surface, except that the areas of the ends or bases are not included.

86. The **volume** of a solid is expressed by the number of times that it will contain another volume, called the unit of volume. Instead of the word *volume*, the expression **cubical contents** is frequently used.

THE PRISM AND CYLINDER

87. A **prism** is a solid whose ends are equal polygons in parallel planes, and whose sides are parallelograms.

FIG. 51

88. A **parallelopipedon**, Fig. 51, is a prism whose bases (ends) are parallelograms.

89. A **cube**, Fig. 52, is a parallelopipedon whose faces and ends are squares.

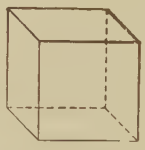

FIG. 52

90. The cube whose edges are equal to the unit of length is taken as the unit of volume when finding the volume of a solid.

Thus, if the unit of length is 1 inch, the unit of volume will be the cube each of whose edges measures 1 inch, or 1 cubic inch; and the number of cubic inches the solid contains will be its volume. If the unit of length is 1 foot, the unit of volume will be 1 cubic foot, etc. Cubic inch, cubic foot, and cubic yard are abbreviated to cu. in., cu. ft., and cu. yd., respectively.

91. Prisms take their names from their bases. Thus, a *triangular prism* is one whose bases are triangles; a *pentagonal prism* is one whose bases are pentagons, etc.

92. A **cylinder,** Fig. 53, is a round body of uniform diameter with circles for its ends.

FIG. 53

93. A **right prism,** or **right cylinder,** is one whose center line (axis) is perpendicular to its bases.

94. The **altitude** of a prism or cylinder is the perpendicular distance between its two ends.

95. To find the convex area of any right prism, or right cylinder:

Rule.—*Multiply the perimeter of the base by the altitude.*

Let p = perimeter of base;
h = altitude;
c = convex area.

Then, $$c = p\,h$$

EXAMPLE 1.—What is the convex area of a right prism whose base is a square, one side of which is 9 inches, and whose altitude is 16 inches?

SOLUTION.— $9 \times 4 = 36$ in., the perimeter of the base. Applying formula of Art. **95,**

$c = 36 \times 16 = 576$ sq. in., the convex area. Ans.

To find the entire area, add the areas of the two ends to the convex area.

EXAMPLE 2.—What is the entire area of the parallelopipedon mentioned in the last question?

SOLUTION.—The area of one end is $9^2 = 81$ sq. in. $81 \times 2 = 162$ sq. in., is the area of both ends. $576 + 162 = 738$ sq. in., the entire area of the parallelopipedon. Ans.

EXAMPLE 3.—What is the entire area of a right cylinder whose base is 16 inches in diameter, and whose altitude is 24 inches?

SOLUTION.— $16 \times 3.1416 = 50.27$ in., or the perimeter (circumference) of the base. $50.27 \times 24 = 1{,}206.48$ sq. in., the convex area.

$16^2 \times .7854 \times 2 = 402.12$ sq. in., the area of the ends.

$1{,}206.48 + 402.12 = 1{,}608.6$ sq. in., the entire area. Ans.

96. To find the volume of a prism, or cylinder:

Rule.—*The volume of any prism or cylinder is equal to the area of the base multiplied by the altitude.*

Let A = area of base;
h = altitude;
V = volume.

Then, $$V = A\,h$$

If the given prism is a cube, the three dimensions are all equal, and the volume equals the cube of one of the edges Hence, if the volume is given, the length of an edge is found by extracting the cube root.

If the volume and the area of the base are given, the altitude is $h = \frac{V}{A}$. If the cylinder or prism is hollow, the volume is equal to the area of the ring or base multiplied by the altitude.

EXAMPLE 1.—What is the volume of a rectangular prism whose base is 6 inches by 4 inches, and whose altitude is 12 inches?

SOLUTION.—The base of a rectangular prism is a rectangle; hence, $6 \times 4 = 24$ sq. in., the area of the base. Applying formula of Art. **96**,

$V = 24 \times 12 = 288$ cu. in., or the volume. Ans.

EXAMPLE 2.—What is the volume of a cube whose edge is 9 inches?

SOLUTION.— $9^3 = 9 \times 9 \times 9 = 729$ cu. in., the volume. Ans.

EXAMPLE 3.—What is the volume of a cylinder whose base is 7 inches in diameter, and whose altitude is 11 inches?

SOLUTION.— $7^2 \times .7854 = 38.48$ sq. in., the area of the base. Applying formula of Art. **96**,

$V = 38.48 \times 11 = 423.28$ cu. in., the volume. Ans.

THE PYRAMID AND CONE

97. A **pyramid,** Fig. 54, is a solid whose base is a polygon, and whose sides are triangles uniting at a common point, called the **vertex.** If the base is a regular polygon, and the sides have the same inclination to the base, the pyramid is a **regular pyramid.**

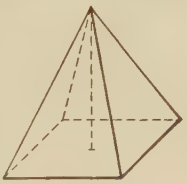

Fig. 54

98. A **cone,** Fig. 55, is a solid whose base is a circle, and whose convex surface tapers uniformly to a point called the **vertex.**

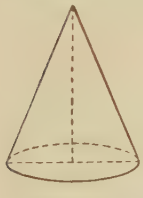

Fig. 55

99. The **altitude** of a pyramid or cone is the perpendicular distance from the vertex to the base.

100. The **slant height** of a regular pyramid is a line drawn from the vertex perpendicular to one of the sides of the base. The slant height of a cone is a straight line drawn from the vertex to the circumference of the base, and lying on the surface of the cone.

101. To find the convex area of a regular pyramid or a cone:

Rule.—*The convex area of a regular pyramid or of a cone is equal to the perimeter of the base multiplied by one-half the slant height.*

Let p = perimeter;
s = slant height;
c = convex area.

Then, $$c = \frac{p\,s}{2}$$

Example 1.—What is the convex area of a regular pentagonal pyramid, if each side of the base measures 6 inches and the slant height measures 14 inches?

Solution.—The base of the pentagonal pyramid is a pentagon, and consequently it has five sides. $6 \times 5 = 30$ in., or the perimeter of the base. Applying formula of Art. **101,**

$$c = \frac{p\,s}{2} = \frac{30 \times 14}{2} = 210 \text{ sq. in., the convex area. Ans.}$$

EXAMPLE 2.—What is the entire area of a cone whose altitude is 15 inches, and whose base is 16 inches in diameter?

SOLUTION.—The slant height of the cone is the hypotenuse of a right triangle whose legs are the radius of the base and altitude of the cone, respectively. Therefore, the slant height is equal to $\sqrt{15^2 + 8^2} = 17$ in. (Art. **56**). The perimeter of the base is $16 \times 3.1416 = 50.2656$ in. Applying formula of Art. **101**,

$$c = \frac{50.2656 \times 17}{2} = 427.26 \text{ sq. in.}$$

The area of the base is $16^2 \times .7854 = 201.06$ sq. in. The entire area is, therefore, $427.26 + 201.06 = 628.32$ sq. in. Ans.

102. To find the volume of any pyramid or cone:

Rule.—*The volume of any pyramid or cone equals the area of the base multiplied by one-third of the altitude.*

Let A = area of base;
h = altitude;
V = volume.

Then, $$V = \frac{A\,h}{3}$$

EXAMPLE 1.—What is the volume of a triangular pyramid, each edge of whose base measures 6 inches, and whose altitude is 8 inches?

SOLUTION.—The base is an equilateral triangle; hence, applying the rule of Art. **68**, the area is $6^2 \times .433 = 15.59$ sq. in. Applying formula of Art. **102**,

$$V = \frac{A\,h}{3} = \frac{15.59 \times 8}{3} = 41.57 \text{ cu. in. Ans.}$$

EXAMPLE 2.—What is the volume of a cone whose altitude is 18 inches, and whose base is 14 inches in diameter?

SOLUTION.— $14^2 \times .7854 = 153.94$ sq. in., the area of the base. Applying formula of Art. **102**,

$$V = \frac{A\,h}{3} = \frac{153.94 \times 18}{3} = 923.64 \text{ cu. in., the volume. Ans.}$$

103. It has been stated that the volume of a cone or a pyramid is equal to one-third the product of the area of the base multiplied by the altitude. Similarly, the volume of any solid whose base is a plane figure and which tapers to a

point like a cone or a pyramid is equal to one-third of the product of its base and altitude.

EXAMPLE.—Find the volume of an elliptical cone, whose base is an ellipse with diameters 8 inches and 6 inches, and the altitude is 7.5 inches.

SOLUTION.—The area of the ellipse at the base is $3.1416 \times 4 \times 3$ The volume is equal to one-third the product of the area of the base and altitude; that is,

$$V = \frac{1}{3} \times 3.1416 \times 4 \times 3 \times 7.5 = 94.248$$

Hence, the volume is 94.248 cu. in. Ans.

EXAMPLES FOR PRACTICE

1. Find the volume of a triangular pyramid of which the altitude is 4 inches and the base is an equilateral triangle having each side 3 inches long. Ans. 5.2 cu. in.

2. Find the weight of a steel bar 16 feet long and 2 inches in diameter, the weight of steel being taken as .28 pound per cubic inch. Ans. 168.89 lb.

3. What is the entire area of a hexagonal prism 12 inches long, each side of the base being 1 inch long? Ans. 77.196 sq. in.

4. (*a*) Find the convex area of a cone whose altitude is 12 inches, and the circumference of whose base is 31.416 inches. (*b*) Find the volume of the cone. Ans. { (*a*) 204.2 sq. in. (*b*) 314.16 cu. in.

THE FRUSTUM OF A PYRAMID OR A CONE

104. If a pyramid is cut by a plane parallel to the base, as in Fig. 56, so as to form two parts, the lower part is called a **frustum** of the pyramid.

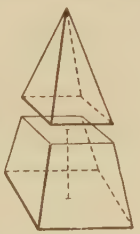

FIG. 56

105. If a cone is cut in a similar manner, as in Fig. 57, the lower part is called a **frustum of the cone.**

106. The upper end of a frustum of a pyramid or cone is called the **upper base**, and the lower end the **lower base.** The **altitude of a** frustum is the perpendicular distance between the bases.

107. To find the convex area of a frustum of a regular pyramid or of a cone:

FIG. 57

Rule.—*The convex area of a frustum of a regular pyramid or of a cone equals one-half the sum of the perimeters of its bases multiplied by the slant height of the frustum.*

Let p = perimeter of lower base;
p' = perimeter of upper base;
s = slant height;
c = convex area.

Then, $$c = \left(\frac{p + p'}{2}\right)s$$

EXAMPLE 1.—Given the frustum of a triangular pyramid in which each side of the lower base measures 10 inches, each side of the upper base measures 6 inches, and whose slant height is 9 inches; find the convex area.

SOLUTION.— 10 in. $\times$ 3 = 30 in., the perimeter of the lower base. 6 in. $\times$ 3 = 18 in., the perimeter of the upper base. Applying formula of Art. **107**,

$$c = \left(\frac{p + p'}{2}\right)s = \frac{30 + 18}{2} \times 9 = 216 \text{ sq. in., the convex area. Ans.}$$

EXAMPLE 2.—If the diameters of the two bases of a frustum of a cone are 12 inches and 8 inches, respectively, and the slant height is 12 inches, what is the entire area of the frustum?

SOLUTION.— $\dfrac{(12 \times 3.1416) + (8 \times 3.1416)}{2} \times 12 = 376.99$ sq. in., the convex area.

$8^2 \times .7854 = 50.27$ sq. in.
$12^2 \times .7854 = 113.1$ sq. in.

113.1 + 50.27 = 163.37 sq. in., the area of the two ends. 376.99 + 163.37 = 540.36 sq. in., the entire area of the frustum. Ans.

108. To find the volume of the frustum of a pyramid or a cone:

Rule.—*Add the areas of the upper base, the lower base, and the square root of the product of the areas of the two bases; multiply this sum by one-third of the altitude.*

Let A = area of lower base;
a = area of upper base;
h = altitude;
V = volume.

Then, $$V = (A + a + \sqrt{A\,a})\frac{h}{3}$$

EXAMPLE 1.—Given a frustum of a hexagonal pyramid in which each edge of the lower base measures 8 inches, and each edge of the upper base measures 5 inches, and whose altitude is 14 inches, what is its volume?

SOLUTION.—A hexagonal pyramid is one whose base is a regular hexagon, as shown in Fig. 58. Hence, applying formula of Art. **68**,

$$A = 8^2 \times 2.5981 = 166.28 \text{ sq. in.}$$

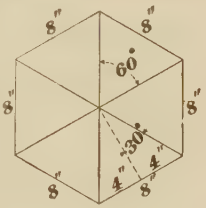

FIG. 58

In a similar way, the area of the upper base is found to be 64.95 sq. in. Then, applying formula of Art. **108**,

$$V = (166.28 + 64.95 + \sqrt{166.28 \times 64.95})\frac{14}{3}$$

$$= 335.15 \times \frac{14}{3} = 1{,}564.03 \text{ cu. in., the volume. Ans.}$$

EXAMPLE 2.—What is the volume of a frustum of a cone whose upper base is 8 inches in diameter, whose lower base is 12 inches in diameter, and whose altitude is 15 inches?

SOLUTION.—The area of the upper base is $8^2 \times .7854 = 50.27$ sq. in. The area of the lower base is $12^2 \times .7854 = 113.1$ sq. in., nearly. The square root of their product is $\sqrt{50.27 \times 113.1} = 75.4$.

Then, $$V = (50.27 + 113.1 + 75.4)\frac{15}{3}$$

$$= 238.77 \times \frac{15}{3} = 1{,}193.85 \text{ cu. in., the volume. Ans.}$$

THE WEDGE

109. A **wedge**, as here considered, is a solid whose base is a rectangle, two of whose opposite faces are parallel triangles, and two are parallelograms whose intersection is called the **edge** of the wedge. A wedge may therefore be defined as a triangular prism having one rectangular face, called the **base**. In Fig. 59, $ABCD$ is the base and EF the edge of the wedge.

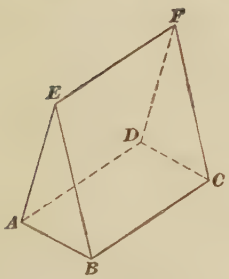

FIG. 59

110. The **altitude** of a wedge is the perpendicular distance between the base and the opposite edge.

111. To find the volume of a wedge:

Rule.—*The volume of any wedge is equal to the area of the base multiplied by one-half the altitude.*

Let A = area of base;
h = altitude;
V = volume.

Then, $$V = \frac{A\,h}{2}$$

Example.—What is the volume of a wedge whose base is a rectangle 6 feet long and 4 feet wide, and whose altitude is 10 feet?

Solution.—The area of the base is $4 \times 6 = 24$ sq. ft. Applying formula of Art. **111**,

$$V = \frac{24 \times 10}{2} = 120 \text{ cu. ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Steel weighs .28 pound per cubic inch; find the weight of a steel wedge whose base is a rectangle 3 inches by $1\frac{1}{2}$ inches and whose altitude is 8 inches. Ans. 5.04 lb.

2. Find the volume of the frustum of a square pyramid of which the larger base is 15 inches square, the smaller base, 14 inches square, and the altitude, 3 inches. Ans. 631 cu. in.

3. A round tank is 8 feet in diameter at the top (inside) and 10 feet at the bottom; if the tank is 12 feet deep, how many gallons will it hold, there being 231 cubic inches in a gallon? Ans. 5,734.2 gal.

4. (*a*) What is the convex area of the frustum of a square pyramid whose altitude is 16 inches, one side of whose lower base is 28 inches long, and of the upper base 10 inches? (*b*) What is the volume of the frustum. Ans. $\begin{cases} (a)\ 1{,}395.18 \text{ sq. in.} \\ (b)\ 6{,}208 \text{ cu. in.} \end{cases}$

THE SPHERE

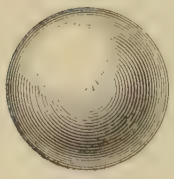

Fig. 60

112. A **sphere**, Fig. 60, is a solid bounded by a uniformly curved surface every point of which is equally distant from a point within, called the **center.**

The word **ball** is commonly used instead of sphere.

113. To find the area of the surface of a sphere

Rule.—*The area of the surface of a sphere equals the square of the diameter multiplied by* π.

Let S = surface;
d = diameter.

Then, $$S = \pi d^2$$

EXAMPLE.—What is the area of the surface of a sphere whose diameter is 14 inches?

SOLUTION.—Applying formula of Art. **113**, $S = 3.1416 \times 14^2 = 3.1416 \times 14 \times 14 = 615.75$ sq. in., the area. Ans.

114. To find the volume of a sphere:

Rule.—*The volume of a sphere equals the cube of the diameter multiplied by* $\frac{\pi}{6}$.

Let V = volume;
d = diameter.

Then, $$V = \frac{\pi}{6} d^3 = .5236 d^3$$

EXAMPLE.—What is the weight of a lead cannon ball 12 inches in diameter, a cubic inch of lead weighing .41 pound?

SOLUTION.—Applying formula of Art. **114**, $V = .5236 \times 12 \times 12 \times 12 = 904.78$ cu. in., the volume of the ball.

$904.78 \times .41 = 370.96$ lb. Ans.

The volume of a spherical shell, or hollow sphere, is equal to the difference in volume between two spheres having, respectively, the outer and the inner diameter of the shell.

115. To find the diameter of a sphere of known volume:

Rule.—*Divide the volume by .5236 and extract the cube root of the quotient. The result is the diameter.*

$$d = \sqrt[3]{\frac{V}{.5236}} = 1.2407 \sqrt[3]{V}$$

EXAMPLE.—The volume of a sphere is 96.1 cubic inches; what is its diameter?

SOLUTION.—Applying formula of Art. **115**,

$$d = \sqrt[3]{\frac{V}{.5236}} = \sqrt[3]{\frac{96.1}{.5236}} = 1.2407 \sqrt[3]{96.1} = 5.68 \text{ in.} \quad \text{Ans.}$$

116. If any solid is cut into two parts by a plane, the surface of either part exposed by the removal of the other part is called a **plane section** of the solid.

Plane sections are divided into three classes: longitudinal sections, cross-sections, and right sections. A **longitudinal section** is any plane section taken lengthwise through the solid. Any other plane section is called a **cross-section.** If the surface exposed by taking a plane section of a solid is perpendicular to the center line of the solid, the section is called a **right section.** The surface exposed by any longitudinal section of a cylinder is a rectangle. The surface exposed by a right section of a cube is a square; of a cylinder or a cone, a circle. An oblique cross-section of a cylinder is an ellipse.

THE CYLINDRICAL RING

117. A **cylindrical ring** is a solid that may be generated by a circle revolving about an external axis in its plane.

118. To find the convex area of a cylindrical ring:

Rule.—*Multiply the circumference of an imaginary cross-section on the line A B, Fig. 61, by the length of the center line D.*

EXAMPLE.—A piece of round iron rod is bent into circular form to make a ring for a chain; if the outside diameter of the ring is 12 inches and the inside diameter is 8 inches, what is its convex area?

SOLUTION.—The diameter of the center circle equals one-half the sum of the inside and outside diameters, $\frac{12+8}{2} = 10$, and $10 \times 3.1416 = 31.416$ in., the length of the center line. The radius of the inside circle is 4 in., of the outside circle 6 in.; therefore, the diameter of the cross-section on the line $A B$ is 2 in. Then, $2 \times 3.1416 = 6.2832$ in., and $6.2832 \times 31.416 = 197.4$ sq. in., or the convex area. Ans.

119. To find the volume of a cylindrical ring:

Rule.—*The volume will be the same as that of a cylinder whose altitude equals the length of the dotted center line D.*

Fig. 61, and whose base is the same as a cross-section of the ring on the line A B, drawn from the center O. Hence, to find the volume of a cylindrical ring, multiply the area of an imaginary cross-section on a line A B, by the length of the center line D.

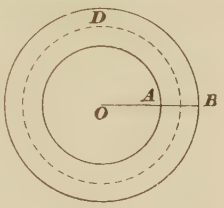

FIG. 61

EXAMPLE.—What is the volume of a cylindrical ring whose outside diameter is 12 inches, and whose inside diameter is 8 inches?

SOLUTION.—The diameter of the center circle equals one-half the sum of the inside and outside diameters, $\frac{12+8}{2} = 10$. $10 \times 3.1416 = 31.416$ in., the length of the center line. The radius of the outside circle is 6 in., of the inside circle, 4 in.; therefore, the diameter of the cross-section on the line $A\ B$ is 2 in. Then, $2^2 \times .7854 = 3.1416$ sq. in., the area of the imaginary cross-section; and $3.1416 \times 31.416 = 98.7$ cu. in., the volume. Ans.

EXAMPLES FOR PRACTICE

1. (*a*) What is the area of the surface of a sphere 30 inches in diameter? (*b*) What is the volume of the sphere?

Ans. { (*a*) 2,827.44 sq. in.
(*b*) 14,137.2 cu. in.

2. (*a*) What is the convex area of a cylindrical ring, the outside diameter of the ring being 10 inches and the inside diameter $7\frac{1}{2}$ inches? (*b*) What is the volume of the ring?

Ans. { (*a*) 107.95 sq. in.
(*b*) 33.734 cu. in.

3. The volume of a sphere is 606.132 cubic inches; what is the convex area of a cone whose slant height is 10 inches, and the diameter of whose base is the same as the diameter of the sphere?

Ans. 164.934 sq. in.

THE PRISMOID

120. A **prismoid** is a solid whose two bases are any polygons in parallel planes, and whose lateral faces may be divided into triangles and trapezoids by lines joining the vertexes of one base with those of the other. Thus, the solid shown in Fig. 62 is a prismoid; its bases are the pentagon $A\ B\ C\ D\ E$ and the quadrilateral $F\ G\ H\ I$, which lie in

parallel planes; and its faces are the triangle GBC and the trapezoids $GCDH$, $HDEI$, $IEAF$, and $FABG$.

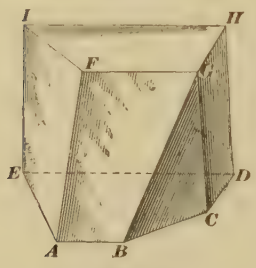

FIG. 62

121. The **altitude** of a prismoid is the perpendicular distance between the bases or parallel faces.

122. The parallel faces or bases of a prismoid are commonly called its **end sections.**

A prismoid is also defined as a solid having two parallel end faces, and composed of any combination of prisms, wedges, and pyramids, whose common altitude is the perpendicular distance between the parallel faces.

123. The **middle section** of a prismoid is the polygon formed by a plane, parallel to the bases, and cutting the prismoid at equal distances from the two bases or end sections. Thus, polygon $PQRS$ is the middle section of the prismoid shown in Fig. 63.

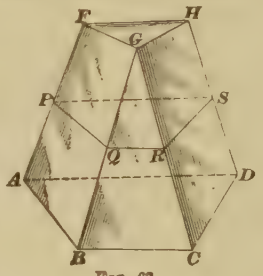

FIG. 63

124. Any dimension of the middle section of a prismoid may be taken equal to one-half the sum of the corresponding dimensions of the two end sections or bases. Thus, in Fig. 63, $PQ = \frac{1}{2}(AB + FG)$, $QR = \frac{1}{2}BC$, $RS = \frac{1}{2}(GH + CD)$, and $SP = \frac{1}{2}(HF + DA)$.

125. The area of the middle section of a prismoid may be measured directly, or calculated from its dimensions as determined from the dimensions of the end sections. It is not, in general, equal to one-half the sum of the areas of the bases.

The area of the middle section of a prism is the same as the area of either base; the area of the middle section of a wedge is equal to one-half the area of the base; the area of the middle section of a pyramid is equal to one-fourth the area of the base.

126. To find the volume of a prismoid:

Rule.—*Multiply the sum of the areas of the two end sections plus four times the area of the middle section by one-sixth the altitude.*

Let A = area of one base or end section;
A' = area of opposite base or end section;
M = area of middle section;
h = altitude;
V = volume of prismoid.

Then, $$V = \frac{h}{6}(A + A' + 4M)$$

This formula for finding the volume of a prismoid is known as the **prismoidal formula.** It is theoretically exact for determining the volumes of those solids to which it applies.

The derivation of this formula is as follows:

A prismoid can always be divided into elementary parts that will be prisms, wedges, and pyramids. From formula of Art. **96,** the volume of a prism is $V = Ah$; from formula of Art. **111,** the volume of a wedge is $V = \frac{Ah}{2}$; and from formula of Art. **102,** the volume of a pyramid is $V = \frac{Ah}{3}$. If these expressions are reduced to a common denominator, there will result,

For a prism, $$V = \frac{6Ah}{6} \qquad (1)$$

For a wedge, $$V = \frac{3Ah}{6} \qquad (2)$$

For a pyramid, $$V = \frac{2Ah}{6} \qquad (3)$$

Since any prism is of uniform cross-section throughout its length, every section will have the same area A, and equation (1) may be written

$$V = \frac{6Ah}{6} = \frac{h}{6}(A + A' + 4M)$$

For a wedge, evidently $A' = 0$, and $M = \frac{1}{2}A$. Hence, equation (2) may be written

$$V = \frac{3Ah}{6} = \frac{h}{6}(A + 0 + 2A) = \frac{h}{6}(A + A' + 4M)$$

For a pyramid, $A' = 0$, and $M = \frac{1}{4}A$. Hence, equation (3) may be written

$$V = \frac{2Ah}{6} = \frac{h}{6}(A + 0 + A) = \frac{h}{6}(A + A' + 4M)$$

Each of these formulas is the same as the formula given in this article; which shows that the latter formula applies correctly to the volume of a prism, pyramid, or wedge, and since it applies to each, it applies also to their sum, or the volume of a prismoid.

EXAMPLE.—Find the volume of the prismoid shown in Fig. 64, whose altitude is 14 inches.

SOLUTION.—Let PQR be the middle section. Then,

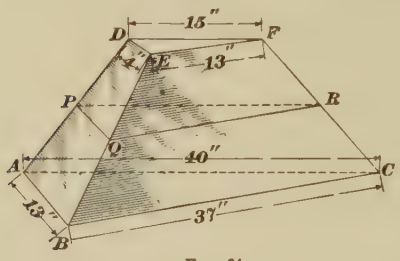

FIG. 64

$PQ = \frac{1}{2}(AB + DE) = \frac{1}{2}(13 + 4) = 8.5$ in.

$QR = \frac{1}{2}(BC + EF) = \frac{1}{2}(37 + 13) = 25$ in.

$RP = \frac{1}{2}(AC + DF) = \frac{1}{2}(40 + 15) = 27.5$ in.

The areas of the triangles are calculated by formula of Art. **47**, which gives the area of $ABC = 240$ sq. in., area of $DEF = 24$ sq. in., and area of $PQR = 105.2$ sq. in., nearly. Hence,

$$V = \frac{14}{6} \times (240 + 24 + 4 \times 105.2) = 1{,}597.9 \text{ cu. in., nearly.} \quad \text{Ans.}$$

127. A familiar example of a prismoid is a railway cutting where the roadway is a horizontal plane, the side slopes are inclined planes, and the original surface of the ground is more or less inclined and irregular.

For calculating the volume of cuts and fills the prismoidal formula, though theoretically exact, gives results that are only approximate, on account of the inequalities of the surface of the ground. The nearer to each other the cross-sections are taken, the more accurate will be the result.

EXAMPLE 1.—Find, by the prismoidal formula, the volume of the frustum of a square pyramid of which the larger base is 2.5 feet square, the smaller base is 1 foot square, and the altitude is 16 feet.

SOLUTION.—The area of the larger base is $2.5 \times 2.5 = 6.25$ sq. ft.; the area of the smaller base is $1 \times 1 = 1$ sq. ft. The middle section is a square whose side is one-half the sum of the side of the upper and lower base; that is, $\frac{1}{2} \times (2.5 + 1) = 1.75$ ft. The area of the middle section is $1.75^2 = 3.0625$ sq. ft. Applying formula of Art. **126**, the volume of the frustum is

$$\tfrac{1}{6} \times 16 \times (6.25 + 1 + 4 \times 3.0625) = 52 \text{ cu. ft.} \quad \text{Ans.}$$

EXAMPLE 2.—In a railway cutting 200 feet long, the following are the areas, in square feet, of the cross-sections taken every 50 feet, namely: 2,700, 2,619, 2,556, 2,484, 2,610. What is its volume?

SOLUTION.—The volume between the first and the third cross-section is, by formula of Art. **126**,

$$V = \frac{100}{6}(2{,}700 + 2{,}556 + 4 \times 2{,}619) = 262{,}200 \text{ cu. ft.}$$

The volume between the third and the fifth section is

$$V = \frac{100}{6}(2{,}556 + 2{,}610 + 4 \times 2{,}484) = 251{,}700 \text{ cu. ft.}$$

The volume of the cutting is the sum of the volumes of the two prismoids, which is 513,900 cu. ft. = 19,033 cu. yd. Ans.

128. Average End Areas.—In practice, the volume of cuts and fills is often calculated by what is known as the **method by average end areas**, or simply as the **end area method**. By this method, the volume of the solid is found by multiplying one-half the sum of the two end areas by the distance between the two sections. Thus, let

A = area of one cross-section;
A' = area of next cross-section;
h = perpendicular distance between sections;
V = volume.

Then,
$$V = \frac{h}{2}(A + A')$$

Results obtained by this formula are approximate and slightly larger than those given by the prismoidal formula. On account of its simplicity, the average end area formula is much used in practical earth-work calculations. The inequalities of the surface of the ground make it impossible to find the exact volume of a cut or fill, however accurate may be the formula applied.

EXAMPLE.—The areas of two cross-sections of a fill 50 feet apart are 2,700 and 2,619 square feet respectively; find the volume of the section, in cubic yards.

SOLUTION.—In this case, $A = 2{,}700$; $A' = 2{,}619$; and $h = 50$; then

$$V = \frac{50}{2}(2{,}700 + 2{,}619) = 132{,}975$$

Hence, the volume is 132,975 cu. ft. = 4,925 cu. yd. Ans

EXAMPLES FOR PRACTICE

1. Find the volume of a right prismoid whose bases are rectangles that measure 10 inches by 8 inches and 8 inches by 6 inches, and whose height is 40 inches. Ans. 2,533.3 cu. in.

2. A railway cutting is 800 feet in length; the areas, in square yards, of cross-sections taken every 100 feet are: 237, 220, 204, 187, 171, 186, 204, 210, 220. Find the number of cubic yards in the cutting: (*a*) by the prismoidal formula; (*b*) by average end areas.

Ans. { (*a*) 53,633 cu. yd.
(*b*) 53,683 cu. yd.

3. Find, by the prismoidal formula, the volume of a frustum of a hexagonal pyramid, each side of the lower base being 12 inches; of the upper base, 8 inches; and the altitude being 12 inches.

Ans. 3,159.3 cu. in.

4. Find, by the prismoidal formula, the volume of a wedge whose base is a rectangle 15 feet in length and 9 feet in width, and whose altitude is 12 feet. Ans. 810 cu. ft.

PLANE TRIGONOMETRY

(PART 1)

THE TRIGONOMETRIC FUNCTIONS

DEFINITIONS

1. Trigonometric Functions and Trigonometry Defined.—Let A, Fig. 1, be any acute angle; AM and AN, its sides; BC, a perpendicular drawn to the side AN from any point on the side AM; and $B'C'$, a perpendicular drawn to the side AM from any point on the side AN. In the right triangle ABC, one of the vertexes of which is the vertex of the angle A, the hypotenuse AB will be referred to as *the hypotenuse;* the perpendicular BC, opposite the vertex of the angle A, as *the side opposite;* and the leg AC, containing the vertex of the angle A, as *the side adjacent.* Likewise, in the right triangle $AB'C'$, the hypotenuse is AB'; the side opposite is $B'C'$; and the side adjacent, or the leg containing the vertex of the angle A, is AC'. It should be borne in mind that these terms are used in connection with, or with reference to, the angle A.

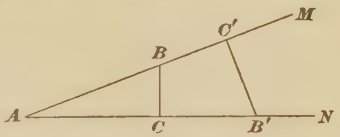

FIG. 1

The two right triangles ABC and $AB'C'$, having the acute angle A in common, are similar. Therefore,

$$\frac{AB}{AC} = \frac{AB'}{AC'}, \quad \frac{BC}{AB} = \frac{B'C'}{AB'}, \quad \frac{BC}{AC} = \frac{B'C'}{AC'}$$

It will be observed that, from whichever side the perpendicular is drawn, and whatever the point from which it is drawn, the ratio of the hypotenuse to the side adjacent

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remains unchanged, or is **constant**. The same is true of the ratio of the side opposite to the side adjacent, and, in general, of the ratio of any two of the three lines—hypotenuse, side adjacent, and side opposite. Evidently, these ratios are different for different angles. Thus, if A is 45°, both acute angles B and B' are also 45°; the triangles ABC and $AB'C'$ are isosceles; and therefore

$$\frac{BC}{AC} = \frac{B'C'}{AC'} = 1$$

If A is greater than 45°, BC is greater than AC, and the ratio $\frac{BC}{AC}$, having its numerator greater than its denominator, is greater than 1.

Confining ourselves to the ratio $\frac{BC}{AC}$ of the side opposite to the side adjacent, it is seen that the value of this ratio depends on the magnitude of the angle, and may, therefore, be used for the determination of the angle. Thus, it has just been shown that when the angle is 45° the ratio is equal to 1; hence, if in the solution of a problem it is found that the two legs of a right triangle are equal, or that their ratio is 1, it can be at once concluded that each of the acute angles is 45°.

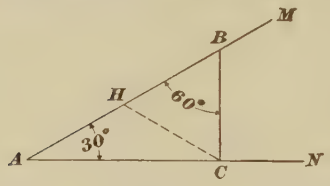

FIG. 2

Consider now an angle A, Fig. 2, of 30°. The right triangle ABC having been constructed, BC is the side opposite and AC the side adjacent. If H is the middle point of the hypotenuse, the line HC is equal to AH, or $\frac{AB}{2}$; for, if a semicircle is described on AB as a diameter, with HA as a radius, that semicircle must pass through C, since the angle ACB is a right angle. Now, HC being equal to HB, the angle HCB is equal to B, or 60°; and, as the sum of the three angles of the triangle BHC is 180°, the angle BHC must be 60°. The triangle HBC being equiangular, it is also equilateral, and therefore $BC = BH = \frac{AB}{2}$, and

the ratio of the side opposite to the hypotenuse is $\frac{BC}{AB} = \frac{\frac{1}{2}AB}{AB} = \frac{1}{2}$. Suppose, now, that in dealing with a right triangle the hypotenuse is found, by measurement, to be 1,500 feet and one of the sides 750 feet. Since the ratio of 750 to 1,500 is $\frac{1}{2}$, we at once conclude that the angle opposite the 750-foot side is 30°, and the other angle of the triangle, 60°.

These illustrations give a general idea of the practical value and use of the ratios under consideration. These ratios are determined for each angle, by methods that will be again referred to further on, and collected together in a table, from which the angle corresponding to any given ratio can be determined. Thus, if in a certain angle the ratio of the opposite side to the hypotenuse is $\frac{1}{2}$, this ratio is looked for in the table, where it is found as that belonging to 30°. In this manner, the value of the angle is determined from the ratio in question, that ratio being obtained from the measured lengths of certain lines.

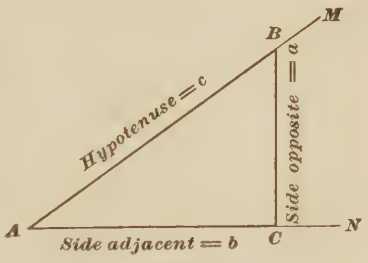

FIG. 3

2. The ratios considered in the preceding article are called **trigonometric functions** of the angle A. In the triangle ABC, Fig. 3, two ratios are obtained by dividing any of three sides by each of the other two. Hence, there are six trigonometric functions of the angle A. This is true of any angle, since A is here used to represent any angle whatever. These functions have very important and useful properties, which make them exceedingly valuable for the solution of geometrical problems by computation.

3. **Trigonometry** is that branch of mathematics that treats of the properties of trigonometric functions and of their application to the solution of triangles.

4. The Sine and the Tangent.—Two of the most important of the trigonometric functions are the ratio of the side opposite to the hypotenuse, and that of the side opposite to the side adjacent; that is, $\frac{a}{c}$ and $\frac{a}{b}$, Fig. 3. They are called, respectively, the **sine** of A and the **tangent** of A. The words *sine* and *tangent* are abbreviated to *sin* and *tan*, respectively, and the expressions *sin A*, *tan A*, are for brevity read *sine A*, *tangent A*, instead of *sine of A*, and *tangent of A*. We have, then,

$$\sin A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{a}{c} \qquad (1)$$

$$\tan A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{a}{b} \qquad (2)$$

If these formulas are fixed in the mind, little difficulty will be experienced in remembering the others that will be given. It should be noticed that the side opposite is the numerator in both ratios. The occurrence of the letter a in both the words *adjacent* and *tangent* will help one to remember which of the two fractions represents the tangent and which the sine.

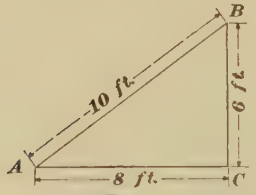

Fig. 4

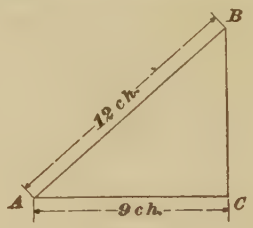

Fig. 5

Example 1.—In the right triangle $A\,B\,C$, Fig. 4, the lengths of the sides are shown; find the sine and the tangent of A.

Solution.—In this case, the hypotenuse $A\,B = 10$; the side adjacent, $A\,C = 8$; side opposite, $B\,C = 6$. These values in formulas **1** and **2** give

$$\sin A = \frac{6}{10} = .6. \quad \text{Ans.}$$

$$\tan A = \frac{6}{8} = .75. \quad \text{Ans.}$$

Example 2.—In the right triangle $A\,B\,C$, Fig. 5, the hypotenuse is 12 chains, and the side $A\,C$ is 9 chains; find: (*a*) the sine and the tangent of A; (*b*) the sine and the tangent of B.

SOLUTION.—(*a*) For the angle A, we have

hypotenuse $AB = 12$

side adjacent, $AC = 9$

side opposite, $BC = \sqrt{AB^2 - AC^2} = \sqrt{12^2 - 9^2} = 7.9372$

Substituting in formulas **1** and **2**,

$$\sin A = \frac{BC}{AB} = \frac{7.9372}{12} = .66143. \quad \text{Ans.}$$

$$\tan A = \frac{BC}{AC} = \frac{7.9372}{9} = .88191. \quad \text{Ans.}$$

(*b*) For angle B, we have

hypotenuse $BA = 12$

side opposite, $AC = 9$

side adjacent, $BC = 7.9372$

Therefore,

$$\sin B = \frac{AC}{AB} = \frac{9}{12} = .75. \quad \text{Ans.}$$

$$\tan B = \frac{AC}{BC} = \frac{9}{7.9372} = 1.1339. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. In a right triangle ABC (make a sketch of this triangle), A and B are the two acute angles; the hypotenuse = 40 feet; side opposite B = 15 feet; find: (*a*) sin A and tan A; (*b*) sin B and tan B.

Ans. $\begin{cases} (a)\ \sin A = .92703,\ \tan A = 2.47207 \\ (b)\ \sin B = .37500,\ \tan B = .40452 \end{cases}$

2. From a point on one side of an angle M, a perpendicular is drawn on the other side; it is found that this perpendicular is 12.5 inches long, and that it meets the other side at a distance of 7.75 inches from the vertex; find the sine and the tangent of the angle M. (Make a sketch of this triangle.)

Ans. $\begin{cases} \sin M = .84988 \\ \tan M = 1.61290 \end{cases}$

3. From a point on one side of an angle A distant 10 inches from the vertex, a perpendicular is drawn on the other side; the distance from the vertex to the foot of the perpendicular is 6 inches; find sin A and tan A.

Ans. $\begin{cases} \sin A = .80000 \\ \tan A = 1.33333 \end{cases}$

4. The two acute angles of a right triangle are P and Q; the side opposite P is 150 feet, and that opposite Q is 225 feet; find: (*a*) sin P and tan P; (*b*) sin Q and tan Q.

Ans. $\begin{cases} (a)\ \sin P = .55469,\ \tan P = .66667 \\ (b)\ \sin Q = .83204,\ \tan Q = 1.50000 \end{cases}$

5. The Cosine and Cotangent.—The **cosine** and **cotangent** of an angle are, respectively, the sine and the tangent of the complement of the angle. The words *cosine* and *cotangent* are abbreviated to *cos* and *cot*, respectively, and the expressions *cos A*, *cot A* are read *cosine A*, *cotangent A*. Denoting any angle by A, its complement is $90° - A$; therefore, according to the definitions just given,

$$\cos A = \sin (90° - A) \qquad (1)$$
$$\cot A = \tan (90° - A) \qquad (2)$$

Since the complement of $90° - A$ is A, it also follows that

$$\cos (90° - A) = \sin A \qquad (3)$$
$$\cot (90° - A) = \tan A \qquad (4)$$

With reference to the angle B, Fig. 3, BC is the side adjacent and AC the side opposite. Therefore, by formulas **1** and **2**, Art. **4**,

$$\sin B = \frac{b}{c}, \tan B = \frac{b}{a}$$

and therefore, since A is the complement of B,

$$\cos A = \sin B = \frac{b}{c}$$
$$\cot A = \tan B = \frac{b}{a}$$

or, again referring to the angle A, which is the angle under consideration,

$$\cos A = \frac{\text{side adjacent}}{\text{hypotenuse}} \qquad (5)$$
$$\cot A = \frac{\text{side adjacent}}{\text{side opposite}} \qquad (6)$$

The student will, after some practice, become familiar with these formulas. Whenever he forgets them, he should refer to the definitions of the cosine and cotangent, which will at once enable him to write down the formulas, provided that he remembers those for the sine and the tangent.

6. The Secant and Cosecant.—The **secant** of an angle is the reciprocal of the cosine of the angle; that is, 1 divided by the cosine.

The word *secant* is abbreviated to *sec*. According to the definition, we have

$$\sec A = \frac{1}{\cos A} \qquad (1)$$

It follows that

$$\cos A = \frac{1}{\sec A} \qquad (2)$$

7. The **cosecant** of an angle is the secant of the complement of the angle. The abbreviations *cosec* and *csc* are used for *cosecant*. According to the definition, we have

$$\csc A = \sec (90° - A) \qquad (1)$$

Since A is the complement of $90° - A$, we have also

$$\csc (90° - A) = \sec A \qquad (2)$$

By means of formula **1**, Art. **6**, this relation may be written

$$\csc A = \sec (90° - A) = \frac{1}{\cos (90° - A)}$$

or, since $\cos (90° - A) = \sin A$ (formula **3**, Art. **5**),

$$\csc A = \frac{1}{\sin A} \qquad (3)$$

Therefore, the cosecant of an angle may also be defined as the reciprocal of the sine. Notice very particularly that

secant = reciprocal of co*sine*
co*secant* = reciprocal of *sine*

From formula **3** above follows

$$\sin A = \frac{1}{\csc A} \qquad (4)$$

8. Cofunctions and Complementary Functions. The functions cosine, cotangent, and cosecant are sometimes called **cofunctions** of the angle considered; while the sine, tangent, and secant are called **fundamental functions**. As has been explained, the cofunctions of an angle are the corresponding fundamental functions of the complement of the angle. Thus, the cosine of A is the sine of $90° - A$; the cotangent of A is the tangent of $90° - A$; etc.

A fundamental function and its corresponding cofunction are called **complementary functions** of each other. The sine, for example, is the complementary function of the cosine; and the cosine is the complementary function of the sine.

EXAMPLE 1.—Find: (*a*) the cosine of the angle A, Fig. 5; (*b*) the cotangent; (*c*) the secant; (*d*) the cosecant.

SOLUTION.—(*a*) The cosine of A is equal to the sine of B, or

$$\frac{AC}{AB} = \frac{9}{12} = .75. \quad \text{Ans.}$$

(*b*) The cotangent of A is equal to the tangent of B, or (see example 2, Art. **4**)

$$\frac{AC}{BC} = \frac{9}{7.9372} = 1.1339. \quad \text{Ans.}$$

(*c*) The secant of A is 1 divided by cos A, or

$$1 \div \frac{9}{12} = \frac{12}{9} = 1.33333. \quad \text{Ans.}$$

(*d*) The cosecant of A is 1 divided by sin A, or

$$1 \div \frac{BC}{AB} = \frac{AB}{BC} = \frac{12}{7.9372} = 1.51187. \quad \text{Ans.}$$

EXAMPLE 2.—Find the functions of 30°.

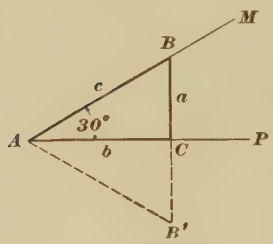

FIG. 6

SOLUTION.—Let the angle MAP, Fig. 6, be 30°. Draw BC perpendicular to AP, produce it to B', making $CB' = CB$, and draw AB'. The triangle BAB' thus formed is isosceles, and angle $CAB' = CAB = 30°$. Therefore, $BAB' = 30° + 30° = 60°$. Also, angle $B = 90° - 30° = 60°$; and angle B' = angle $B = 60°$. As the three angles of ABB' are equal, the sides are also equal, and $c = BB' = 2a$. Now, the figure gives,

$$b = \sqrt{c^2 - a^2} = \sqrt{(2a)^2 - a^2} = \sqrt{3a^2} = a\sqrt{3}$$

Bearing these values in mind, we have

$$\sin 30° = \frac{a}{c} = \frac{a}{2a} = \frac{1}{2}. \quad \text{Ans.}$$

$$\tan 30° = \frac{a}{b} = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}. \quad \text{Ans.}$$

$$\cos 30° = \frac{b}{c} = \frac{a\sqrt{3}}{2a} = \frac{\sqrt{3}}{2}. \quad \text{Ans.}$$

$$\cot 30° = \frac{b}{a} = \frac{a\sqrt{3}}{a} = \sqrt{3}. \quad \text{Ans.}$$

$$\sec 30° = \frac{1}{\cos 30°} = 1 \div \frac{\sqrt{3}}{2} = \frac{2}{\sqrt{3}} = \frac{2}{3}\sqrt{3}. \quad \text{Ans.}$$

$$\csc 30° = \frac{1}{\sin 30°} = 1 \div \frac{1}{2} = 2. \quad \text{Ans.}$$

NOTE.—It is only in a few cases that the values of the trigonometric functions of an angle can be derived by elementary principles, as above. The general method for determining the functions of any angle is comparatively complicated, and is beyond the scope of this work. The trigonometric functions of any angle can be obtained from a table, as will be presently explained.

EXAMPLES FOR PRACTICE

1. The acute angles of a right triangle are B and C; the side opposite B is 1,200 feet; and that opposite C is 1,500 feet; find the fundamental functions of B, and from them the cofunctions of C.

Ans. $\begin{cases} \sin B = .62471, \tan B = .8, \sec B = 1.2806 \\ \cos C = .62471, \cot C = .8, \csc C = 1.2806 \end{cases}$

2. From example 2, Art. 8, derive the functions of 60° ($= 90° - 30°$).

Ans. $\begin{cases} \sin 60° = \frac{\sqrt{3}}{2}, \tan 60° = \sqrt{3}, \cos 60° = \frac{1}{2} \\ \cot 60° = \frac{\sqrt{3}}{3}, \sec 60° = 2, \csc 60° = \frac{2}{3}\sqrt{3} \end{cases}$

3. Given $\sin A = \frac{2}{3}$ and $\cos B = \frac{4}{5}$, find $\csc A$ and $\sec B$.

Ans. $\begin{cases} \csc A = 1.5 \\ \sec B = 1.25 \end{cases}$

4. Find the trigonometric functions of 45°. (Notice that here the side opposite is equal to the side adjacent. Denote the hypotenuse by c, and express the other two sides in terms of c.)

Ans. $\begin{cases} \sin 45° = \cos 45° = \frac{1}{2}\sqrt{2} \\ \tan 45° = \cot 45° = 1 \\ \sec 45° = \csc 45° = \sqrt{2} \end{cases}$

9. The Versed Sine and Coversed Sine.—The **versed sine** (*vers*) of an angle is 1 minus the cosine; and the **coversed sine** (*covers*) is 1 minus the sine.

$$\text{vers } A = 1 - \cos A \qquad (1)$$

$$\text{covers } A = 1 - \sin A \qquad (2)$$

These two functions are not much used, except in railroad work.

10. Summing Up.—The foregoing definitions are summed up in the table given below, which contains the expressions for the functions of the angle A, Fig. 7, in terms of the hypotenuse c, the side opposite, a, and the side adjacent, b.

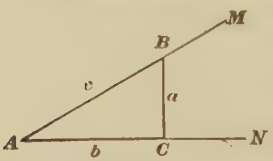

FIG. 7

TABLE I

Function	sin	tan	cos	cot	sec	csc	vers	covers
Value . . .	$\frac{a}{c}$	$\frac{a}{b}$	$\frac{b}{c}$	$\frac{b}{a}$	$\frac{c}{b}$	$\frac{c}{a}$	$1-\frac{b}{c}$	$1-\frac{a}{c}$

The ratios $\frac{c}{b}$ and $\frac{c}{a}$ for the secant and cosecant are obtained from the formulas $\sec A = 1 \div \cos A = 1 \div \frac{b}{c} = \frac{c}{b}$, $\csc A = 1 \div \sin A = 1 \div \frac{a}{c} = \frac{c}{a}$.

11. Representation of the Trigonometric Functions by Lines.—Let A, Fig. 8, be any angle. From its

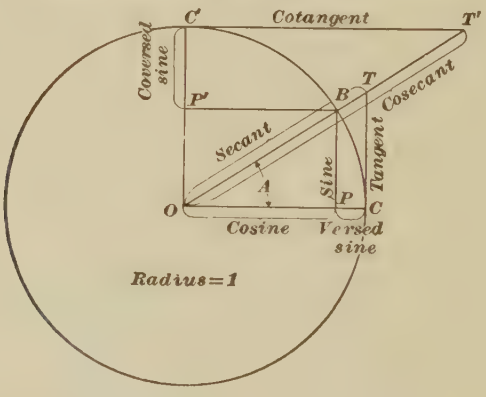

FIG. 8

vertex O, describe a circle of radius 1; or, otherwise, describe any circle and take its radius as unity. This circle intersects

the sides of the angle at B and C. Draw the tangent CT, meeting OB produced at T; the radius OC' perpendicular to OC; the lines BP and BP' perpendicular to OC and OC', respectively; and the tangent $C'T'$, meeting OB produced at T'.

Since the angle A is measured by the arc CB, the trigonometric functions of the angle are said to be likewise the trigonometric functions of the arc. It is, for instance, immaterial whether we say that 1 is the tangent of an angle of 45° or of an arc of 45°.

In the figure constructed as just explained, the trigonometric functions of the angle A, or of the arc CB, may be represented by lines, as marked. For, in the right triangle OPB, in which BP, OP, and OB are, respectively, the side opposite, the side adjacent, and the hypotenuse, we have

$$\sin A = \frac{BP}{OB}, \cos A = \frac{OP}{OB}$$

or, since $OB = 1$,

$$\sin A = \frac{BP}{1} = BP, \cos A = \frac{OP}{1} = OP$$

In the triangle OCT, in which CT and OC are, respectively, the side opposite and the side adjacent, and OT is the hypotenuse,

$$\tan A = \frac{CT}{OC} = \frac{CT}{1} = CT$$

$$\sec A = \frac{OT}{OC} = \frac{OT}{1} = OT$$

By the same reasoning, it can be shown that $C'T'$ and OT' are, respectively, the tangent and the secant of the angle $C'OT'$, or the cotangent and the cosecant of A, since $C'OT'$ is the complement of A.

Let the student verify that, according to the definitions of the versed sine and coversed sine, these functions are represented by PC and $P'C'$, respectively.

RELATIONS AMONG THE FUNCTIONS OF AN ANGLE

12. Method of Marking a Triangle.—The triangle ABC, Fig. 7, has the angles marked by the capital letters A, B, and C and the sides opposite these angles marked by the small letters a, b, and c, respectively. This method of marking a triangle is very useful and convenient, as it points out at once the relative position of the sides and the angles. In a right triangle, the right angle is usually designated by C. In the figures that follow, when only the angles are marked, the sides opposite are taken as marked by the small letters corresponding to the capital letters that mark the angles.

13. Relation Between Tangent and Cotangent. In Fig. 7,

$$\tan A = \frac{a}{b}, \cot A = \frac{b}{a}$$

Multiplying these equations together gives

$$\tan A \times \cot A = \frac{a}{b} \times \frac{b}{a} = 1$$

whence,

$$\cot A = \frac{1}{\tan A}$$

$$\tan A = \frac{1}{\cot A}$$

That is, the tangent and cotangent are each the reciprocal of the other. This is a very important relation, and should be committed to memory, together with those given in the two articles following.

14. Tangent and Cotangent in Terms of Sine and Cosine.—In Fig. 7,

$$\sin A = \frac{a}{c}, \cos A = \frac{b}{c}$$

Dividing these equations member by member gives

$$\frac{\sin A}{\cos A} = \frac{a}{c} \div \frac{b}{c} = \frac{a}{b}$$

that is, since $\frac{a}{b} = \tan A$,

$$\tan A = \frac{\sin A}{\cos A} \qquad (1)$$

Also, because the cotangent is the reciprocal of the tangent,

$$\cot A = \frac{\cos A}{\sin A} \qquad (2)$$

15. Relations Between the Squares of Certain Functions.—A power of a trigonometric function is indicated by writing the exponent immediately after the abbreviation used for the function. Thus, the square of the sine of A, or of $\sin A$, is written $\sin^2 A$, and read *sine square A*. Similarly, the cube of $\tan A$ is written $\tan^3 A$, and read *tangent cube A*, etc.

In the right triangle ABC, Fig. 7, we have

$$a^2 + b^2 = c^2$$

Dividing both members of this equality by c^2 gives

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1$$

that is, $$\sin^2 A + \cos^2 A = 1 \qquad (1)$$

Again, dividing both members of the equation $c^2 = a^2 + b^2$ by b^2,

$$\frac{c^2}{b^2} = \frac{a^2}{b^2} + 1 = 1 + \frac{a^2}{b^2}$$

that is, $$\sec^2 A = 1 + \tan^2 A \qquad (2)$$

Similarly, if both members of the equation $c^2 = a^2 + b^2$ are divided by a^2,

$$\frac{c^2}{a^2} = 1 + \frac{b^2}{a^2}$$

that is, $$\csc^2 A = 1 + \cot^2 A \qquad (3)$$

16. To Express Any Function in Terms of Any Other Function.—In the triangle ABC, Fig. 7, we have

$$a^2 + b^2 = c^2 \qquad (1)$$

Dividing both members of this equation by c^2 gives

$$\frac{a^2}{c^2} + \frac{b^2}{c^2} = 1 \qquad (2)$$

From these two equations, any of the six ratios $\frac{a}{b}, \frac{a}{c}, \frac{b}{a}, \frac{b}{c}$,

$\frac{c}{a}$, $\frac{c}{b}$ can be found when one of them is given. If, for instance, $\frac{a}{c}$ is given, $\frac{c}{a}$ is obtained by dividing 1 by $\frac{a}{c}$; $\frac{b}{c}$, by solving equation (2) for $\frac{b^2}{c^2}$ and taking the square root; $\frac{c}{b}$, by taking the reciprocal of the value just found for $\frac{b}{c}$. To find $\frac{a}{b}$, divide both members of equation (1) by b^2, which gives

$$\frac{a^2}{b^2} + 1 = \frac{c^2}{b^2}$$

whence, multiplying through by $\frac{b^2}{c^2}$,

$$\frac{b^2}{c^2}\left(\frac{a^2}{b^2} + 1\right) = \frac{c^2}{b^2} \times \frac{b^2}{c^2} = 1$$

and hence, dividing through by $\frac{a^2}{b^2} + 1$,

$$\frac{b^2}{c^2} = \frac{1}{\frac{a^2}{b^2} + 1}$$

Substituting this value of $\frac{b^2}{c^2}$ in equation (2) and solving for $\frac{a}{b}$, the latter ratio is obtained in terms of $\frac{a}{c}$.

Table II gives the relation between any two functions of any angle A.

TABLE II

RELATIONS BETWEEN THE FUNCTIONS OF AN ANGLE

In Terms of	$\sin A$	$\cos A$	$\tan A$	$\cot A$	$\sec A$	$\csc A$
$\sin A =$	$\sin A$	$\sqrt{1-\cos^2 A}$	$\dfrac{\tan A}{\sqrt{1+\tan^2 A}}$	$\dfrac{1}{\sqrt{1+\cot^2 A}}$	$\dfrac{\sqrt{\sec^2 A-1}}{\sec A}$	$\dfrac{1}{\csc A}$
$\cos A =$	$\sqrt{1-\sin^2 A}$	$\cos A$	$\dfrac{1}{\sqrt{1+\tan^2 A}}$	$\dfrac{\cot A}{\sqrt{1+\cot^2 A}}$	$\dfrac{1}{\sec A}$	$\dfrac{\sqrt{\csc^2 A-1}}{\csc A}$
$\tan A =$	$\dfrac{\sin A}{\sqrt{1-\sin^2 A}}$	$\dfrac{\sqrt{1-\cos^2 A}}{\cos A}$	$\tan A$	$\dfrac{1}{\cot A}$	$\sqrt{\sec^2 A-1}$	$\dfrac{1}{\sqrt{\csc^2 A-1}}$
$\cot A =$	$\dfrac{\sqrt{1-\sin^2 A}}{\sin A}$	$\dfrac{\cos A}{\sqrt{1-\cos^2 A}}$	$\dfrac{1}{\tan A}$	$\cot A$	$\dfrac{1}{\sqrt{\sec^2 A-1}}$	$\sqrt{\csc^2 A-1}$
$\sec A =$	$\dfrac{1}{\sqrt{1-\sin^2 A}}$	$\dfrac{1}{\cos A}$	$\sqrt{1+\tan^2 A}$	$\dfrac{\sqrt{1+\cot^2 A}}{\cot A}$	$\sec A$	$\dfrac{\csc A}{\sqrt{\csc^2 A-1}}$
$\csc A =$	$\dfrac{1}{\sin A}$	$\dfrac{1}{\sqrt{1-\cos^2 A}}$	$\dfrac{\sqrt{1+\tan^2 A}}{\tan A}$	$\sqrt{1+\cot^2 A}$	$\dfrac{\sec A}{\sqrt{\sec^2 A-1}}$	$\csc A$

TRIGONOMETRIC TABLES

TABLES OF NATURAL FUNCTIONS

17. To facilitate calculations, tables of the trigonometric functions are used. The tables give values for the sines, cosines, tangents, and cotangents of angles from 0° to 90°. The values of the secant and cosecant are not generally given in tables; they are obtained by dividing 1 by the cosine and the sine, respectively, according to formula **1**, Art. **6**, and formula **3**, Art. **7**.

There are two kinds of trigonometric tables; namely, the table of *natural functions* and the table of *logarithmic functions*. The table of natural functions gives the actual values of the functions, while the table of logarithmic functions gives the logarithms of the functions. It may be remarked that, except in making a table, the values of the functions are never calculated directly because the process is so long and laborious that it would require considerable time to calculate even the value of one function of an angle; nor is there a simple method of calculating the angle corresponding to a given function.

NOTE.—In all that follows, the number of seconds by which an angle exceeds a whole number of degrees and minutes will be referred to as *the odd seconds*, or *the number of odd seconds*, or simply *the number of seconds* in the angle; while the expression *total number of seconds* will be applied to the number obtained by reducing the degrees and minutes to seconds, and adding the odd seconds. Thus, the odd seconds, or, for shortness, the seconds in 34° 36′ 16″ are 16; while the total number of seconds is the number of seconds in 34°, plus the number of seconds in 36′, plus 16; that is, $34 \times 60 \times 60 + (36 \times 60) + 16 = 124{,}576$. A similar notation will be used with regard to minutes. The explanations that follow refer to the *Trigonometric Tables* used with this Course.

18. To Find the Natural Functions of an Angle Less Than 45° and Containing No Odd Seconds.—The required function is found in the double column marked at

the top with the given number of degrees, in the subdivision of that column headed by the name of the given function, and horizontally opposite the number in the left-hand column (marked ′) that expresses the number of odd minutes in the angle. When the function considered is a sine or a cosine, it is taken from the table headed Natural Sines and Cosines; when a tangent or cotangent, from the table headed Natural Tangents and Cotangents.

EXAMPLE.—Find the natural functions of an angle of 37° 23′.

SOLUTION.—On page 30 of the table headed Natural Sines and Cosines, the double column headed 37° is found. Looking in the left-hand minute column for 23 (number of odd minutes in the given angle), and glancing along the horizontal row to the right of 23, the number .60714 is found in the single column marked Sine under 37°; and the number .79459 is found in the column marked Cosine. Therefore,

$$\sin 37° 23′ = .60714. \quad \text{Ans.}$$
$$\cos 37° 23′ = .79459. \quad \text{Ans.}$$

The tangent and cotangent are taken in a similar manner from the table headed Natural Tangents and Cotangents, page 39. The results are:

$$\tan 37° 23′ = .76410. \quad \text{Ans.}$$
$$\cot 37° 23′ = 1.30873. \quad \text{Ans}$$

EXAMPLES FOR PRACTICE

Verify the following values:

(*a*) $\sin 39° 55′ = .64167$; $\cos 39° 55′ = .76698$; $\tan 39° 55′ = .83662$; $\cot 39° 55′ = 1.19528$.

(*b*) $\tan 16° 32′ = .29685$; $\cos 16° 32′ = .95865$; $\sec 16° 32′ = 1.04313$; $\csc 16° 32′ = 3.51407$.

(*c*) $\cot 43° 2′ = 1.07112$; $\csc 43° 2′ = 1.46537$; $\tan 43° 2′ = .93360$; $\cos 43° 2′ = .73096$.

19. To Find the Natural Functions of an Angle Greater Than 45° and Containing No Odd Seconds. The required function is found in the double column marked at the bottom with the given number of degrees, in the subdivision of that column having at the bottom the name of the given function, and horizontally opposite the number in the right-hand column (marked ′) that expresses the odd minutes in the angle. It will be observed that the number of degrees at the bottom of the pages decrease as the pages increase,

and that the number of minutes in the right-hand column increase from bottom to top.

EXAMPLE.—Find the functions of 53° 43′.

SOLUTION.—The double column marked 53° at the bottom is found on page 30 of Natural Sines and Cosines. Looking along the horizontal row determined by the number 43 in the right-hand minute column, the number .80610 is found in the single column marked Sine at the bottom, and the number .59178 in the single column marked Cosine at the bottom, these two columns forming the double column marked 53° at the bottom. Therefore,

$$\sin 53° 43′ = .80610. \quad \text{Ans.}$$
$$\cos 53° 43′ = .59178. \quad \text{Ans.}$$

The tangent and cotangent are similarly taken from page 39 of Natural Tangents and Cotangents. The results are:

$$\tan 53° 43′ = 1.36217. \quad \text{Ans.}$$
$$\cot 53° 43′ = .73413. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

Verify the following values:

(*a*) $\sin 67° 45′ = .92554$; $\cos 67° 45′ = .37865$; $\tan 67° 45′ = 2.44433$; $\cot 67° 45′ = .40911$.

(*b*) $\cot 74° 3′ = .28580$; $\csc 74° 3′ = 1.04004$; $\sin 74° 3′ = .96150$.

(*c*) $\cos 48° 9′ = .66718$; $\cot 48° 9′ = .89567$; $\csc 48° 9′ = 1.34248$.

20. To Find the Natural Functions of an Angle Containing Odd Seconds.—The method of solving this problem by means of the table is founded on the following principle, which applies within the limits of approximation with which the table is constructed:

If several angles are taken within an interval not greater than 1′; that is, so that the difference between the greatest and the smallest shall not exceed 1′, the ratio of the difference between any two of these angles to the difference between any other two is the same as the ratio obtained by dividing the difference between the values of any trigonometric function for the first pair of angles, by the difference between the values of the same function for the second pair of angles. For instance, if the angles 43° 46′ 32″, 43° 46′ 34″, 43° 46′ 40″, and 43° 47′ are taken between 43° 46′ and 43° 47′, then

$$\frac{43^\circ\ 47' - 43^\circ\ 46'\ 40''}{43^\circ\ 46'\ 34'' - 43^\circ\ 46'\ 32''} = \frac{\sin 43^\circ\ 47' - \sin 43^\circ\ 46'\ 40''}{\sin 43^\circ\ 46'\ 34'' - \sin 43^\circ\ 46'\ 32''}$$

In general, if A, B, C, D are any angles within an interval of 1′, then

$$\frac{A-B}{C-D} = \frac{\sin A - \sin B}{\sin C - \sin D} = \frac{\cos A - \cos B}{\cos C - \cos D}$$
$$= \frac{\tan A - \tan B}{\tan C - \tan D} = \frac{\cot A - \cot B}{\cot C - \cot D}$$

Similarly,

$$\frac{A-B}{B-C} = \frac{\sin A - \sin B}{\sin B - \sin C} = \frac{\cos A - \cos B}{\cos B - \cos C}, \text{ etc.}$$

Let A be the number of degrees and minutes in any angle, and s the number of odd seconds. Then the angle, which will be represented by $A + s''$, lies between A and $A + 1'$ or between A and $A + 60''$. For instance, if the angle is 25° 15′ 37″, it lies between 25° 15′, which is represented by A, and 25° 16′, which is 25° 15′ + 1′, or $A + 1'$, or $A + 60''$. In this case s represents 37″. From the principle stated above we have,

$$\frac{(A + 60'') - A}{(A + s'') - A} = \frac{\sin (A + 60'') - \sin A}{\sin (A + s'') - \sin A}$$

or,

$$\frac{60}{s} = \frac{\sin (A + 1') - \sin A}{\sin (A + s'') - \sin A}$$

whence, solving this equation for $\sin (A + s'')$,

$$\sin (A + s'') = \sin A + [\sin (A + 1') - \sin A]\frac{s}{60} \qquad (1)$$

Similarly,

$$\tan (A + s'') = \tan A + [\tan (A + 1') - \tan A]\frac{s}{60} \qquad (2)$$

For the cosine, we have

$$\cos (A + s'') = \cos A + [\cos (A + 1') - \cos A]\frac{s}{60}$$

but, since the cosine of an angle decreases as the angle increases, $\cos A$ is greater than $\cos (A + 1')$, and therefore it is better to write the formula thus,

$$\cos (A + s'') = \cos A - [\cos A - \cos (A + 1')]\frac{s}{60} \qquad (3)$$

Similarly,

$$\cot (A + s'') = \cot A - [\cot A - \cot (A + 1')]\frac{s}{60} \qquad (4)$$

The functions of A and $A + 1'$ can be readily taken from the table, as explained in the preceding articles, and from them the functions of $A + s''$ are determined by the formulas just given, or by the following rule, which states in words what the formulas express in symbols:

Rule.—*Find, in the table, the sine, cosine, tangent, or cotangent corresponding to the degrees and minutes in the angle.*

For the seconds, find the difference between this value and the value of the sine, cosine, tangent, or cotangent of an angle 1 minute greater; multiply this difference by a fraction whose numerator is the number of seconds in the given angle and whose denominator is 60.

If the sine or tangent is sought, add this correction to the value first found; if the cosine or cotangent is sought, subtract the correction.

EXAMPLE.—Find: (*a*) the sine of 56° 43′ 17″; (*b*) the cosine; (*c*) the tangent; and (*d*) the cotangent.

SOLUTION.—(*a*) Here $A = 56° 43'$, $s = 17$, $A + 1' = 56° 44'$.

$$\begin{aligned} \sin (A + 1') &= \sin 56° 44' = .83613 \\ \sin A &= \sin 56° 43' = .83597 \\ \text{Difference} &= .00016 \\ &\times \tfrac{17}{60} \\ &.00005, \text{ nearly} \end{aligned}$$

Adding this product to sin A, we have

$$\sin 56° 43' 17'' = .83597 + .00005 = .83602. \quad \text{Ans.}$$

(*b*)
$$\begin{aligned} \cos A &= \cos 56° 43' = .54878 \\ \cos (A + 1') &= \cos 56° 44' = .54854 \\ \text{Difference} &= .00024 \\ &\times \tfrac{17}{60} \\ &.00007, \text{ nearly} \end{aligned}$$

Subtracting this product from cos A, we have

$$\cos 56° 43' 17'' = .54878 - .00007 = .54871. \quad \text{Ans.}$$

(*c*)
$$\begin{aligned} \tan (A + 1') &= \tan 56° 44' = 1.52429 \\ \tan A &= \tan 56° 43' = 1.52332 \\ \text{Difference} &= .00097 \\ &\times \tfrac{17}{60} \\ &.00027, \text{ nearly} \end{aligned}$$

Adding this product to tan A, we have

$$\tan 56^\circ\ 43'\ 17'' = 1.52332 + .00027 = 1.52359. \quad \text{Ans.}$$

(*d*)
$$\cot A = \cot 56^\circ\ 43' = .65646$$
$$\cot (A + 1') = \cot 56^\circ\ 44' = .65604$$
$$\text{Difference} = .00042$$
$$\times \frac{17}{60}$$
$$.00012, \text{ nearly}$$

Subtracting this product from cot A, we have

$$\cot 56^\circ\ 43'\ 17'' = .65646 - .00012 = .65634. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

Verify the following values:

(*a*) sin 18° 54′ 45″ = .32412; tan 18° 54′ 45″ = .34262.
(*b*) cos 34° 17′ 18″ = .82621; cot 34° 17′ 18″ = 1.46659.
(*c*) sin 72° 26′ 20″ = .95340; cot 72° 26′ 20″ = .31647.
(*d*) cos 65° 6′ 9″ = .42100; tan 65° 6′ 9″ = 2.15457.
(*e*) sin 80° 0′ 3″ = .98481; cot 80° 0′ 3″ = .17631.
(*f*) tan 14° 14′ 14″ = .25373; cos 14° 14′ 14″ = .96928.

21. To Find the Angle Corresponding to a Given Function, When the Function Is in the Table.—This case does not present any difficulty. Having found the given function in the table, the degrees in the angle are taken from the top or the bottom, and the minutes from the left- or the right-hand column, according as the name of the function is at the top or at the bottom of the page.

EXAMPLE 1.—The sine of an angle is .47486; what is the angle?

SOLUTION.—Glancing down the columns marked Sine in the table of Natural Sines and Cosines, .47486 is found (on page 28) in the column headed 28°. The number of minutes, 21, is found in the left-hand minute column, horizontally opposite .47486. Therefore, .47486 = sin 28° 21′. Ans.

EXAMPLE 2.—Find the angle whose cosine is .27032.

SOLUTION.—Looking in the columns marked Cosine at the top of the page, the given cosine is not found; hence, the angle is greater than 45°. Consequently, looking in the columns marked Cosine at the bottom of the page, .27032 is found (on page 26) in the double column marked 74° at the bottom, and in the horizontal row beginning with 19 in the right-hand minute column. Therefore, the angle whose cosine is .27032 is 74° 19′; or, .27032 = cos 74° 19′. Ans.

EXAMPLE 3.—Find the angle whose tangent is 2.15925.

SOLUTION.—On searching the table of Natural Tangents, the given tangent is found to belong to an angle greater than 45°, so that it must be looked for in the column marked Tangent at the bottom. It is found in the column having 65° at the bottom and opposite 9′ in the right-hand minute column. Therefore, 2.15925 = tan 65° 9′. Ans.

EXAMPLE 4.—Find the angle whose cotangent is .43412.

SOLUTION.—From the table of Natural Cotangents, it is found that this value is less than the cotangent of 45°, so it must be found in the column marked Cotangent at the bottom. Looking there, it is found in the column having 66° at the bottom, and opposite 32′, in the right-hand column of minutes. Therefore, the angle whose cotangent is .43412 is 66° 32′, or .43412 = cot 66° 32′. Ans.

EXAMPLES FOR PRACTICE

1.	Find the angle whose sine is .47486.	Ans. 28° 21′
2.	Find the angle whose cosine is .74353.	Ans. 41° 58′
3.	Find the angle whose tangent is 2.06247.	Ans. 64° 8′
4.	Find the angle whose cotangent is 1.20665.	Ans. 39° 39′
5.	Find the angle whose sine is .76903.	Ans. 50° 16′
6.	Find the angle whose tangent is 9.93101.	Ans. 84° 15′

22. To Find the Angle Corresponding to a Given Function, When the Function Is Not in the Table. Since the table includes the functions of all angles containing no odd seconds, a function not found in the table must correspond to an angle having odd seconds. Let the odd seconds that are to be determined be denoted by s, and the degrees and minutes by A, as in Art. **20.** Now, two consecutive functions including the given function can always be found in the table; that is, two consecutive functions of which one is greater and the other less than the given function. The required angle must, therefore, lie between the two angles corresponding to these two consecutive functions, and its number of degrees and minutes, A, is the number of degrees and minutes in the smaller of the two angles. The larger angle is $A + 1'$, or $A + 60''$, while the required angle is $A + s''$. Having determined A, it only remains to determine the number of odd seconds, or s. This is done by means of

the following formulas, obtained by solving for s the formulas found in Art. **20.**

If the given function is a sine or tangent,

$$s = \frac{\sin (A + s'') - \sin A}{\sin (A + 1') - \sin A} \times 60 \qquad (1)$$

$$s = \frac{\tan (A + s'') - \tan A}{\tan (A + 1') - \tan A} \times 60 \qquad (2)$$

If the given function is a cosine or cotangent,

$$s = \frac{\cos A - \cos (A + s'')}{\cos A - \cos (A + 1')} \times 60 \qquad (3)$$

$$s = \frac{\cot A - \cot (A + s'')}{\cot A - \cot (A + 1')} \times 60 \qquad (4)$$

Observe that, although $A + s''$ is not known, its sine, cosine, etc., as the case may be, is known, or given. Thus, if the problem is to find the angle whose cotangent is .97888, we have cot $(A + s'')$ = .97888.

The foregoing formulas lead to the following general rule for finding the angle corresponding to a given function:

Rule.—*Find the difference of the two numbers in the table between which the given function lies, and use that difference as the denominator of a fraction.*

Find the difference between the function belonging to the smaller angle and the given function, and use that difference as the numerator of the fraction mentioned above. Multiply this fraction by 60. The result will be the number of seconds to be added to the smaller angle in order to obtain the required angle.

EXAMPLE 1.—Find the angle whose sine is .57698.

SOLUTION.—Looking in the table of Natural Sines, in the columns marked Sine, it is found that the given sine lies between .57691 (= sin 35° 14′) and .57715(= sin 35° 15′). The difference between them is .57715 − .57691 = .00024. The difference between the sine of the smaller angle, or .57691, and the given sine, or .57698, is .57698 − .57691 = .00007. Then, $\frac{.00007}{.00024} \times 60 = \frac{7}{24} \times 60 = 18''$, nearly, and the required angle is 35° 14′ 18″; or, .57698 = sin 35° 14′ 18″. Ans.

NOTE.—In practice, only the significant figures of the differences forming the terms of the function are used, the decimal point being dispensed with. Thus, .57715 − .57691 = 24, it being understood that this means 24 units of the fifth decimal order, or .00024.

EXAMPLE 2.—Find the angle whose cosine is .27052.

SOLUTION.—Looking in the table of Cosines, the given cosine is found to belong to a greater angle than 45° and therefore it must be looked for in the columns marked Cosine at the bottom of the page. It is found between the numbers .27060 (= cos 74° 18′) and .27032 (= cos 74° 19′). The difference between the two numbers is .27060 − .27032 = 28 units of the fifth order. The cosine of the smaller angle, or 74° 18′, is .27060, and the difference between this and the given cosine is .27060 − .27052 = 8 units of the fifth order. Hence, $\frac{8}{28} \times 60 = 17''$; and, therefore, .27052 = cos 74° 18′ 17″. Ans.

EXAMPLE 3.—Find the angle whose tangent is 2.15841.

SOLUTION.— 2.15841 falls between 2.15760 (= tan 65° 08′) and 2.15925 (= tan 65° 9′). The difference between these numbers is 2.15925 − 2.15760 = 165 units of the fifth order; 2.15841 − 2.15760 = 81 units of the fifth order. Hence, $\frac{81}{165} \times 60 = 30''$, nearly, and therefore 2.15841 = tan 65° 8′ 30″. Ans.

EXAMPLE 4.—Find the angle whose cotangent is 1.26342.

SOLUTION.— 1.26342 falls between 1.26395 (= cot 38° 21′) and 1.26319 (= cot 38° 22′). The difference between these numbers is 1.26395 − 1.26319 = .00076. Also, 1.26395 − 1.26342 = .00053. $\frac{53}{76} \times 60 = 42''$, and therefore 1.26342 = cot 38° 21′ 42″. Ans.

EXAMPLES FOR PRACTICE

1. Find: (*a*) the sine of 48° 17′; (*b*) the cosine; (*c*) the tangent.

Ans. (*a*) .74644; (*b*) .66545; (*c*) 1.12172

2. Find: (*a*) the sine of 13° 11′ 6″; (*b*) the cosine; (*c*) the tangent.

Ans. (*a*) .22810; (*b*) .97364; (*c*) .23427

3. Find: (*a*) the sine of 72° 0′ 2″; (*b*) the cosine; (*c*) the tangent.

Ans. (*a*) .95106; (*b*) .30901; (*c*) 3.07778

4. (*a*) Of what angle is .26489 the sine? (*b*) Of what angle is it the cosine?

Ans. (*a*) 15° 21′ 37″; (*b*) 74° 38′ 23″

5. (*a*) Of what angle is .688 the sine? (*b*) Of what angle is it the cosine? (*c*) Of what angle is it the tangent?

Ans. (*a*) 43° 28′ 20″; (*b*) 46° 31′ 40″; (*c*) 34° 31′ 40″

TABLE OF LOGARITHMIC FUNCTIONS

23. The student is already familiar with the use of the table of logarithms of numbers. As stated in Art. **17**, a **table of logarithmic functions** is a table containing the logarithms of the natural functions, these logarithms being, for convenience, called **logarithmic functions.** Thus, the logarithm of the sine of an angle is referred to as the **logarithmic sine** of the angle.

The connection between the tables can be seen from the following:

From table of natural functions, cot 44° . . . . . = 1.03553
From table of logarithms, log 1.03553 . . . . . = .01516
From table of logarithmic functions, log cot 44° = .01516

Few tables give the logarithmic secants and cosecants. These logarithmic functions may be obtained from the relations,

$$\sec A = \frac{1}{\cos A}, \csc A = \frac{1}{\sin A}$$

which give,

$$\log \sec A = -\log \cos A, \log \csc A = -\log \sin A$$

That is, instead of adding the logarithmic secant or cosecant, the logarithmic cosine or sine, respectively, may be subtracted. Likewise, instead of subtracting the logarithmic secant, the logarithmic cosine may be added, and instead of subtracting the logarithmic cosecant, the logarithmic sine may be added.

24. Description of the Table.—The table of logarithmic functions contains for every minute the logarithms, to five decimal places, of the trigonometric sines, cosines, tangents, and cotangents of angles from 0° to 90°. From 0° to 45°, the degrees are placed at the top of the page and the minutes in the column headed ′ on the left. From 45° to 90°, the degrees are at the bottom of the page, the minutes in the last whole column at the right, and the name of the trigonometric function is placed at the bottom of the column.

This arrangement is similar to that in the table of natural functions. It will be observed that the numbers of degrees at the top of the pages increase in the order of the pages from 0° to 44°, while those at the bottom decrease from 89° to 44°.

The general description of the table will be better understood by referring to one of its pages. Take, for instance, the page marked 11° at the top and 78° at the bottom. The first column on the left (marked ′) contains the natural numbers from 1 to 60. These numbers represent minutes. Horizontally opposite to these numbers, and in the columns marked at the top log sin, log tan, etc., are printed the logarithmic functions, each function being in the same horizontal line as the number of minutes by which the corresponding angle exceeds 11°. Thus, the logarithmic tangent of 11° 39′, which is $\bar{1}.31425$, is found in the column marked log tan at the top, and in the same horizontal line as the number 39 in the left-hand column. Similarly, the number $\bar{1}.99072$, being in the column marked at the top log cos, and in the same horizontal line as 48 in the left-hand column, is the logarithmic cosine of 11° 48′. In some tables, several mantissas are printed under and to the right of the same characteristic, and are understood to belong with that characteristic. Thus, in the logarithm just considered, only the mantissa .99072 is printed, the characteristic being the same as the first one found above that mantissa.

The last column but one (marked ′ at the bottom) contains the natural numbers from 1 to 60, increasing from bottom to top. It will be observed that any angle determined by the number of degrees at the bottom (78 in this case) and any number of minutes in the right-hand minute column, is the complement of the angle determined by the number of degrees at the top (11 in this case) and the number of minutes in the left-hand minute column, horizontally opposite the number of minutes in the right-hand minute column. Thus, the number 18 in the right-hand minute column is horizontally opposite the number 42 in the left-hand column, and we have, $78° 18′ + 11° 42′ = 90°$. Therefore,

since the fundamental functions of an angle are equal to the cofunctions of its complement,

$$\sin 11° 42' = \cos 78° 18'$$
$$\cot 11° 42' = \tan 78° 18', \text{ etc.}$$

and $$\log \sin 11° 42' = \log \cos 78° 18', \text{ etc.}$$

For this reason, the notation log tan is written at the bottom of the column headed log cot, to indicate that the logarithms in this column are the logarithmic tangents of angles whose number of degrees is the number (78 in this case) at the bottom of the page, and whose number of minutes is opposite those logarithms in the right-hand minute column. Similarly, the columns marked log sin, log tan, and log cos at the top are marked, respectively, log cos, log cot, and log sin at the bottom.

25. After the column marked log sin there is a column marked d. This column contains the differences, expressed in units of the fifth decimal order, between the consecutive logarithmic sines given in the sine column. Thus, referring to the page headed 11°, the first number in the d-column following the sine column is 65; it will be observed that this number is opposite the space between the logarithmic sines $\bar{1}.28125$ and $\bar{1}.28060$, and is the difference, in units of the fifth decimal order, or expressed in hundred thousandths, between these two logarithmic sines. These differences are called **tabular differences.** Similar differences are printed in the column marked d after the cosine column, and in the column marked c. d. between the tangent and the cotangent column. The notation c. d. means *common difference*, as the differences between the successive logarithmic tangents are the same as those between the corresponding cotangents, although obtained by reversing the order in which the functions are subtracted; that is to say, $\log \tan A - \log \tan B = \log \cot B - \log \cot A$.

The tabular differences for the cosines are not given in the first ten pages, both for want of space and because they are so small that they can be readily determined by mental subtraction.

The use of the tabular differences, the use and contents of the column marked p. p. in all pages but the first three, and the peculiarities and applications of these first three pages of the table will be explained further on.

26. To Find the Logarithmic Functions of an Angle Having No Odd Seconds.

Rule.—*For an angle less than 45°, look for the degrees at the top of the page and for the minutes in the column (marked ′) at the left of the page on which the number of degrees is found. Then look across the page along the horizontal row containing the given number of minutes, into the column headed by the name of the function whose logarithm is required. The desired logarithm is found in this row and column.*

For an angle between 45° and 90°, find the degrees at the bottom of the page and the minutes in the column (marked ′) at the right of the page. Then look across the page, along the horizontal row containing the given number of minutes, into the column marked at the bottom with the name of the function whose logarithm is to be found. The row and column thus determined contain the desired logarithm.

Example 1.—Find the logarithmic sine and the logarithmic tangent of 15° 24′.

Solution.—On the page marked 15° at the top, in the column headed log sin, and in the same horizontal row with 24, the number $\bar{1}.42416$ is found; and in the column headed log tan, the number $\bar{1}.44004$ is found. Hence,

$$\log \sin 15° 24' = \bar{1}.42416. \quad \text{Ans.}$$
$$\log \tan 15° 24' = \bar{1}.44004. \quad \text{Ans.}$$

Example 2.—Find the logarithmic tangent and cosine of 73° 10′.

Solution.—As 73 is greater than 45, it is found at the bottom of the page. Looking for the number of minutes (10′) in the right-hand minute column, and following the horizontal row determined by this number into the column marked log tan at the bottom, the number .51920 is found. Likewise, the number $\bar{1}.46178$ is found in the column marked log cos at the bottom, and horizontally opposite the number 10 in the right-hand minute column. Therefore,

$$\log \tan 73° 10' = .51920. \quad \text{Ans.}$$
$$\log \cos 73° 10' = \bar{1}.46178. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Find: (*a*) the logarithmic cosine of 36° 58′; (*b*) the logarithmic tangent.

Ans. { (*a*) $\bar{1}.90254$ (*b*) $\bar{1}.87659$

2. Find: (*a*) the logarithmic tangent of 23° 39′; (*b*) the logarithmic cotangent.

Ans. { (*a*) $\bar{1}.64140$ (*b*) .35860

3. Find: (*a*) the logarithmic sine of 79° 45′; (*b*) the logarithmic cosine.

Ans. { (*a*) $\bar{1}.99301$ (*b*) $\bar{1}.25028$

4. Find: (*a*) the logarithmic tangent of 46° 59′; (*b*) the logarithmic cotangent.

Ans. { (*a*) .03009 (*b*) $\bar{1}.96991$

27. To Find the Logarithmic Functions of an Angle Containing an Odd Number of Seconds.—Let the number of degrees and minutes in an angle any of whose logarithmic functions is required be denoted by A, and the number of odd seconds by s. Thus, if the angle is 37° 43′ 19″, A will equal 37° 43′, and s will equal 19″; also, $A + 1'$, or $A + 60''$, will equal 37° 43′ + 1′, or 37° 44′. (See Art. **20.**) Since the table gives the logarithmic functions of any angle containing no odd seconds, the logarithmic functions of A and $A + 1'$ may be readily found, as explained in the last article. Let these logarithmic functions be denoted by l and l', respectively, and the required logarithmic function by L. In the general theory of logarithms, treated in advanced works on mathematics, it is shown that if two consecutive angles (as 37° 43′ and 37° 44′) are taken from the table, the difference between any logarithmic function of the greater and the same logarithmic function of the smaller angle is to the difference between the same logarithmic function of any intermediate angle (as 37° 43′ 19″) and the same function of the smaller angle, as the difference between the greater and the smaller angle is to the difference between the intermediate and the smaller angle. If the notation $F(A)$, read *function of A*, is employed to denote any logarithmic function of an angle A, we have, writing $A + 60''$ instead of $A + 1'$,

$$\frac{F(A+60'')-F(A)}{F(A+s)-F(A)}=\frac{(A+60'')-A}{(A+s)-A}=\frac{60}{s}$$

that is,
$$\frac{l'-l}{L-l}=\frac{60}{s}$$

whence,
$$L-l=(l'-l)\frac{s}{60}$$

and
$$L=l+(l'-l)\frac{s}{60}$$

The difference between l' and l, being the difference between two consecutive logarithmic functions, may be taken from the column of tabular differences in the table. (See Art. **25.**) Denoting the tabular difference $l'-l$ by D, the preceding equation becomes

$$L=l+D\times\frac{s}{60}$$

It should be observed that, since the sine and the tangent increase with the angle, while the cosine and cotangent decrease as the angle increases, $l'-l$ is positive or negative according as the functions considered are fundamental functions (sine, tangent) or cofunctions (cosine, cotangent). In the latter case, D in the formula should be treated as negative; that is, the product $D\times\frac{s}{60}$ should be subtracted from l.

It should also be borne in mind that the tabular difference D is expressed in units of the fifth order of decimals, or hundred thousandths. Thus, if the number of seconds s is 15, and the tabular difference is 36, the quantity to be added to l is $.00036\times\frac{15}{60}=.00009$.

If $l=\bar{1}.59812$, the work is arranged as follows:

$$l=\bar{1}.59812$$
$$D\times\frac{s}{60}=\quad 9$$
$$L=\bar{1}.59821$$

When, as in this case, the product $D\times\frac{s}{60}$ is small, it can readily be added or subtracted mentally. Only the significant figures of D (those given in the d-column) are used, it being understood that the result expresses units of

the fifth order of decimals. Thus, instead of writing $D = .00036$, and $D \times \frac{s}{60} = .00036 \times \frac{s}{60}$, the following abbreviated notation is used: $D = 36$; $D \times \frac{s}{60} = 36 \times \frac{s}{60}$, the latter product expressing decimal units of the fifth order, or hundred thousandths.

The foregoing formula indicates the process by which the logarithmic functions of an angle containing odd seconds are obtained. It may be stated in words as follows:

Rule.—*Drop the seconds, and find the logarithmic function of the remaining angle. Find the tabular difference between this logarithmic function and the same function of the angle next higher in the table. Multiply this tabular difference by the number of seconds in the angle and divide the product by 60. Add this result to or subtract it from the logarithm found, according as the logarithm to be determined is that of a fundamental function or that of a cofunction. The result thus obtained is the required logarithmic function.*

EXAMPLE 1.—Find: (*a*) the logarithmic sine of 15° 40′ 32″; (*b*) the logarithmic cosine.

SOLUTION.—(*a*) Dropping the seconds, 15° 40′ is obtained, whose logarithmic sine, found as in Art. **25**, is $\bar{1}.43143$; that is, $l = \bar{1}.43143$. Opposite the space between this logarithm and the following, and in the column marked d, is found the tabular difference 45($= D$). Applying the formula given in Art. **27**,

$$L = \bar{1}.43143 + .00045 \times \tfrac{32}{60}$$

$$l = \bar{1}.43143$$

$$D \times \frac{s}{60} = 45 \times \frac{32}{60} = \quad 24$$

$$L = \bar{1}.43167$$

that is, $\log \sin 15° 40' 32'' = \bar{1}.43167$. Ans.

In practice, it is not necessary to write all the figures of l before adding the correction $D \times \frac{s}{60}$. Having found the value of l in the table, one places and keeps the finger on that value and calculates the correction $D \times \frac{s}{60}$. In the majority of cases, this correction can be added mentally to l. Thus, in the example just explained, the correction is 24, which, being mentally added to the number 43 formed by the

last two figures of l, gives 67 as the last two figures of L. The other figures of L are the same as those of l.

(*b*) The logarithmic cosine of 15° 40′ is $\bar{1}.98356 (= l)$. Horizontally opposite the space between this logarithm and the following, the tabular difference 4 $(= D)$ is found in the column marked d on the right of the cosine column. As the function under consideration is a cofunction, the correction $D \times \frac{s}{60}$ must be subtracted for l. We have, then,

$$l = \bar{1}.98356$$

$$D \times \frac{s}{60} = 4 \times \frac{32}{60} = \qquad 2, \text{ to the nearest unit}$$

$$L = \bar{1}.98354$$

Therefore, $\log \cos 15° 40' 32'' = \bar{1}.98354$. Ans.

In practice, the correction 2 would be subtracted mentally, without previously writing the value of l.

EXAMPLE 2.—Find the logarithmic tangent of 63° 39′ 27″.

SOLUTION.—Dropping the seconds, and referring to the page marked 63° at the bottom, the logarithmic tangent of 63° 39′ is found to be .30512 $(= l)$. Since in this case the angles increase from bottom to top, the tabular difference to be used is that horizontally opposite the space between the logarithm just taken and the one immediately above it in the column (that is, .30543). This difference is 31, printed in the column marked c. d. on the left of the cotangent column. We have, therefore,

$$l = .30512$$

$$\frac{s}{60} \times D = \frac{27}{60} \times 31 = \qquad 14, \text{ to the nearest unit}$$

$$L = .30526$$

Therefore, $\log \tan 63° 39' 27'' = .30526$. Ans.

EXAMPLE 3.—Find the logarithmic cotangent of 54° 8′ 9″.

SOLUTION.—Dropping the seconds, the value of l is found to be $\bar{1}.85913$. The tabular difference in the c. d. column and horizontally opposite the space between this logarithm and the one immediately above it is 26. As the cotangent is a cofunction, the correction $\frac{s}{60} \times D$ is to be subtracted from l. Then,

$$l = \bar{1}.85913$$

$$\frac{s}{60} \times D = \frac{9}{60} \times 26 = \qquad 4$$

$$L = \bar{1}.85909$$

Therefore, $\log \cot 54° 8' 9'' = \bar{1}.85909$. Ans.

EXAMPLES FOR PRACTICE

1. Find the logarithmic sine, tangent, and cosine of 33° 21′ 46″.

Ans. $\begin{cases} \log \sin = \bar{1}.74032 \\ \log \tan = \bar{1}.81852 \\ \log \cos = \bar{1}.92179 \end{cases}$

2. Find the logarithmic sine and cotangent of 23° 3′ 17″.

Ans. $\begin{cases} \log \sin = \bar{1}.59286 \\ \log \cot = .37100 \end{cases}$

3. Find the logarithmic tangent and cosine of 49° 12′ 12″.

Ans. $\begin{cases} \log \tan = .06395 \\ \log \cos = \bar{1}.81516 \end{cases}$

4. Find the logarithmic sine, tangent, and cosine of 72° 52′ 49″.

Ans. $\begin{cases} \log \sin = \bar{1}.98031 \\ \log \tan = .51143 \\ \log \cos = \bar{1}.46890 \end{cases}$

5. Find the logarithmic sine and cotangent of 81° 38′ 28″.

Ans. $\begin{cases} \log \sin = \bar{1}.99536 \\ \log \cot = \bar{1}.16712 \end{cases}$

6. Find the logarithmic tangent and cosine of 65° 0′ 47″.

Ans. $\begin{cases} \log \tan = .33159 \\ \log \cos = \bar{1}.62574 \end{cases}$

7. Find the logarithmic secant and cosecant of 59° 0′ 9″.

Ans. $\begin{cases} \log \sec = .28819 \\ \log \csc = .06692 \end{cases}$

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28. Use of the Column of Proportional Parts.—The method described in the preceding article can be applied to any table of logarithmic functions. Some tables, however, among them the table furnished with this Course, contain a column giving the products of the tabular differences by the fractions $\frac{6}{60}$, $\frac{7}{60}$, $\frac{8}{60}$, $\frac{9}{60}$, $\frac{10}{60}$, $\frac{20}{60}$, $\frac{30}{60}$, $\frac{40}{60}$, and $\frac{50}{60}$. These products are called **proportional parts,** and are given in the right-hand column (marked p. p. at the top) of each page, beginning with 3°. The tabular differences are here printed in heavy figures. Under each tabular difference are given the products of it by $\frac{6}{60}$, $\frac{7}{60}$, etc., the number of sixtieths being printed horizontally opposite the product, on the left of a vertical line. Thus, referring to the right-hand column of the page marked 13° at the top, the numbers 54, 53, 52, printed in heavy type, are tabular differences. The number 27, directly under 54, and horizontally opposite the

number 30 on the left of the vertical line, is the product of 54 by $\frac{30}{60}$. Likewise, 17.3, found under 52, and horizontally opposite 20, is the product of 52 by $\frac{20}{60}$. The proportional parts for 1, 2, 3, 4, 5 are obtained from those for 10, 20, 30, etc., by moving the decimal point one place to the left. Thus, the proportional part for 20, under the tabular difference 52, is 17.3, as just explained. The proportional part for 2, that is, the product of 52 by $\frac{2}{60}$, is 1.73.

In the first three pages of the logarithmic table, no proportional parts are given, the use of these pages being different from that of the others. In pages 45, 46, and 47, not all the tabular differences are given in the p. p. column, owing to want of space; but the proportional part for any tabular difference is easily obtained by means of the proportional parts for digits given at the bottom of the p. p. column. Referring, for example, to page 45, the tabular difference 215, which is found in the c. d. column, does not appear in the p. p. column. If we wish to find the product of 215 by $\frac{30}{60}$, we look in the p. p. column for the tabular difference next lower than 215, which is 212. Horizontally opposite 30, and under 212, we find 106; that is, $212 \times \frac{30}{60} = 106$. As $215 = 212 + 3$, we must add to the product just found (106), the product of $3 \times \frac{30}{60}$. This is taken from the column headed 3 near the bottom of the p. p. column: there we find 1.5 horizontally opposite 30; that is, $3 \times \frac{30}{60} = 1.5$. Therefore, $215 \times \frac{30}{60} = 106 + 1.5 = 107.5$. The addition of these two products can usually be effected mentally.

The correction $D \times \frac{s}{60}$ to be applied to l in order to find L (formula of Art. **27**) is found from the table of proportional parts as follows:

Rule.—*Having found the tabular difference D, look for this difference in the column of proportional parts. If this difference is found in that column and the number of seconds is a digit greater than 5 or a digit followed by a cipher, look for it on the left of the vertical line under D; the correction is then found horizontally opposite this number, and directly under D. If the*

number of seconds is a digit less than 6, add a cipher, find the proportional part corresponding to the resulting number, and move the decimal point one place to the left. If the number of seconds consists of two significant digits (as 39), find the correction for the first digit followed by a cipher, and that for the second digit, and add the two corrections. (Thus, if the number of seconds is 43, the correction is found by adding the corrections for 40 and 3.)

If the tabular difference D is not found in the p. p. column (which may happen only on pages 45 to 47), take, as just explained, the proportional part corresponding to the next lower tabular difference found in the p. p. column; then, from the digit columns found at the bottom of the p. p. column, find the proportional part corresponding to the difference between D and the tabular difference just used. Add the two proportional parts thus found.

EXAMPLE 1.—Find: (*a*) the logarithmic tangent of 22° 17′ 8″; (*b*) the logarithmic cosine.

SOLUTION.—(*a*) Dropping the seconds, we find log tan 22° 17′ = $\bar{1}.61256 (= l)$; $D = 36$. Turning to the column of proportional parts, 36 is found in heavy type near the top of the page. Following the horizontal row that begins with 8 (number of seconds) at the left of the vertical line under 36, we find in that row, and directly under 36, the correction 4.8, which may be called 5, as there are no other numbers to be combined with it. Therefore,

$$l = \bar{1}.61256$$

$$\frac{s}{60} \times D = \text{p. p.} = \quad 5$$

$$L = \bar{1}.61261$$

That is, log tan 22° 17′ 8″ = $\bar{1}.61261$. Ans.

(*b*) $l = \log \cos 22° 17' = \bar{1}.96629$; $D = 5$. Looking for the column headed 5 among the proportional parts, the correction .7 (or say 1) is found directly under 5 and horizontally opposite 8. Therefore,

$$l = \bar{1}.96629$$

$$\frac{s}{60} \times D = \text{p. p.} = \quad 1$$

$$L = \bar{1}.96628$$

That is, log cos 22° 17′ 8″ = $\bar{1}.96628$. Ans.

EXAMPLE 2.—Find the logarithmic sine of 3° 18′ 9″.

SOLUTION.— $l = \sin 3° 18' = \bar{2}.76015$; $D = 219$. The difference 219 is not found in the p. p. column; the tabular difference in the p. p. column next lower is 216. Under 216, and horizontally opposite 9, is

found 32.4. The difference between 219 and 216 is 3. Looking for 3 in the digit columns at the bottom of the p. p. column, .5 is found under 3, and horizontally opposite 9. Therefore, $219 \times \frac{9}{60} = 32.4 + .5 = 33$, nearly.

$$l = \bar{2}.76015$$
$$219 \times \frac{9}{60} = \quad 33$$
$$L = \bar{2}.76048$$

That is, $\log 3^\circ\, 18'\, 9'' = \bar{2}.76048$. Ans.

Example 3.—Find: (*a*) the logarithmic tangent of $53^\circ\, 47'\, 04''$; (*b*) the logarithmic cosine.

Solution.—(*a*) $l = \log \tan 53^\circ\, 47' = .13529$; $D = 26$; the proportional part for 40, under D, that is, under 26, is 17.3; the proportional part for 4 is $\frac{17.3}{10}$, or 2, nearly.

$$l = .13529$$
$$26 \times \frac{4}{60} = \quad 2$$
$$L = .13531$$

That is, $\log \tan 53^\circ\, 47'\, 4'' = .13531$. Ans.

(*b*) $l = \log \cos 53^\circ\, 47' = \bar{1}.77147$; $D = 17$. The number horizontally opposite 40, in the column headed 17 among the proportional parts, is 11.3; the proportional part for 4 is, therefore, $\frac{11.3}{10} = 1$, nearly.

$$l = \bar{1}.77147$$
$$17 \times \frac{4}{60} = \quad 1$$
$$L = \bar{1}.77146$$

That is, $\log \cos 53^\circ\, 47'\, 4'' = \bar{1}.77146$. Ans.

Example 4.—To find the logarithmic cotangent of $72^\circ\, 35'\, 47''$.

Solution.— $l = \log \cot 72^\circ\, 35' = \bar{1}.49652$; $D = 45$. Looking among the proportional parts for the column headed 45, the correction for 40 is found to be 30, and that for 7 is found to be 5.3. Therefore,

$$l = \bar{1}.49652$$
$$\text{p. p. for } 40 = 30.0$$
$$\text{p. p. for } 7 = \underline{5.3}$$
$$\text{p. p. for } 47 = \quad 35$$
$$L = \bar{1}.49617$$

That is, $\log \cot 72^\circ\, 35'\, 47'' = \bar{1}.49617$. Ans.

In practice, it would not be necessary to write down the corrections 30 and 5.3, which would be added mentally. The same remark applies to all similar cases.

EXAMPLES FOR PRACTICE

1. Find the logarithmic sine and cotangent of 9° 39′ 17″.

Ans. $\begin{cases} \log \sin = \bar{1}.22456 \\ \log \cot = .76924 \end{cases}$

2. Find the logarithmic sine, tangent, and cosine of 39° 8′ 52″.

Ans. $\begin{cases} \log \sin = \bar{1}.80025 \\ \log \tan = \bar{1}.91065 \\ \log \cos = \bar{1}.88959 \end{cases}$

3. Find the logarithmic cotangent and cosecant of 80° 3′ 46″.

Ans. $\begin{cases} \log \cot = \bar{1}.24352 \\ \log \csc = .00657 \end{cases}$

4. Find the logarithmic sine, secant, and tangent of 49° 0′ 54″.

Ans. $\begin{cases} \log \sin = \bar{1}.87788 \\ \log \sec = .18319 \\ \log \tan = .06197 \end{cases}$

5. Find the logarithmic tangent and cosine of 4° 2′ 4″.

Ans. $\begin{cases} \log \tan = \bar{2}.84838 \\ \log \cos = \bar{1}.99892 \end{cases}$

29. To Find the Angle Corresponding to Any Logarithmic Function When the Given Function Is Found in the Table.—In this case, the angle, which contains no odd seconds, is found as follows:

Rule.—*Find the given logarithm in the column marked by the name of the function whose logarithm is given. Then, if the name of the given function is at the top of the column, the number of degrees in the angle is that at the top of the page, and the number of minutes is horizontally opposite the logarithm, in the left-hand minute column. If the name of the function is at the foot of the column, the number of degrees in the angle is that at the foot of the page, and the number of minutes is in the right-hand minute column, horizontally opposite the given logarithm.*

In searching the table for a given logarithm, it should be borne in mind that the logarithmic sines and tangents increase, and the cosines and cotangents decrease, from 0° to 90°. Therefore, in the columns marked log sin and log tan at the top, the logarithms increase, and in the columns headed log cos and log cot the logarithms decrease, from the first to the last page. The sines and tangents continue to

increase, and the cosines and cotangents to decrease, from the last page to the first, in the columns marked with the names of these functions, respectively, at the bottom. Thus, the last page contains, in the column headed log sin, the logarithmic sines of the angles between 44° and 45°. The sines are continued in the column marked log sin at the bottom, which contains the logarithmic sines of the angles between 45° and 46°; the preceding page contains the sines of angles between 46° and 47°, etc. Here the logarithmic sines increase from bottom to top, and in the inverse order of the pages.

When looking for a given logarithmic sine, open the table at random. Glance at both of the sine columns, that is, the column marked log sin at the top and the column marked log sin at the bottom, and compare the logarithms in them with the given logarithm. If the given logarithm is less than those found in the column marked log sin at the top, said given logarithm must be in that column, but in a preceding page. If the given logarithm is greater than those in the column marked log sin at the bottom, said given logarithm must be in that column, but in a preceding page. If neither of these is the case, the given logarithm must be in a subsequent page. Turn a few pages forwards or backwards, as the case may be, and repeat the operation. The comparison of the two columns, however, is not usually necessary after the first three figures of the given logarithm have been found in one of them, as that logarithm is then found in that column, and can be readily seen among the logarithms beginning with those three figures.

Proceed exactly in the same manner when the given function is a cosine; that is, treat the cosine as though it were a sine; but, having found the given logarithm, treat it as that of a cosine and take the angle accordingly.

As the tangents of angles less than 45° are less than 1, their logarithmic tangents have negative characteristics, and as the tangents of angles greater than 45° are greater than 1, their logarithmic tangents have positive characteristics. Therefore, a logarithmic tangent should be looked for in the

column marked log tan at the top or at the bottom, according as its characteristic is negative or positive. For a logarithmic cotangent, the rule should be reversed.

EXAMPLE 1.—Find the angle whose logarithmic sine is $\bar{1}.57669$.

SOLUTION.—Opening the table at random, say at the page marked 36° at the top, it is at once seen that the logarithms in the column marked log sin at the top are greater than the given logarithm. This logarithm must, therefore, be in that column, but in a preceding page. Turning the pages backwards, a few at a time, the given logarithm is found on page 64, among those logarithms whose first three figures are $\bar{1}.57$. As the name of the function is at the head of the column, the number of degrees (22) is taken from the top of the page, and that of minutes (10) from the left-hand minute column. Therefore, the angle whose logarithmic sine is $\bar{1}.57669$ is 22° 10′, or $\bar{1}.57669 = \log \sin 22° 10'$.

Suppose that the table had first been opened at page 56. Since the given logarithm is greater than those in the column marked log sin at the top and less than those in the column marked log sin at the bottom (or log cos at the top), the given logarithm is to be found in a subsequent page. Suppose also that, turning the pages forwards, a few at a time, we come to page 63, and find the first three figures ($\bar{1}.57$) of the given logarithm in the column marked log sin at the top. Then, without consulting the other column, we follow the former column to the bottom, and into the next page, where we find the given logarithm, and take the corresponding angle as before.

EXAMPLE 2.—To find the angle whose logarithmic sine is $\bar{1}.89810$.

SOLUTION.—Open the table at random, say at page 73. Since the given logarithm is greater than those in the column marked log sin at the top, and less than those in the column marked log sin at the bottom, it must be found in a subsequent page. Suppose that we turn next to page 85. We see at once that the given logarithm is greater than those in the column headed log sin, and also than those in the column marked log sin at the bottom. Therefore, it must be in the latter column in some preceding page. Turning the pages backwards, we find the first three figures ($\bar{1}.89$) of the given logarithm on page 79, and among the logarithms to which these three figures are common, we find $\bar{1}.89810$. As this is a logarithmic sine, and the name sine is at the bottom of the column, the degrees in the corresponding angle are taken from the bottom of the page, and the minutes from the right-hand minute column. Therefore, 52° 16′ is the angle whose logarithmic sine is $\bar{1}.89810$; that is, $\bar{1}.89810 = \log \sin 52° 16'$. Ans.

EXAMPLE 3.—Find the angle whose logarithmic cosine is $\bar{1}.86924$.

SOLUTION.—Treating this as though it were a logarithmic sine, it is found, as explained above, on page 34, in the column marked log sin

at the bottom. Since the name cosine is at the top of the column, the required angle is 42° 16′. That is, $\bar{1}.86924 = \log \cos 42° 16'$. Ans.

EXAMPLE 4.—Find the angle whose logarithmic cotangent is .15639.

SOLUTION.—As the characteristic is positive, the logarithm should be looked for in the column marked log cot at the top. After looking in a few pages, the first three figures (0.15) of the logarithm are found on page 76, and among them is found the given logarithm. The name of the function being at the head of the column, the degrees in the angle are taken from the top of the page, and the minutes from the left-hand minute column. Therefore, $.15639 = \log \cot 34° 54'$. Ans.

EXAMPLES FOR PRACTICE

1. Find the angle whose logarithmic sine is $\bar{1}.57885$. Ans. 22° 17′
2. Find the angle whose logarithmic sine is $\bar{1}.66731$. Ans. 27° 42′
3. Find the angle whose logarithmic sine is $\bar{2}.93740$. Ans. 4° 58′
4. Find the angle whose logarithmic sine is $\bar{1}.98345$. Ans. 74° 17′
5. Find the angle whose logarithmic cosine is $\bar{1}.92086$. Ans. 33° 33′
6. Find the angle whose logarithmic cosine is $\bar{1}.57232$. Ans. 68° 4′
7. Find the angle whose logarithmic cosine is $\bar{1}.84949$. Ans. 45° 0′
8. Find the angle whose logarithmic tangent is $\bar{1}.97649$. Ans. 43° 27′
9. Find the angle whose logarithmic cotangent is $\bar{2}.89274$. Ans. 85° 32′
10. Find the angle whose logarithmic tangent is .67377. Ans. 78° 2′
11. Find the angle whose logarithmic cotangent is .35517. Ans. 23° 49′
12. Find the angle whose logarithmic tangent is 1.28060. Ans. 87° 0′

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30. To Find the Angle Corresponding to a Given Logarithmic Function When the Function Is Not in the Table.—*Without the Use of Proportional Parts.*—From the formula given in Art. **27**, the following may be obtained:

$$s = \frac{(L - l) \times 60}{D} = \frac{(L - l) \times 60}{l' - l}$$

Therefore, if the function L is given and it is found to lie between the consecutive logarithms l and l', the corresponding angle $A + s$ is that corresponding to l increased by the number of seconds determined by the formula just given. It will be remembered (see Art. **27**) that l and l' are, respectively, the logarithmic functions of two angles (A and $A + 1'$) differing by one minute. If the function is a fundamental function (sine or tangent) l' is greater than l; and since L lies between l and l', L is also greater than l; therefore, both $L - l$ and $l' - l$ are positive. If the function is a cofunction, l is greater than l', and also greater than L; therefore, both $L - l$ and $l' - l$ are negative, and $\frac{L - l}{l' - l}$ is positive. In such case, however, it is better to write this fraction in the form $\frac{l - L}{l - l'}$.

From the formula and the explanations just given, the following rule is derived for finding the angle corresponding to any given logarithmic function:

Rule.—*Find in the table the two consecutive logarithmic functions between which the given function lies. The degrees and minutes in the smaller of the angles corresponding to these two functions are the degrees and minutes in the required angle.*

Find the difference between the given function and that of the smaller angle; multiply that difference by 60, and divide the product by the tabular difference between the two functions in the table. The result will be the number of odd seconds in the required angle.

As the tabular difference is expressed in units of the fifth decimal order, the difference $L - l$ should be likewise expressed. Thus, if $L = \bar{1}.25198$, and $l = \bar{1}.25168$, the difference $L - l$ will be called 30.

EXAMPLE 1.—Find the angle whose logarithmic sine is $\bar{1}.47867$ $(= L)$.

SOLUTION.—The first three figures of the given logarithm are always found in the table, and this makes it easy to determine the functions between which the given logarithm lies. Searching the sine columns of the table, it is found that $\bar{1}.47867$ lies between $\bar{1}.47854 (= l)$ and

$\bar{1}.47894 (= l')$ on page 59. The smaller of the two angles corresponding to these two logarithms is $17° 31' (= A)$. Now, $L - l = 13$, $l' - l$ (tabular difference taken from table) $= 40$. Therefore,

$$s = \frac{13 \times 60}{40} = 19.5'', \text{ or, say, } 20''$$

and $A + s = 17° 31' + 20'' = 17° 31' 20''$

that is, $\bar{1}.47867 = \log \sin 17° 31' 20''$. Ans.

EXAMPLE 2.—Find the angle whose logarithmic tangent is .27743 $(= L)$.

SOLUTION.—As the characteristic is positive, the logarithms between which L lies should be looked for in the column marked log tan at the bottom. These two logarithms are $.27738 (= l)$ and $.27769 (= l')$. The smaller angle corresponds to .27738, and is $62° 10' (= A)$. Also,

$$L - l = 5,\ l' - l (= D) = 31$$

$$A + s = A + \frac{5 \times 60}{31} = 62° 10' + 10'', \text{ nearly, } = 62° 10' 10''$$

that is, $.27743 = \log \tan 62° 10' 10''$. Ans.

EXAMPLE 3.—Find the angle whose logarithmic cotangent is $\bar{1}.85899 (= L)$.

SOLUTION.— L is found to lie between $\bar{1}.85887 (= l')$ and $\bar{1}.85913 (= l)$. It will be noticed that here l is the greater, and l' the smaller of the two logarithms. Angle corresponding to $l = 54° 8' (= A)$.

$$l = \bar{1}.85913$$
$$L = \bar{1}.85899$$
$$l - L = 14;\ l - l' = 26$$

$$A + s = 54° 8' + \frac{14 \times 60}{26} = 54° 8' 32'', \text{ nearly}$$

that is, $\bar{1}.85899 = \log \cot 54° 8' 32''$. Ans.

EXAMPLES FOR PRACTICE

1. Find the angle whose logarithmic sine is $\bar{1}.45566$.
   Ans. $16° 35' 27''$
2. Find the angle whose logarithmic tangent is $\bar{1}.33471$.
   Ans. $12° 11' 44''$
3. Find the angle whose logarithmic sine is $\bar{1}.89798$.
   Ans. $52° 14' 42''$
4. Find the angle whose logarithmic cosine is $\bar{1}.67412$.
   Ans. $61° 49' 23''$
5. Find the angle whose logarithmic cosine is $\bar{1}.92386$.
   Ans. $32° 56' 45''$

6. Find the angle whose logarithmic cotangent is .54139.
Ans. 16° 2′ 20″

7. Find the angle whose logarithmic tangent is $\bar{1}$.86712.
Ans. 36° 22′ 7″

8. Find the angle whose logarithmic cosine is $\bar{1}$.99785.
Ans. 5° 42′ 0″

9. Find the angle whose logarithmic cotangent is $\bar{1}$.12345.
Ans. 82° 25′ 52″

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31. *With the Use of Proportional Parts.*—Having found the degrees and minutes in the angle as in the preceding case, the number s of odd seconds may be conveniently found from the column of proportional parts. In order to facilitate the explanations that follow, the proportional parts corresponding to the tabular difference 105 are here copied from page 48 of the table. It will, therefore, be assumed that the value of D is 105, and, for what is said below, the student should refer to these proportional parts. Such being the case, the formula given at the beginning of the preceding article may be written,

	105
6	10.5
7	12.3
8	14.0
9	15.8
10	17.5
20	35.0
30	52.5
40	70.0
50	87.5

$$s = \frac{(L - l) \times 60}{105}$$

The value $L - l$, which is the difference between the given logarithm and the logarithm of the degrees and minutes (A) in the required angle, is readily determined, as already explained. It is only necessary to repeat that, if the function is a cofunction, $l - L$ should be used instead of $L - l$. Since the numbers on the right of the vertical line are the products of $\frac{105}{60}$ by the numbers on the left, it follows that the numbers on the left are the products of those on the right by $\frac{60}{105}$. Thus, $52.5 = \frac{105}{60} \times 30$, and $30 = 52.5 \times \frac{60}{105} = \frac{52.5 \times 60}{105}$. Therefore, if $L - l$ is found among the numbers directly under 105, the value of s is the number on the left of the vertical column horizontally opposite $L - l$. For example, if $L - l = 35$, then $s = 20''$. If $L - l = 16$, then

$s = 9''$, the number 9 being opposite 15.8, which, to the nearest unit, may be called 16.

It will be remembered that the proportional parts opposite 10, 20, 30, 40, 50, when divided by 10 (that is, when the period is moved one place to the left), give the products of $\frac{10.5}{60}$ by 1, 2, 3, 4, and 5. From those parts we may, therefore, find by inspection the products of $\frac{10.5}{60}$ by all the digits from 1 to 9; and, in what follows, we shall proceed as if the products 1.75, 3.50, 5.25, 7.00, 8.75 of $\frac{10.5}{60}$ by 1, 2, 3, 4, and 5 were actually printed in the table opposite those digits; that is, it will be assumed that the proportional parts run in this order: 1.75, 3.50, 5.25, 7.00, 8.75, 10.5, 12.3, 14.0, etc., up to 87.5, the corresponding numbers on the left being, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 20, 30, 40, 50. The proportional parts 1.75, 3.50, 5.25, 7.00, 8.75 will be referred to as proportional parts found in the table, corresponding to 1, 2, 3, 4, and 5 seconds, respectively.

This being understood, the number s of odd seconds in the angle is determined as follows:

Rule.—*Find* $l, l', L - l$, and $l' - l$ (= *tabular difference, or* D), *as before. Look for the tabular difference* D *in the column of proportional parts. Look for* $L - l$ *in the column of proportional parts directly under* D. *If* $L - l$ *is found there, the number horizontally opposite it on the left of the vertical line is the required number of seconds* s. *If* $L - l$ *is not found under* D, *take the proportional part next lower, which call* p. *Find the difference between* $L - l$ *and* p, *and look among the proportional parts under* D *for this difference, or the part nearest to it, whether higher or lower. Call this part* p'. *Add the numbers horizontally opposite* p *and* p' *on the left of the vertical line. The result will be the required number of seconds* s.

Example 1.—Find the angle whose logarithmic tangent is $\bar{1}.42822 (= L)$.

Solution.— $l = 1.42805$, $A = 15° 0'$, $L - l = 17$, $D = 51$. Looking in the column marked p. p. for 51, the number $17 (= L - l)$ is found under it, horizontally opposite the number 20 on the left of the vertical column. Therefore, $s = 20''$, and

$\bar{1}.42822 = \log \tan 15° 0' 20''$. **Ans.**

EXAMPLE 2.—Find the angle whose logarithmic cosine is $\bar{1}.52783 (= L)$.

SOLUTION.— $l = \bar{1}.52811$, $A = 70° 17'$, $l - L = 28$, $D = 36$. The proportional part under 36 next lower than 28 is 24; $28 - 24 = 4$; the proportional part nearest 4 is 4.2; the number horizontally opposite 24 is 40; and the number horizontally opposite 4.2 is 7; hence, $s = 40 + 7 = 47''$, and therefore

$$\bar{1}.52783 = \log \cos 70° 17' 47''. \quad \text{Ans.}$$

EXAMPLE 3.—Find the angle whose logarithmic sine is $\bar{1}.66191 (L)$.

SOLUTION.— $l = \bar{1}.66173$; $A = 27° 19'$; $L - l = 18$; $D = 24$. Looking in the p. p. column for 24, the proportional part next lower than 18 is $16 (= p)$, horizontally opposite which is 40. $18 - p = 18 - 16 = 2$. This difference is found among the proportional parts in the table (since it is the same as 20 with the decimal point moved one place to the left), and corresponds to $5''\left(= \frac{50}{10}\right)$. Therefore, $s = 40 + 5 = 45''$, and

$$\bar{1}.66191 = \log \sin 27° 19' 45''. \quad \text{Ans.}$$

EXAMPLE 4.—Find the angle whose logarithmic cotangent is $\bar{1}.00375 (= L)$.

SOLUTION.— $l = \bar{1}.00427$; $A = 84° 14'$; $l - L = 52$; $D = 126$. The proportional part under 126 next lower than 52 is $42 (= p)$, which corresponds to $20''$; $52 - 42 = 10$. The proportional part nearest to 10 is $10.50\left(= \frac{105.0}{10}\right)$, which corresponds to $5''\left(= \frac{50}{10}\right)$. Therefore, $s = 20'' + 5'' = 25''$, and

$$\bar{1}.00375 = \log \cot 84° 14' 25''. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Find the angle whose logarithmic sine is $\bar{1}.78988$. Ans. $38° 3' 20''$
2. Find the angle whose logarithmic tangent is $\bar{1}.78540$. Ans. $31° 23' 15''$
3. Find the angle whose logarithmic sine is $\bar{1}.77777$. Ans. $36° 49' 56''$
4. Find the angle whose logarithmic cosine is $\bar{1}.87341$. Ans. $41° 39' 21''$
5. Find the angle whose logarithmic cotangent is .31789. Ans. $25° 41' 9''$
6. Find the angle whose logarithmic cosine is $\bar{1}.34567$. Ans. $77° 11' 38''$

7. Find the angle whose logarithmic cotangent is $\bar{1}.00381$.
Ans. 84° 14′ 22″

8. Find the angle whose logarithmic tangent is 1.00300.
Ans. 84° 19′ 42″

9. Find the angle whose logarithmic sine is $\bar{2}.99001$.
Ans. 5° 36′ 30″

32. Tabular Values Increased by 10.—To avoid calculating with negative characteristics, they may be made positive by increasing them by 10. Thus, log sin 27° may be given as 9.65705 instead of $\bar{1}.65705$. The true logarithm is, therefore, $9.65705-10$; the -10 is usually not written, but is implied. In many books this method is used for the logarithms of trigonometric functions. In applying such logarithms to the solution of a problem, the characteristic in the final result must be corrected to agree with the conditions of the problem.

GENERAL PRINCIPLE OF INTERPOLATION

33. It has been explained in some of the preceding articles how to determine the natural or the logarithmic functions of any angle containing an odd number of seconds, and therefore, not found in the table; also, how to find the angle corresponding to a given function, when that function is not in the table but lies between two values given in the table. The operation by which such intermediate values are determined from a table is called **interpolation.** The values that are actually given in the table are called **tabular values.** For example, in the table of logarithmic functions already described are found all angles that lie between 0° and 90° and contain no odd seconds, and also the logarithmic sines, cosines, etc. of such angles; those are all tabular values. Angles containing odd seconds are not in the table, nor are their logarithmic functions. Both these angles and their functions are intermediate values, and it is in connection with them that interpolation is used.

34. The general principle of interpolation, to be explained presently, is of the utmost importance, and of great value to the engineer, whose work requires the frequent use of tables of various kinds. That principle, although only approximately true, applies to nearly all tables with which the engineer has to deal, and the student should endeavor to make himself thoroughly familiar with it.

Let a table be constructed on the general type shown on the margin, the left-hand column containing values of a quantity X, and the right-hand column corresponding values of some quantity whose values depend on the values of X. Thus, the values of X may be the natural numbers 1, 2, 3, 4, etc., and the corresponding values of F may be the logarithms or the square roots of those numbers; or the values of X may be angles, and those of F may be sines, cosines, etc., either natural or logarithmic. So far as the principle of interpolation is concerned, it is immaterial what kind of quantity is represented by X, and what kind of quantity is tabulated under F. It should be stated, however, that the principle applies only to tables in which the differences between consecutive values of X and the differences between the corresponding values of F do not vary very rapidly.

X	F
—	—
—	—
—	—
x_1	f_1
x	f
x_2	f_2
—	—

Let x_1 and x_2, as shown in the above general form, be two consecutive values of X given in the table, and f_1 and f_2 the corresponding values of F. Let x be a value of X lying between x_1 and x_2, and f the corresponding value of F. Neither x nor f is in the table, but one of them is given, and the problem is to find the other by interpolation. For instance, if the table is one of natural tangents in which the angles increase by whole minutes, x_1 and x_2 may be, respectively, 31° 42′ and 31° 43′, and f_1 and f_2 their corresponding tangents; while x may be any angle between 31° 42′ and 31° 43′, and f its tangent. Either x may be given to find f; or f may be given to find x.

The quantity by which the tabular value x_1 must be algebraically increased in order to obtain x will be called the **increment** of x_1, and denoted by $i(x_1)$, read *increment of* x_1 (mathematicians use the notation Δx_1, read *delta* x_1). We have, then,

$$x = x_1 + i(x_1) \qquad (1)$$

Using a similar notation for f_1,

$$f = f_1 + i(f_1) \qquad (2)$$

If x is given, $i(x_1)$ may be assumed as given, since $i(x_1) = x - x_1$. Then $i(f_1)$ is determined by interpolation, as explained below, and f is found from formula 2. Similarly, if f is given, $i(f_1)$ is likewise given, and x is found by interpolation.

The difference, as $x_2 - x_1$, of two consecutive values of X, will be called the **interval** of X; and that between two consecutive values of F, the interval of F. The notation $I(x_1)$, read *interval of* x_1, will be used to denote the interval $x_2 - x_1$. Similarly, $I(f_1)$ will denote the interval $f_2 - f_1$.

The principle of interpolation is this: *The increments* $i(x_1)$ *and* $i(f_1)$ *are to each other as the corresponding intervals* $I(x_1)$ *and* $I(f_1)$; *or, algebraically,*

$$\frac{i(x_1)}{i(f_1)} = \frac{I(x_1)}{I(f_1)} \qquad (3)$$

This formula is very easily remembered on account of its symmetry. The following, derived from it, serve, respectively, to find $i(f_1)$ when x is given, and $i(x_1)$ when f is given:

$$i(f_1) = I(f_1) \times \frac{i(x_1)}{I(x_1)} \qquad (4)$$

$$i(x_1) = I(x_1) \times \frac{i(f_1)}{I(f_1)} \qquad (5)$$

The last two formulas may be stated in the form of a general principle, as follows: *Either increment is equal to the corresponding interval multiplied by the ratio of the other increment to the other interval.* It is easy to remember what the numerator of this ratio is, by noticing that the ratio is

always less than 1, and that, since the increment is always less than the interval, the former must be the numerator and the latter the denominator. It should be noted that $i(x_1)$, $i(f_1)$, $I(x_1)$, and $I(f_1)$ may be expressed in any convenient units, it being understood that $i(f_1)$, as determined from formula **4**, is in the same units as $I(f_1)$; and that $i(x_1)$, as determined from formula **5**, is in the same units as $I(x_1)$. Thus, if the values of f_2 and f_1 in the table are, respectively, 4.3476 and 4.3463, then, $I(f_1) = f_2 - f_1 = .0013$, or, if one ten-thousandth is taken as the unit, we may write $I(f_1) = 13$. The value of $i(f_1)$, determined from formula **4**, must be understood to express ten-thousandths. For instance, if $\frac{i(x_1)}{I(x_1)} = .3$, then, $i(f_1) = 13 \times .3 = 3.9$ (ten-thousandths) $= 4$ (ten-thousandths), nearly.

The value of f is then found thus,

$$\begin{aligned} f_1 &= 4.3463 \\ i(f_1) &= \quad\ 4 \\ f &= 4.3467 \end{aligned}$$

Usually, the correction $i(f_1)$ can be added to f_1 mentally, in order to find f.

EXAMPLE 1.—Find the logarithm of 57,846 by means of a five-place table giving the logarithms of numbers consisting of four figures.

SOLUTION.—Only the mantissas will be considered, since the characteristics are determined by inspection. The given number lies between 57,840 $(= x_1)$ and 57,850 $(= x_2)$, whose logarithms are, respectively, .76223 $(= f_1)$ and .76230 $(= f_2)$. We have, therefore, expressing $f_2 - f_1$, or $I(f_1)$, in units of the fifth order,

$$\begin{aligned} x &= 57846 & f_2 &= .76230 \\ x_1 &= 57840 & f_1 &= .76223 \\ i(x_1) &= \quad 6 & I(f_1) &= \quad\ 7 \end{aligned}$$

$$I(x_1) = x_2 - x_1 = 10$$

Then (formula **4**),

$$i(f_1) = 7 \times \frac{6}{10} = 4.2 = 4, \text{ nearly}$$

and

$$f = \begin{cases} f_1 \\ + i(f_1) \end{cases} = \begin{cases} .76223 \\ \quad +4 \end{cases} = .76227. \quad \text{Ans.}$$

EXAMPLE 2.—Find, by means of a five-place table, the number the mantissa of whose logarithm is .47693.

SOLUTION.—Here $f(= .47693)$ lies between the tabular values $.47683(= f_1)$ and $.47698(= f_2)$, which are, respectively, the logarithms of $29{,}980(= x_1)$ and $29{,}990(= x_2)$. We have, then,

$$\begin{aligned} f_2 &= .47698 \\ f &= .47693 \\ f_1 &= .47683 \\ I(f_1) = f_2 - f_1 &= 15 \\ i(f_1) = f - f_1 &= 10 \end{aligned} \qquad \begin{aligned} x_2 &= 29{,}990 \\ x_1 &= 29{,}980 \\ I(x_1) &= 10 \end{aligned}$$

Then (formula 5),

$$i(x_1) = 10 \times \frac{10}{15} = 7, \text{ nearly}$$

and $$x = x_1 + i(x_1) = 29{,}980 + 7 = 29{,}987. \text{ Ans.}$$

This gives the significant figures of the number. The decimal point should be placed according to the characteristic of the given logarithm.

EXAMPLE 3.—Find the angle whose natural tangent is $.56781(= f)$ by means of a table giving the natural tangents of angles varying by minutes.

SOLUTION.—Here f is found to lie between $.56769(= \tan 29^\circ 35' = f_1)$ and $.56808(= \tan 29^\circ 36' = f_2)$. Expressing $x_2 - x_1$, or $I(x_1)$, in seconds, we have

$$\begin{aligned} x_2 &= 29^\circ\ 36' \\ x_1 &= 29^\circ\ 35' \\ I(x_1) &= 60'' \end{aligned} \qquad \begin{aligned} f_2 &= .56808 \\ f &= .56781 \\ f_1 &= .56769 \\ I(f_1) &= 39 \\ i(f_1) &= 12 \end{aligned}$$

Then (formula 5),

$$i(x_1) = 60'' \times \frac{12}{39} = 18'', \text{ nearly}$$

and $$x = x_1 + i(x_1) = 29^\circ\ 35'\ 18''. \text{ Ans.}$$

EXAMPLE 4.—In Searles' field book is given a table of lengths of arcs for different degrees of curvature. Part of it is as follows (lengths in feet):

Degree of Curve $(=X)$	Length of Arc for One Station $(=F)$
10° 10′	100.131
10° 20′	100.136
10° 30′	100.140

Find the length of the arc between two stations for a 10° 26′ curve

SOLUTION.—Here we have, $x = 10° 26'$, which lies between $10° 20'$ $(=x_1)$ and $10° 30'(= x_2)$. Expressing $I(x_1)$ and $i(x_1)$ in minutes, and $I(f_1)$ and $i(f_1)$ in thousandths, we have $I(x_1) = 10$, $i(x_1) = 6$, $I(f_1) = 140 - 136 = 4$.

Therefore (formula 4),

$$i(f_1) = 4 \times \frac{6}{10} = 2, \text{ nearly}$$

and $$f = f_1 + i(f_1) = \left\{\begin{matrix}100.136\\+2\end{matrix}\right\} = 100.138. \quad \text{Ans.}$$

In all simple cases like this the operations can be performed mentally and very rapidly.

EXAMPLES FOR PRACTICE

1. From the following table, find, by interpolation, the cube root of 347.3 and that of 349.7.

Number	Cube Root
347	7.0271
348	7.0338
349	7.0406
350	7.0473

Ans. $\left\{\begin{matrix}7.0291\\7.0453\end{matrix}\right.$

2. Find, from the following table, the diameter of a circle whose circumference is 63.57318.

Diameter	Circumference
20.1	63.14601
20.2	63.46017
20.3	63.77433

Ans. 20.236

SOLUTION OF RIGHT TRIANGLES

35. Fundamental Equations.—Let $A\ B\ C$, Fig. 9, be a right triangle, in which A, B, and C are the angles and a, b, and c are the lengths of the sides, c being the hypotenuse. Since A and B are complementary angles, we have

$$\sin A = \cos B \qquad \tan A = \cot B$$
$$\cos A = \sin B \qquad \cot A = \tan B$$

Also, from the definitions of the trigonometric functions, $\sin A = \frac{a}{c}$, $\tan A = \frac{a}{b}$, $\cos B = \sin A = \frac{a}{c}$, $\cot A = \frac{b}{a}$; whence, expressing the value of a from each of these equations,

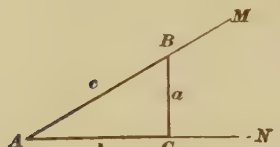

Fig. 9

$$a = c \sin A \qquad (1)$$
$$a = b \tan A \qquad (2)$$
$$a = c \cos B \qquad (3)$$
$$a = b \cot B \qquad (4)$$

From formulas **1** and **3**, the following values are found for c:

$$c = \frac{a}{\sin A} = a \csc A \qquad (5)$$

$$c = \frac{a}{\cos B} = a \sec B \qquad (6)$$

Finally, from geometry,

$$c^2 = a^2 + b^2 \qquad (7)$$

Of the trigonometric formulas just given, it is only necessary to commit to memory formulas **1** and **2**, as the others are immediate consequences of these. These two formulas may be stated in words thus:

Either leg of a right triangle is equal to the hypotenuse multiplied by the sine, or to the other leg multiplied by the tangent, of the opposite angle.

It should be observed that, since a is either leg whose opposite angle is A, and adjacent angle B, the letters a and b may be interchanged in the preceding formulas, provided that A and B are likewise interchanged. Thus, by interchanging a and b, A and B in formulas **1** and **5**, we obtain,

$$b = c \sin B, \; c = \frac{b}{\sin B} = b \csc B$$

36. Solution of a Right Triangle.—In general, when some of the parts of a triangle are given, the process of determining the others is called **solving the triangle**, or the **solution of the triangle**. The latter expression is applied also to the triangle determined in accordance with the given data.

In order to solve a right triangle, two parts, one at least of which should be a side, must be known in addition to the right angle. The two parts may be either (1) one side and one of the acute angles, or (2) two sides.

37. Case I.—*Given a Side and an Acute Angle.* The other acute angle is found from the relation $A + B = 90°$, and the other two sides by means of formulas **1** to **7**, Art. **35**, as illustrated by the following examples:

EXAMPLE 1.—In Fig. 10, the length of the hypotenuse $A\,B$ of the right triangle $A\,C\,B$, right-angled at C, is 24 feet, and the angle A is 29° 31′; find the sides $A\,C$ and $B\,C$, and the angle B.

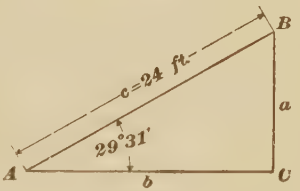

FIG. 10

NOTE.—When working examples of this kind, make a sketch and mark the known parts, as shown in the figure.

SOLUTION WITHOUT LOGARITHMS.— $B = 90° - A = 90° - 29°\,31′ = 60°\,29′$. By formula **3**, Art. **35**, interchanging a and b, and A and B,

$b = c \cos A = 24 \cos 29°\,31′ = 24 \times .87021 = 20.89$ ft., nearly.

By formula **1**, Art **35**,

$a = 24 \sin 29°\,31′ = 24 \times .49268 = 11.82$ ft., nearly.

Ans. $\begin{cases} B = 60°\,29′ \\ A\,C = 20.89 \text{ ft.} \\ B\,C = 11.82 \text{ ft.} \end{cases}$

SOLUTION BY LOGARITHMS.—By formulas **3** and **1**, Art. **35**,

$$b = 24 \cos 29° 31' \quad (1)$$
$$a = 24 \sin 29° 31' \quad (2)$$

LOGARITHMS FOR (1)	LOGARITHMS FOR (2)
$\log 24 = 1.38021$	$\log 24 = 1.38021$
$\log \cos 29° 31' = \bar{1}.93963$	$\log \sin 29° 31' = \bar{1}.69256$
$\log b = 1.31984$	$\log a = \overline{1.07277}$
$b = 20.89$	$a = 11.82$

In working examples of this kind, the two logarithmic functions should be taken from the table at the same time. It saves time and space to arrange the operations as follows:

$$\log a = \overline{1.07277};\ a = 11.82$$
$$\log \sin 29° 31' = \bar{1}.69256$$
$$\log 24 = 1.38021$$
$$\log \cos 29° 31' = \bar{1}.93963$$
$$\log b = \overline{1.31984};\ b = 20.89. \quad \text{Ans.}$$

The logarithm of 24 is written first, and then the logarithms of the sine and cosine, one over, the other under, log 24, the addition being performed upwards in one case and downwards in the other.

EXAMPLE 2.—One leg of a right triangle $A\ C\ B$, Fig. 11, is 37 feet 7 inches long; the angle opposite is 25° 33′ 7″; what are the lengths of the hypotenuse and the side adjacent, and what is the other angle?

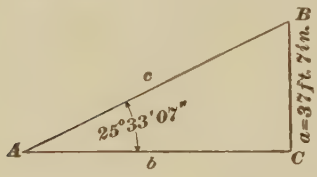

FIG. 11

SOLUTION WITHOUT LOGARITHMS. $B = 90° - 25° 33' 07'' = 64° 26' 53''$. Reducing 37 ft. 7 in. to ft., we have, $a = 37.583$ ft., nearly.

By formula **5**, Art. **35**,

$$c = \frac{37.583}{\sin 25° 33' 07''} = \frac{37.583}{.43133} = 87.133 \text{ ft., nearly.}$$

By formula **4**, Art. **35**, interchanging a and b, and A and B,

$$b = a \cot A = 37.583 \times 2.09166 = 78.611 \text{ ft., nearly.}$$

Ans. $\begin{cases} B = 64° 26' 53'' \\ A\ C = 78.611 \\ A\ B = 87.133 \text{ ft.} \end{cases}$

SOLUTION BY LOGARITHMS.—As before,

$$c = \frac{37.583}{\sin 25° 33' 7''}$$

Also,

$$b = 37.583 \cot 25° 33' 7''$$
$$\log b = \overline{1.89548};\ b = A\ C = 78.611 \text{ ft.}$$
$$\log \cot 25° 33' 7'' = \overline{.32049}$$
$$\log 37.583 = 1.57499$$
$$\log \sin 25° 33' 7'' = \bar{1}.63483$$
$$\log c = \overline{1.94018};\ c = A\ B = 87.132 \text{ ft.} \quad \textbf{Ans.}$$

It is to be noted that the value of A B given by logarithms is different in the fifth figure from the result given by natural functions. This is due to the fact that in using five-place tables the results can be depended on to be correct to only four figures, and to have a very close approximation to the fifth figure.

NOTE.—In the majority of cases, the solution by logarithms is far more expeditious than the solution by natural functions. The student is strongly advised to form the habit of solving all trigonometric problems by means of logarithms and the logarithmic functions, whenever these functions can be used.

38. Case II.—*Given Two Sides.* If the given sides are the two legs a and b, A is found from formula **2**, Art. **35**, and B from the relation $A + B = 90°$. To find c, formula **7**, Art. **35**, may be used; but, unless a and b are convenient numbers to square, it is preferable to determine c by formula **5**, Art. **35**, after having determined A.

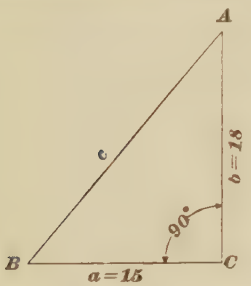

FIG. 12

If the given sides are the hypotenuse c and one leg, say a, the

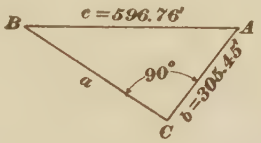

FIG. 13

angle A is found by formula **1**, Art. **35**, B from the relation $A + B = 90°$, and b from either formula **4**, or formula **7**, Art. **35**. The latter gives

$$b = \sqrt{c^2 - a^2}$$

Unless c and a are convenient numbers to square, the quantity under the radical should be replaced by the product $(c + a)\ (c - a)$, and then

$$\log b = \tfrac{1}{2}\ [\log\ (c + a) + \log\ (c - a]$$

from which b can be readily determined.

EXAMPLE 1.—Given a and b as shown in Fig. 12, to find A, B, and c

SOLUTION.—Formula **2**, Art. **35**,

$$\tan A = \frac{a}{b} = \frac{15}{18} = \frac{5}{6} = .83333$$

$$A = 39° 48' 20''$$

$$B = 90° - 39° 48' 20'' = 50° 11' 40''$$

Formula **5**, Art. **35**,

$$c = \frac{15}{\sin A} = \frac{15}{\sin 39^\circ\ 48'\ 20''}$$

$$\log 15 = 1.17609$$

$$\log \sin 39^\circ\ 48'\ 20'' = \bar{1}.80630$$

$$\log c = 1.36979;\quad c = 23.431$$

Otherwise,

$$c = \sqrt{15^2 + 18^2} = \sqrt{(3 \times 5)^2 + (3 \times 6)^2} = 3\sqrt{5^2 + 6^2} = 3\sqrt{61} = 23.431.$$

Ans.

EXAMPLE 2.—The hypotenuse c and the leg b having the values shown in Fig. 13, find the acute angles and the leg a.

SOLUTION.—By formula **3**, Art. **35**, interchanging a and b, A and B,

$$\cos A = \frac{b}{c} = \frac{305.45}{596.76}$$

$\log 305.45 = 2.48494$ $\qquad$ $90^\circ = 89^\circ\ 59'\ 60''$

$\log 596.76 = 2.77580$ $\qquad$ $A = 59^\circ\ 12'\ 46''$

$\log \cos A = \bar{1}.70914;\ A = 59^\circ\ 12'\ 46''$ $\qquad$ $B = 30^\circ\ 47'\ 14''$

Formula **2**, Art. **35**,

$$a = 305.45 \tan 59^\circ\ 12'\ 46''$$

$$\log 305.45 = 2.48494$$

$$\log \tan 59^\circ\ 12'\ 46'' = .22489$$

$$\log a = 2.70983;\quad a = 512.66$$

Otherwise, $\quad a = \sqrt{c^2 - b^2} = \sqrt{(c+b)(c-b)}$

$c + b = 902.21$ $\qquad$ $\log (c+b) = 2.95531$

$c = 596.76$ $\qquad$ $\log (c-b) = 2.46435$

$b = 305.45$ $\qquad$ $2)5.41966$

$c - b = 291.31$ $\qquad$ $\log a = 2.70983;\ a = 512.66$

Ans. $\begin{cases} A = 59^\circ\ 12'\ 46'' \\ B = 30^\circ\ 47'\ 14'' \\ a = 512.66 \text{ ft.} \end{cases}$

EXAMPLES FOR PRACTICE

1. In a right triangle $A\,C\,B$, right-angled at C (let the student make a sketch), the hypotenuse $A\,B = 40$ inches and angle $A = 28^\circ\ 14'\ 14''$; solve the triangle.

Ans. $\begin{cases} \text{Angle } B = 61^\circ\ 45'\ 46'' \\ A\,C = 35.239 \text{ in.} \\ B\,C = 18.925 \text{ in.} \end{cases}$

2. In a right triangle $A\,C\,B$, right-angled at C, the side $B\,C = 10$ feet 4 inches; if angle $A = 26^\circ\ 59'\ 6''$, what are the other parts?

Ans. $\begin{cases} \text{Angle } B = 63^\circ\ 0'\ 54'' \\ A\,B = 22 \text{ ft. } 9\frac{1}{4} \text{ in., nearly} \\ A\,C = 20 \text{ ft. } 3\frac{1}{2} \text{ in., nearly} \end{cases}$

3. In a right triangle ACB, the hypotenuse $AB = 60$ feet and the side $AC = 22$ feet; solve the triangle.

Ans. $\begin{cases} \text{Angle } A = 68^\circ\ 29'\ 22'' \\ \text{Angle } B = 21^\circ\ 30'\ 38'' \\ BC = 55.821 \text{ ft.} \end{cases}$

4. In a right triangle ACB, right-angled at C, side $AC = .364$ foot and side $BC = .216$ foot; solve the triangle.

Ans. $\begin{cases} \text{Angle } A = 30^\circ\ 41'\ 6'' \\ \text{Angle } B = 59^\circ\ 18'\ 54'' \\ AB = .423 \text{ ft.} \end{cases}$

PRACTICAL EXAMPLES

39. When an object is viewed by an observer, the object may be either above or below a horizontal plane passing through the observer's eye. The angle made with this plane by the line of sight, that is, by the line from the observer's eye to the object, is called an **angle of elevation** if the object is above that plane; an **angle of depression** if the object is below that plane. The object is said to be seen at an angle of elevation or at an angle of depression according as it is above or below the plane in question. For example, a lighthouse is seen from a ship at sea at an angle of elevation, while the ship is seen from the lighthouse at an angle of depression.

EXAMPLE 1.—The angle of elevation of the top of a vertical cliff, CB, Fig. 14, at a point 100 feet from its base, is 36° 50′; find the height of the cliff.

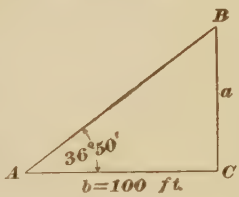

FIG. 14

SOLUTION.—By formula **2**, Art. **35**, required height $= a = 100 \times \tan 36^\circ 50' = 100 \times .74900 = 74.9$ ft. Ans.

EXAMPLE 2.—A statue is placed on the top of a column. At a point on the ground 130 feet from the base of the column, the angle of elevation of the top of the statue and that of the column are 43° 38′ and 40° 58′, respectively; find the height of the statue and column. (Let the student make a sketch.)

SOLUTION.—Let h = height of column;
h' = height of column and statue.

Then, $\tan 40^\circ 58' = \dfrac{h}{130}$

Whence, $h = 130 \times \tan 40^\circ 58' = 112.875$

Also, $\tan 43^\circ 38' = \dfrac{h'}{130}$

Whence, $h' = 130 \times \tan 43^\circ 38' = 123.942$

Therefore, the height of the column is 112.875 ft. Ans.

The height of the statue is $123.942 - 112.875 = 11.07$ ft. Ans.

EXAMPLE 3.—The top and bottom of a lighthouse $L\ T$, Fig. 15, located on a hill $M\ N$, are observed from a ship S with a sextant. It is found that the angles of elevation of T and L are, respectively, 6° 27′ and 5° 15′. If the height of the lighthouse is 128 feet, and the surface of the sea is assumed to be plane, what are: (a) the height $h(= C\ L)$ of the hill above sea level? (b) the horizontal distance $d(= S\ C)$ of the ship from the lighthouse?

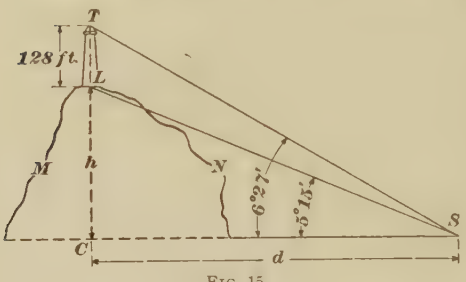

FIG. 15

SOLUTION.—(a) In the right triangles $L\ C\ S$ and $T\ C\ S$, we have

$$d = h \cot 5^\circ 15', \text{ and } d = (h + 128) \cot 6^\circ 27'$$

Equating the two values of d,

$$h \cot 5^\circ 15' = (h + 128) \cot 6^\circ 27'$$

whence,

$$h = \frac{128 \cot 6^\circ 27'}{\cot 5^\circ 15' - \cot 6^\circ 27'} = \frac{128 \times 8.84551}{10.8829 - 8.84551} = 555.72. \text{ Ans.}$$

(b) From (a),

$$d = h \cot 5^\circ 15' = 555.72 \cot 5^\circ 15' = 6{,}047.8 \text{ ft. Ans.}$$

EXAMPLE 4.—In Fig. 16, $P_1\ T_1$ is the track of a railroad that curves into a circular arc $T_1\ M\ T_2$ at T_1. The chord $T_1\ T_2$ of the whole arc is found, by measurement, to be 764.7 feet, and the chord $T_1\ M$ of half the arc, 393.2 feet. Find: (a) the external angle I between $P_2\ T_2$ and $P_1\ T_1$ produced; (b) the radius $r(= C\ T_1)$ of the curve $T_1\ M\ T_2$.

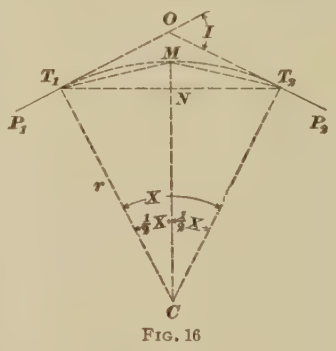

FIG. 16

SOLUTION.—(a) Draw $C\ T_1$, $C\ M$, and $C\ T_2$, as shown. Since $P_1\ O$ and $P_2\ O$ are tangent to the circle, the angles $O\ T_1\ C$ and $O\ T_2\ C$ are right angles; and as the sum of the angles in the quadrilateral $O\ T_1\ C\ T_2$ is four right angles, we must have $X + T_1\ O\ T_2 = 2$ right angles $= 180^\circ$; we have also, $I + T_1\ O\ T_2 = 180^\circ$; therefore, $I = X$. The line $C\ M$ bisects both the angle X and the chord $T_1\ T_2$. As the angle $M\ T_1\ T_2$ is measured by one-half the

arc $M T_2$, it is equal to one-half of $M C T_2$, or to $\frac{1}{4} X$. The right triangle $M T_1 N$ gives,

$$\cos \frac{1}{4} X \,(= \cos M T_1 N) = \frac{T_1 N}{T_1 M} = \frac{\frac{1}{2} T_1 T_2}{T_1 M} = \frac{382.35}{393.2}$$

whence, by either logarithms or natural functions (logarithms are far preferable in this case),

$\frac{1}{4} X = 13° 29' 20''$; $I (= X) = 4 \times 13° 29' 20'' = 53° 57' 20''$. Ans.

(b) In the right triangles $C T_1 N$,

$$r (= C T_1) = \frac{T_1 N}{\sin \frac{1}{2} X} = \frac{382.35}{\sin 26° 58' 40''} = 842.83 \text{ ft.} \quad \text{Ans.}$$

EXAMPLE 5.—Fig. 17 is a cross-section of a dam, the dimensions being as shown. The batter of the face $A B$ is 30 in 100, or .3. Find: (a) the width $w_1 (= A B)$ of the face; (b) the batter of the back $C D$; (c) the width $w_2 (= C D)$ of the back.

NOTE.—By the batter of one of the sides of an inclined wall is meant the rate at which that side deviates from the vertical. Thus, in Fig. 17, the side $B A$ deviates from the vertical by the amount $M N$ in the vertical distance $B N$, or by the amount $A P$ in the distance $B P$. Either of the ratios $\frac{M N}{B N}$ or $\frac{A P}{B P}$ expresses the batter of the wall. A batter of 30 in 100 is the same as $\frac{30}{100}$, or .3. It will be noticed that the batter is equal to the tangent of the inclination of the side of the wall to the vertical.

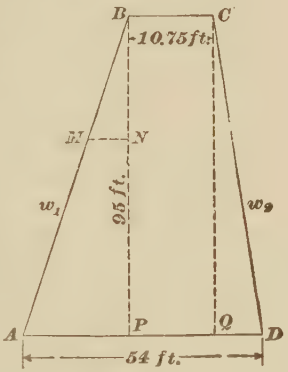

FIG. 17

SOLUTION.—(a) As just explained, tan $A B P$ = batter = .3; whence $A B P = 16° 41' 58''$. The triangle $A B P$ gives,

$$w_1 = \frac{B P}{\cos A B P} = \frac{95}{\cos 16° 41' 58''} = 99.182 \text{ ft.} \quad \text{Ans.}$$

(b) The triangle $A P B$ gives,

$$A P = P B \tan A B P = 95 \times .3 = 28.5 \text{ ft.}$$

From the figure,

$$Q D = A D - A P - P Q = 54 - 28.5 - 10.75 = 14.75 \text{ ft.}$$

The triangle $C Q D$ gives,

$$\text{batter of } C D = \tan Q C D = \frac{Q D}{Q C} = \frac{14.75}{95} = .15526$$

or, say, 15.5 in 100; also, $Q C D = 8° 49' 31''$. Ans.

(c) $$w_2 = \frac{C Q}{\cos Q C D} = \frac{95}{\cos 8° 49' 31''} = 96.139 \text{ ft.} \quad \text{Ans.}$$

EXAMPLE 6.—Fig. 18 represents a derrick; the dimensions being as shown, determine: (a) the inclination A of the boom $Q R$ to the vertical; (b) the inclination M of the rod $P R$ to the vertical; (c) the point U at which the guy rope $P U$ must be tied, that it may make an angle of 60° with the horizontal; (d) the length $P U$ of the guy rope.

SOLUTION.—(*a*) The triangle RQS gives,

$$\sin A = \frac{QS}{QR} = \frac{28}{42} = \frac{2}{3}; \text{ whence, } A = 41° 48' 38''. \quad \text{Ans.}$$

(*b*) The same triangle gives,

$$RS = \sqrt{(42 + 28)(42 - 28)} = \sqrt{70 \times 14} = 31.305$$

The triangle PTR gives,

$$RT = RS - ST = RS - QP = 31.305 - 11.5 = 19.805 \text{ ft.}$$

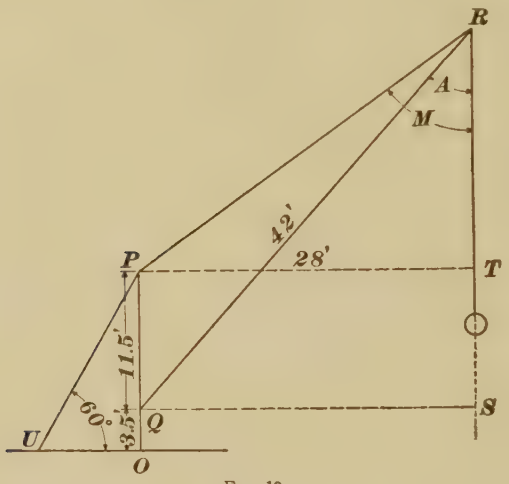

FIG. 18

$$\tan M = \frac{PT}{RT} = \frac{28}{19.805}; \text{ whence, } M = 54° 43' 38''. \quad \text{Ans.}$$

(*c*) In the triangle POU,

$$OU = OP \cot 60° = 15 \cot 60° = 8.660 \text{ ft.} \quad \text{Ans.}$$

(*d*) In the same triangle,

$$PU = \frac{OP}{\sin 60°} = \frac{15}{\sin 60°} = 17.320 \text{ ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. In order to determine the distance CB, Fig. 19, across an intervening stream, a line CA, at right angles to CB, was measured; the angle CAB was also measured, and found to be 50° 16′. If CA = 100 feet, what is the distance CB? Ans. CB = 120.31 ft.

2. A ship was observed from the top of a lighthouse under an angle of depression of 50°; if the top of the lighthouse is 250 feet above sea level, what was the horizontal distance of the ship from the lighthouse? Ans. 209.78 ft.

3. From two points P_1, P_2, Fig. 20, assumed to be on the same horizontal line, the angles of elevation of the top O of a column were found to be as shown. If $P_1 P_2 = 300$ feet, and the points P_1 and P_2 are 9 feet higher than the base of the column, find: (a) the height h $(= OH)$ of the column; (b) the horizontal distance d from P_1 to the axis of the column.

Ans. $\begin{cases} (a)\ h = 366.77 \text{ ft.} \\ (b)\ d = 292.31 \text{ ft.} \end{cases}$

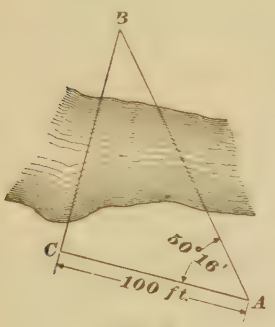

FIG. 19

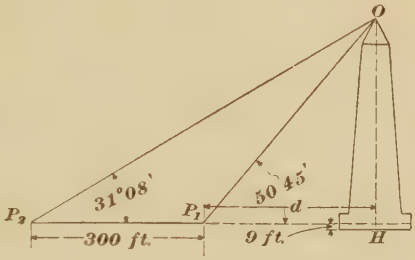

FIG. 20

4. A water pipe has a grade of 5.5 per cent. (that is, the pipe drops or rises 5.5 feet in every hundred feet measured horizontally); find: (a) the inclination of the pipe to the horizontal; (b) the length of pipe required for a horizontal distance of 2,764 feet.

Ans. $\begin{cases} (a)\ 3° 8' 54'' \\ (b)\ 2{,}768.2 \text{ ft.} \end{cases}$

5. The face $A\ B$, Fig. 17, and back $C\ D$ of a dam 80 feet high are to have a batter of 26 and 12 in 100, respectively; if the base $A\ D$ is to be 45 feet wide, find: (a) the angles A and D at the base; (b) the width $B\ C$ of the top.

Ans. $\begin{cases} (a)\ A = 75° 25' 33'',\ D = 83° 9' 26'' \\ (b)\ B\ C = 14.6 \text{ ft.} \end{cases}$

6. Show that the base of an isosceles triangle is equal to twice one of the equal sides multiplied by the sine of one-half the vertical angle (angle opposite base).

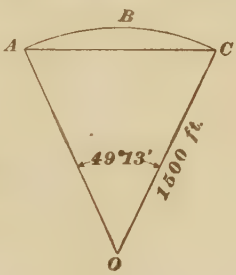

FIG. 21

7. A railroad curve $A\ B\ C$, Fig. 21, radius 1,500 feet, subtends a central angle of 49° 13'. (a) Find the length of the chord $A\ C$. (b) What will be the error in taking the length of the chord for the length of the arc? (Determine the latter length by the rules of geometry).

Ans. $\begin{cases} (a)\ A\ C = 1{,}249.2 \text{ ft.} \\ (b)\ 39.3 \text{ ft.} \end{cases}$

PLANE TRIGONOMETRY

(PART 2)

LOGARITHMIC FUNCTIONS OF SMALL ANGLES

1. Angles less than 3° are of comparatively rare occurrence in practice. When, however, they do occur, and they contain odd seconds, their logarithmic sines, tangents, and cotangents cannot be accurately determined by the general formulas and rules given in *Plane Trigonometry*, Part 1. These functions are found from a special table, which covers the first three pages of the general table of logarithmic functions furnished with this Course. These pages differ from the others in several respects, namely:

(*a*) The column of seconds on the left, marked $''$ at the top, gives the total number of seconds in all angles between 0° and 3°, at intervals of 1 minute. Thus, on page 43, the number 6,360 in the column of seconds is horizontally opposite 46 in the minute column, and is, therefore, the total number of seconds in 1° 46′.

(*b*) The column headed S T, between the sine and the tangent column, contains the values of $\log \tan A - \log A''$, and $\log \sin A - \log A''$ for all values of A between 0° and 3°, varying from minute to minute; A'' is the total number of seconds in the angle A. The first four figures of these differences are common to the tangent and the sine and are printed near the head of the column; the other two figures are printed under S for the sine and under T for the tangent. The two figures corresponding to any angle are horizontally opposite the total number of seconds in the

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angle, this total number of seconds being given in the left-hand column. Thus, for 1° 45′ (= 6,300″), the value of S, or of log sin 1° 45′ − log 6,300, is $\bar{6}.68551$; and the value of T, or of log tan 1° 45′ − log 6,300, is $\bar{6}.68571$.

(*c*) Next to the cotangent column, there is a column marked C, containing the values of $-T$. The first four figures of these values are common to all angles between 0° and 3°, and are printed but once; the other two are printed horizontally opposite the number of seconds in the corresponding angles. Thus, for 1° 51′ (= 6,660″), the value of C is 5.31427. The values of S, T, and C will here be referred to as **corrections.**

2. To Find the Logarithmic Sine or Tangent of an Angle Between 0° and 3°.—If there are no odd seconds in the angle, the logarithm may be at once taken from the table, as in *Plane Trigonometry*, Part 1. Here it will be assumed that the angle contains a number of odd seconds. Let the angle be denoted by A, and the total number of seconds in it by A''; that is, let A'' be the angle reduced to seconds. (See Art. **1.**)

Rule.—*Open the table at the page headed by the number of degrees in the given angle. Look in the minute column for the number of minutes nearest (whether greater or less) to the number of odd minutes and seconds in the given angle. (Thus, if the given angle is 2° 36′ 40″, look for 2° 37′; if the given angle is 2° 36′ 21″, look for 2° 36′.) Take from the column headed* S T *the correction horizontally opposite the number of minutes found as just described, using the correction under* S *for the sine, and that under* T *for the tangent. Look in the column of seconds at the left of the page for the number horizontally opposite the number of minutes in the given angle, and to it add the number of odd seconds in that angle. The result will be the total number of seconds (A'') in the given angle. Find the logarithm of this number of seconds from the table of logarithms of numbers. Add to this logarithm the correction found as above. The result will be the required logarithmic sine or tangent, according to the correction used.*

EXAMPLE 1.—To find the logarithmic sine of 1° 3′ 45″ (= A).

SOLUTION.—Opening the table at page 43 (headed 1°), we look for 4′ in the minute column, since 3′ 45″ is nearer to 4′ than to 3′. Horizontally opposite 4, and in the column headed S T, the sine correction $\bar{6}.68555$ (= S) is found. We now look in the minute column for the number of minutes (3) in the given angle; horizontally opposite it in the left-hand column is the number 3,780, number of seconds in 1° 3′; adding 45″, we obtain 3,825 (= A'') for the total number of seconds in the given angle.

$$\log A'' = \log 3{,}825 = 3.58263$$
$$S = \bar{6}.68555$$
$$\log \sin A = \bar{2}.26818$$

that is, $\log \sin 1° 3' 45'' = \bar{2}.26818$. Ans.

EXAMPLE 2.—To find the logarithmic tangent of 2° 36′ 17″.

SOLUTION.—On page 44, the correction for the tangent, opposite 36′, is $\bar{6}.68587$ (= T). Number of seconds opposite 36′ in the left-hand column, 9,360; $A'' = 9{,}360 + 17 = 9{,}377$.

$$\log 9{,}377 = 3.97206$$
$$T = \bar{6}.68587$$
$$\log \tan 2° 36' 17'' = \bar{2}.65793.$$ Ans.

3. To Find the Logarithmic Cotangent of an Angle Between 0° and 3°.

Rule.—*Find C, A'', and $\log A''$ exactly as in the last article, C being taken from the correction column next to the cotangent column. Subtract $\log A''$ from C. The result will be the required logarithmic cotangent.*

EXAMPLE.—To find the logarithmic cotangent of 1° 52′ 37″.

SOLUTION.—On page 43, the correction under C, and horizontally opposite 53′, is 5.31427; $A'' = 6{,}720 + 37 = 6{,}757$.

$$C = 5.31427$$
$$\log A'' = \log 6{,}757 = 3.82975$$
$$C - \log A'' = 1.48452$$

that is, $\log \cot 1° 52' 37'' = 1.48452$. Ans.

4. To Find the Logarithmic Tangent, Cosine, or Cotangent of an Angle Between 87° and 90°.—These functions also are to be taken from the first three pages of the table of logarithmic functions. The simplest way to proceed is to subtract the angle from 90° and look for the

corresponding complementary function as explained in Arts. **2** and **3**. Thus, log cos 88° 55′ 38″ is obtained by looking for log sin (90° − 88° 55′ 38″) = log sin 1° 4′ 22″.

EXAMPLES FOR PRACTICE

1.	Find the logarithmic sine of 1° 6′ 19″.	Ans. $\bar{2}.28532$
2.	Find the logarithmic sine of 0° 2′ 41″.	Ans. $\bar{4}.89240$
3.	Find the logarithmic tangent of 2° 56′ 57″.	Ans. $\bar{2}.71196$
4.	Find the logarithmic cotangent of 1° 30′ 18″.	Ans. 1.58049
5.	Find the logarithmic cosine of 88° 50′ 49″.	Ans. $\bar{2}.30370$
6.	Find the logarithmic tangent of 89° 3′ 9″.	Ans. 1.78151
7.	Find the logarithmic cotangent of 88° 0′ 25″.	Ans. $\bar{2}.54157$

5. To Find the Angle Corresponding to a Given Logarithmic Function, When the Function Lies Between Two of the Functions in the First Three Pages of the Table.—I. *Sine and Tangent.*—As explained in Art. **1**, log sin $A = S + \log A''$; therefore,

$$\log A'' = \log \sin A - S \qquad (1)$$

Likewise, when log tan A is given,

$$\log A'' = \log \tan A - T \qquad (2)$$

From these formulas is derived the following

Rule.—*Find in the table the logarithm nearest to the given one. Take the correction horizontally opposite this logarithm, and subtract it from the given logarithm. The result will be the logarithm of the total number of seconds (A'') in the given angle. Find the number corresponding to this logarithm, and reduce it to degrees, minutes, and seconds.*

It is here assumed that the given function lies between two functions in the column marked log sin or log tan, as the case may be, at the top. If the names of the functions are at the bottom, the sine should be treated as in *Plane*

Trigonometry, Part 1; the tangent should be treated as if it were a cotangent, according to the directions to be given presently, and when the angle corresponding to that cotangent is found, it should be subtracted from 90°.

II. *Cotangent.*—Since $\log \cot A = C - \log A''$ **(Art. 3)**, we have

$$\log A'' = C - \log \cot A \qquad (3)$$

From this formula is derived the following

Rule.—*Find in the table the logarithmic function nearest the given cotangent. Take from the* C *column the correction horizontally opposite the logarithm just found, and from it subtract the given logarithmic cotangent. The result will be the logarithm of the total number of seconds in the angle.*

Here, as before, it is assumed that the given cotangent lies between two of those marked log cot at the top. If it lies between two logarithms in the column marked log cot at the bottom, it should be treated as if it were a tangent, and having found the angle corresponding to this tangent, it should be subtracted from 90° to obtain the required angle.

III. *Cosine.*

Rule.—*If the given cosine lies between two of those in the column headed log cos, apply the general rule given in Plane Trigonometry, Part 1. If it lies between two of the logarithms in the column marked log cos at the bottom, treat it as if it were a sine, find the angle corresponding to that sine as above, and subtract the result from 90°.*

EXAMPLE 1.—To find the angle whose logarithmic tangent is $\bar{2}.32803$.

SOLUTION.—The logarithmic tangent nearest to $\bar{2}.32803$ is $\bar{2}.32711$, found in the column headed log tan on page 43. The T correction horizontally opposite $\bar{2}.32711$ is $\bar{6}.68564$.

$$\log \tan A = \bar{2}.32803$$
$$T = \bar{6}.68564$$
$$\log A'' = 3.64239$$

From the table of logarithms of numbers,

$$A'' = 4{,}389'' = 1^\circ\ 13'\ 9''. \quad \text{Ans.}$$

EXAMPLE 2.—To find the angle whose logarithmic cotangent is 2.49567.

SOLUTION.—The nearest logarithmic cotangent found in the table is 2.49488. The number opposite this logarithm in the C column is 5.31442.

$$C = 5.31442$$
$$\log \cot A = 2.49567$$
$$\log A'' = 2.81875;$$
$$A'' = 659'' = 0^\circ\ 10'\ 59''. \quad \text{Ans.}$$

NOTE.—Angles are here given to the nearest whole second.

EXAMPLE 3.—To find the angle whose logarithmic cosine is $\bar{2}.63723$.

SOLUTION.—The nearest logarithm, $\bar{2}.63678$, is found on page 44, in the column headed log sin. The given function is, therefore, to be treated as if it were a logarithmic sine, and the angle A, corresponding to this sine is to be subtracted from 90° to obtain the required angle A. The correction horizontally opposite $\bar{2}.63678$, in the S column, is $\bar{6}.68544$.

$$\log \sin A_1 = \bar{2}.63723$$
$$S = \bar{6}.68544$$
$$\log A_1'' = 3.95179;$$
$$A_1 = 8{,}949'' = 2^\circ\ 29'\ 9''$$
$$A = 90^\circ - 2^\circ\ 29'\ 9'' = 87^\circ\ 30'\ 51''. \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

Verify the following values:

(*a*) $\bar{2}.17645 = \log \sin 0^\circ\ 51'\ 37''$ (*e*) $\bar{2}.48790 = \log \cot 88^\circ\ 14'\ 19''$

(*b*) $\bar{3}.94316 = \log \sin 0^\circ\ 30'\ 10''$ (*f*) $2.47608 = \log \cot 0^\circ\ 11'\ 29''$

(*c*) $\bar{2}.65783 = \log \cos 87^\circ\ 23'\ 36''$ (*g*) $1.31009 = \log \tan 87^\circ\ 11'\ 48''$

(*d*) $\bar{2}.58349 = \log \tan 2^\circ\ 11'\ 41''$ (*h*) $\bar{3}.95377 = \log \cos 89^\circ\ 29'\ 6''$

6. Use of the Column of Seconds for Obtaining the Angle Corresponding to a Given Function.—In order to avoid confusing the student by too many rules, the reduction of A'' to degrees, minutes, and seconds was, in the preceding articles, effected by the ordinary rules of arithmetic, without any reference to the table. The following is a more expeditious method:

Let the given function lie between the functions of two consecutive angles, A_1 and $A_1 + 1'$. Then, the degrees and minutes in the required angle are those in A_1, and may be at once written down. The number in the column of seconds on the left, horizontally opposite the number of minutes in A_1, gives the total number of seconds in A_1. Denoting that

number by A_1'' and the number of odd seconds in the required angle by s, we have

$$s = A'' - A_1''$$

EXAMPLE.—To find the angle whose logarithmic tangent is $\bar{2}.30217$.

SOLUTION.—The given function lies between $\bar{2}.29629$ and $\bar{2}.30263$. The angle corresponding to the first of these two functions is $1° 8'$ $(= A_1)$; $A_1'' = 4{,}080''$.

$$\log \tan A = \bar{2}.30217$$
$$T = \bar{6}.68563$$
$$\log A'' = \overline{3.61654};\quad A'' = 4{,}136$$
$$s = A'' - A_1'' = 4{,}136 - 4{,}080 = 56''$$
$$A = A_1 + s = 1° 8' 56''.\quad \text{Ans.}$$

The subtraction $A'' - A_1''$ can usually be effected mentally.

EXAMPLES FOR PRACTICE

Apply the method just described to the Examples for Practice given after Art. **4**.

GENERAL TRIGONOMETRIC FORMULAS

ANGLES AND THEIR TRIGONOMETRIC FUNCTIONS

7. Angle of Any Magnitude.—In trigonometry, an angle is considered as being generated by a straight line turning about one of its ends, which is the vertex of the angle. In this motion, any point in the turning line describes a circular arc, whose number of degrees is the measure of the angle. The turning line is called the **generating line.** The position that this line occupies before it begins to turn, and from which arcs are measured, is called the **initial line,** or the **initial position** of the generating line; and the position it occupies after turning through a certain angle

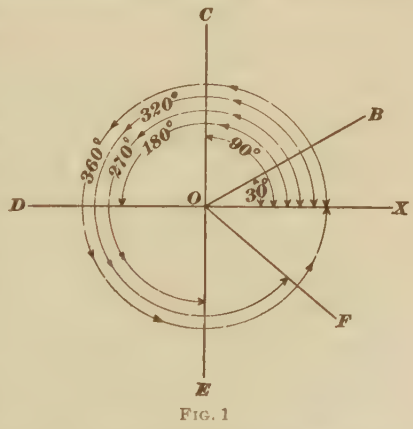

FIG. 1

is called the **final position.** In Fig. 1, for example, the initial position of the generating line is OX. The turning is supposed to take place about the point O and in a direction opposite to that in which the hands of a clock move. When the line turns from the position OX to the final positions OB, OC, OD, OE, OF, it generates angles of 30°, 90°, 180°, 270°, 320°, respectively, as indicated on the figure. If the line makes a complete turn, so that its final position coincides with its initial position OX, the angle generated is 360°.

8. Positive and Negative Angles.—When an angle is described by a line turning in a direction contrary to that in which the hands of a watch move, the angle is considered **positive;** if described in the opposite direction, it is considered **negative.** Referring to Fig. 2, the angle XOB, whose supplement is A, may be regarded as having been described in any of the following manners:

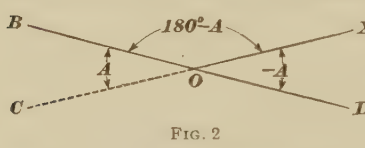

FIG. 2

(a) By turning the generating line about O from the position OX in the positive direction through $(180 - A)$ degrees to the position OB.

(b) By turning the generating line about O in a positive direction through an angle of 180°, when it will be in the position OC, and then turning it back from OC in the negative direction through the angle $-A$ (negative, because turned in the negative direction) into the position OB.

(c) By turning the generating line about O in the negative direction through the angle $-A$, into the position OD, and then turning it back in the positive direction through 180° into the position OB.

It is to be noticed that, however the angle $(180° - A)$ may be regarded as described, the resulting angle XOB is the same.

9. Quadrants.—Let OX, Fig. 3, be the initial position of the generating line, and OM_1, OM_2, OM_3, OM_4 final

positions, determining, respectively, the angles A_1, A_2, A_3, A_4, all measured from OX upwards and toward the left. Producing XO and drawing through O a perpendicular YY' to OX, the plane of the figure is divided into four right angles, called **quadrants.** Taking them in order, following the direction in which positive angles are reckoned, they are distinguished as follows: XOY is the **first quadrant;** YOX', the **second quadrant;** $X'OY'$, the **third quadrant;** and $Y'OX$, the **fourth quadrant.**

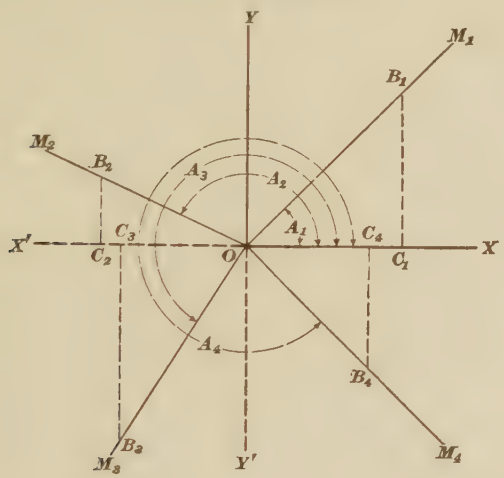

FIG. 3

10. Trigonometric Functions of Any Angle.—In the definitions given in *Plane Trigonometry*, Part 1, only acute angles were considered. Referring to Fig. 3, in which B_1C_1 is perpendicular to OX, the trigonometric functions of the acute angle A_1 were defined by the following equations:

$$\sin A_1 = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{B_1C_1}{OB_1} \qquad \tan A_1 = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{B_1C_1}{OC_1}$$

$$\cos A_1 = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{OC_1}{OB_1} \qquad \cot A_1 = \frac{\text{side adjacent}}{\text{side opposite}} = \frac{OC_1}{B_1C_1}$$

$$\sec A_1 = \frac{\text{hypotenuse}}{\text{side adjacent}} = \frac{OB_1}{OC_1} \qquad \csc A_1 = \frac{\text{hypotenuse}}{\text{side opposite}} = \frac{OB_1}{B_1C_1}$$

These formulas serve as the definitions of the trigonometric functions of any angle; that is, the **sine** of any angle

is the ratio of the side opposite to the hypotenuse; the **tangent** is the ratio of the side opposite to the side adjacent, etc. But, in order that these definitions may be correct, it is necessary to apply to them some algebraic rules relating to signs.

In Fig. 3, the hypotenuse used for the determination of the functions of A_1 is any portion OB_1 of the side OM_1, which is the final position of the generating line. From B_1, a perpendicular B_1C_1 is drawn on the initial line OX, thus determining the right triangle OB_1C_1. The length of the perpendicular B_1C_1, which is the side opposite the vertex of the angle, is the distance of B_1 above the initial line OX, and the length of the adjacent side OC_1 is the distance of the point B_1 to the right of the vertex, measured along the initial line; or, what is the same thing, OC_1 is the distance of B_1C_1 from the vertex, measured toward the right.

Consider now the angle XOM_2, or A_2, in which the final position OM_2 of the generating line lies in the second quadrant. As before, the hypotenuse to be used in the definitions of the trigonometric functions of A_2 is any portion OB_2 of the side OM_2, which is the final position of the generating line. As before, also, a perpendicular from B_2 is drawn on the initial line OX; but, in this case, the perpendicular falls on OX produced. In the right triangle OB_2C_2, the perpendicular B_2C_2 is the side opposite the vertex of the angle A_2, and OC_2 is the side adjacent. It should be noted very particularly that the terms *side opposite* and *side adjacent* are used to describe the positions of the legs of the right triangle with reference to the *vertex* of the angle considered, not to the angle itself. Thus, B_2C_2 is not opposite the angle A_2, but opposite the vertex O of that angle. The length of the side opposite, B_2C_2, measures the distance of B_2 above the initial line; and the length of OC_2, or the side adjacent, measures the distance of the opposite side B_2C_2 to the left of the vertex; or, in the language of algebra, it may be said that $-OC_2$ is the distance of B_2C_2 to the *right* of O.

Having defined the cosine of any angle as the ratio of the side adjacent to the hypotenuse, and the side adjacent as the

distance of the side opposite from the vertex, measured toward the right of the vertex, it is necessary, when the side opposite is to the left of the vertex, to consider its distance from the vertex, or the side adjacent, as negative. This is in accordance with the general principle of algebra, that, if distances counted in one direction are treated as positive, distances in the opposite direction must be treated as negative. In the triangle OB_2C_2, therefore, OC_2 should be treated as negative, and therefore, the cosine of A_2 is $\frac{-OC_2}{OB_2}$.

Considering now the angle A_3, the hypotenuse is, as above, any portion OB_3 of the side OM_3, which is the final position of the generating line. From B_3, the perpendicular B_3C_3 on the initial line (produced) is drawn, and thus a right triangle is determined, in which B_3C_3 is the side opposite, and OC_3 the side adjacent. As previously explained, OC_3 should be treated as negative. The opposite side B_3C_3, which is the distance of B_3 below the initial line, should also be treated as negative; for if distances above the initial line are treated as positive, those below the initial line must be treated as negative.

Finally, in the angle A_4, which terminates in the fourth quadrant, OC_4, the side adjacent, is positive, while B_4C_4, the side opposite, is negative.

The foregoing explanations may be summed up as follows: The side opposite is positive or negative according as the hypotenuse is above or below the initial line. The side adjacent is positive or negative according as it extends toward the right or toward the left of the vertex. The hypotenuse is always positive.

11. Algebraic Signs of the Functions.—Referring again to Fig. 3, it will be observed that, for any angle, as A_1, terminating in the first quadrant, both the side adjacent and the side opposite, or OC_1 and B_1C_1, are positive, and therefore all the functions are positive; for any angle, as A_2, terminating in the second quadrant, the side adjacent, or OC_2, is negative, and the side opposite, or B_2C_2, is positive. Therefore,

$$\sin A_2 = \frac{+ B_2 C_2}{+ O B_2}, \text{ positive} \qquad \tan A_2 = \frac{+ B_2 C_2}{- O C_2}, \text{ negative}$$

$$\cos A_2 = \frac{- O C_2}{+ O B_2}, \text{ negative} \qquad \sec A_2 = \frac{+ O B_2}{- O C_2}, \text{ negative}$$

$$\csc A_2 = \frac{+ O B_2}{+ B_2 C_2}, \text{ positive}$$

The signs of the functions of angles terminating in the third and in the fourth quadrant are similarly determined. The results are tabulated below.

TABLE I

Function	Quadrant			
	First	Second	Third	Fourth
	Sign of Function			
Sine	+	+	−	−
Cosine	+	−	−	+
Tangent	+	−	+	−
Cotangent	+	−	+	−
Secant	+	−	−	+
Cosecant	+	+	−	−

12. Trigonometric Functions of 0° and 90°.—In the right triangles $A\,C\,B$, Fig. 4, the hypotenuse $A\,B$ may be taken to have any value whatever. It is evident that $B\,C$, the side opposite, decreases as the angle $C\,A\,B$ decreases, and becomes zero when the angle becomes zero; and that $B\,C$ coincides with the hypotenuse $A\,B$ when the angle $C\,A\,B$ is 90°. Again, $A\,C$, the adjacent side, increases as the angle decreases, and is equal to the hypotenuse $A\,B$ when the angle $C\,A\,B$ is 0°. Also, $A\,C$ becomes zero when $C\,A\,B$ is 90°. Now, from the definitions of the trigonometric functions,

FIG. 4

$$\sin CAB = \frac{\text{side opposite}}{\text{hypotenuse}}, \text{ whence} \begin{cases} \sin\ 0° = \dfrac{0}{AB} = 0 \\ \sin\ 90° = \dfrac{AB}{AB} = 1 \end{cases}$$

$$\cos CAB = \frac{\text{side adjacent}}{\text{hypotenuse}}, \text{ whence} \begin{cases} \cos\ 0° = \dfrac{AB}{AB} = 1 \\ \cos\ 90° = \dfrac{0}{AB} = 0 \end{cases}$$

In like manner,

$$\tan 0° = \frac{0}{AC} = 0 \qquad \tan 90° = \frac{BC}{0} = \infty$$

$$\cot 0° = \frac{AC}{0} = \infty \qquad \cot 90° = \frac{0}{CB} = 0$$

NOTE.—The cotangent of CAB is equal to $\frac{CA}{CB}$. Now, as the angle decreases, the side CB becomes less and less, and it is evident that, as the denominator of a fraction becomes less and less, the numerator remaining the same, the value of the fraction increases. As the denominator decreases indefinitely, the value of the fraction increases indefinitely, and when the value of the fraction exceeds any known quantity, however great, it is said to be infinite. The sign ∞ is used to express an infinite number.

13. Functions of $(180° - A)$.—Let XOM, Fig. 5, be any angle, and A $(= MOX')$ its supplement. Draw OM' making with OX an angle equal to A, as shown. Take any part OB of OM for the hypotenuse, and draw BC perpendicular to OX produced; draw BB' parallel to OX, and $B'C'$ perpendicular to OX. Then, $BC = B'C'$; $OB = OB'$; $OC = -OC'$ (Art. **10**); and, by the definitions of the functions,

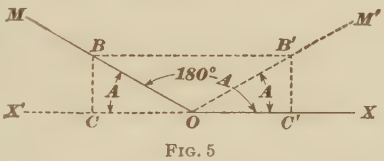

FIG. 5

$$\sin XOM = \frac{BC}{OB} = \frac{B'C'}{OB'} = \sin A$$

$$\cos XOM = \frac{OC}{OB} = \frac{-OC'}{OB'} = -\cos A$$

that is,
$$\sin (180° - A) = \sin A \qquad (1)$$
$$\cos (180° - A) = -\cos A \qquad (2)$$

Similarly, $\tan (180° - A) = -\tan A$ **(3)**

$$\cot (180° - A) = -\cot A \quad \textbf{(4)}$$

These relations are especially useful for finding the logarithmic functions of angles greater than 90°, since these functions are arithmetically equal to those of the supplements of the angles; that is, when signs are disregarded, any function of an angle and that of its supplement are equal. For example, $\sin 105° = \sin (180° - 105°) = \sin 75°$; $\cos 105° = -\cos (180° - 105°) = -\cos 75°$.

14. Functions of $(90° + A)$.—By formula 1 of Art. **13**, $\sin (90° + A) = \sin [180° - (90 + A)] = \sin (90° - A)$ or, since $\sin (90° - A) = \cos A$,

$$\sin (90° + A) = \cos A \quad \textbf{(1)}$$

The following formulas may be derived in a similar manner:

$$\tan (90° + A) = -\cot A \quad \textbf{(2)}$$

$$\cos (90° + A) = -\sin A \quad \textbf{(3)}$$

$$\cot (90° + A) = -\tan A \quad \textbf{(4)}$$

15. Functions of Negative Angles.—The complement of an angle is the algebraic difference between the angle and 90°. If the angle is greater than 90°, its complement is negative. Thus, the complement of 95° is $90° - 95° = -5°$. The cofunctions of an angle are the corresponding fundamental functions of its complement, whether that complement be positive or negative. Thus, $\cos 85° = \sin (90° - 85°) = \sin 5°$; $\cos 95° = \sin (90° - 95°) = \sin (-5°)$. Similarly, $\sin 95° = \cos (90° - 95°) = \cos (-5°)$. It is, therefore, necessary to know how to determine the functions of negative angles.

If $90° + A$ is any angle, its complement is $90° - (90° + A) = -A$; and, therefore,

$$\cos (90° + A) = \sin (-A), \quad \cot (90° + A) = \tan (-A)$$
$$\sin (90° + A) = \cos (-A), \quad \tan (90° + A) = \cot (-A)$$

whence, replacing the values of cos $(90° + A)$, cot $(90° + A)$, etc. from the preceding article,

$$\sin(-A) = -\sin A \qquad (1)$$

$$\tan(-A) = -\tan A \qquad (2)$$

$$\cos(-A) = \cos A \qquad (3)$$

$$\cot(-A) = -\cot A \qquad (4)$$

ADDITION OF ANGLES

16. To Express the Sine or Cosine of the Sum or Difference of Two Angles in Terms of the Sine and Cosine of the Angles.—The following formulas are fundamental; being of frequent occurrence, they are very important, and should be committed to memory:

$$\sin(A + B) = \sin A \cos B + \cos A \sin B \qquad (1)$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B \qquad (2)$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B \qquad (3)$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B \qquad (4)$$

NOTE.—The derivation of these formulas is given in the Appendix at the end of this Section, under the Roman numeral I. That Appendix contains this and a few other demonstrations that are comparatively laborious and may be found irksome by some. They are not essential to the understanding of the formulas, and the student is not required to learn them. He is, however, advised to peruse them carefully, as they are good exercises in the handling and transforming of both algebraic and trigonometric expressions.

These formulas are not used, as they seem to imply, to determine the sine or the cosine of the sum or difference of two angles, when the sine and cosine of those angles are given. They can be used for this purpose, but there would be no advantage in so doing. Their main value consists in their application to transforming complicated trigonometric expressions into simpler ones. The student will often have occasion to employ them in this manner. In order that he may have an idea of this application of the formulas, two examples are given here.

EXAMPLE 1.—To determine the angle A from the relation

$$\frac{\sin(A+28^\circ)}{\sin A} = .95$$

SOLUTION.—Applying formula **1**, we have

$$\frac{\sin(A+28^\circ)}{\sin A} = \frac{\sin A \cos 28^\circ + \cos A \sin 28^\circ}{\sin A}$$

$$= \frac{\sin A \cos 28^\circ}{\sin A} + \frac{\cos A \sin 28^\circ}{\sin A} = \cos 28^\circ + \cot A \sin 28^\circ$$

replacing $\frac{\cos A}{\sin A}$ by its equal $\cot A$ (see *Plane Trigonometry*, Part 1). Substituting this value of the quotient $\frac{\sin(A+28^\circ)}{\sin A}$ in the given equation, we have,

$$\cos 28^\circ + \cot A \sin 28^\circ = .95$$

whence $$\cot A = \frac{.95 - \cos 28^\circ}{\sin 28^\circ} = \frac{.95 - .88295}{.46947} = .14282$$

and, therefore, $$A = 81^\circ\ 52'\ 19''.\ \text{Ans.}$$

EXAMPLE 2.—To transform the expression $\tan A + \tan B$ into the expression $\frac{\sin(A+B)}{\cos A \cos B}$.

NOTE.—Transformations of this kind are very often useful, when logarithms are employed. Thus, if $\tan A + \tan B$ were to be multiplied by 39.578, it would be necessary first to find the natural tangent of A, then that of B, add the two together, take the logarithm of the sum thus obtained, and add this logarithm to that of 39.578. If, however, the expression $\frac{\sin(A+B)}{\cos A \cos B}$ is used, the logarithms of $\sin(A+B)$, $\cos A$ $\cos B$ can be taken from the table, and the operation performed without having recourse to natural functions, which are often inconvenient.

SOLUTION.—We have (*Plane Trigonometry*, Part 1),

$$\tan A + \tan B = \frac{\sin A}{\cos A} + \frac{\sin B}{\cos B} = \frac{\sin A \cos B + \cos A \sin B}{\cos A \cos B}$$

According to formula **1**, the numerator of this last fraction is equal to $\sin(A+B)$. Therefore,

$$\tan A + \tan B = \frac{\sin(A+B)}{\cos A \cos B}.\quad \text{Ans.}$$

17. Sine and Cosine of 2 A and of $\frac{1}{2}$ A.—From the formulas for the sine and cosine of the sum of two angles, the following are deduced:

$$\sin 2A = 2 \sin A \cos A \qquad (1)$$

$$\cos 2A = \cos^2 A - \sin^2 A \qquad (2)$$

$$\cos 2A = 1 - 2 \sin^2 A \qquad (3)$$

$$\cos 2A = 2 \cos^2 A - 1 \qquad (4)$$

$$\sin A = 2 \sin \tfrac{1}{2} A \cos \tfrac{1}{2} A \qquad (5)$$

$$\cos A = \cos^2 \tfrac{1}{2} A - \sin^2 \tfrac{1}{2} A \qquad (6)$$

$$\cos A = 1 - 2 \sin^2 \tfrac{1}{2} A \qquad (7)$$

$$\cos A = 2 \cos^2 \tfrac{1}{2} A - 1 \qquad (8)$$

As in the case of formulas **1** to **4**, Art. **16**, these formulas are used mainly for the purposes of transformation. They are very simply derived as follows:

When B is made equal to A, formula **1**, Art. **16**, becomes

$$\sin (A + A) = \sin A \cos A + \cos A \sin A$$

that is, $$\sin 2A = 2 \sin A \cos A$$

Similarly, formula **2**, Art. **16**, becomes

$$\cos (A + A) = \cos A \cos A - \sin A \sin A$$

that is, $$\cos 2A = \cos^2 A - \sin^2 A$$

Formula **3** follows from this, by writing $1 - \sin^2 A$ instead of $\cos^2 A$ (since $\sin^2 A + \cos^2 A = 1$); and formula **4**, by writing $1 - \cos^2 A$ instead of $\sin^2 A$.

Formulas **1** to **4** give the sine and cosine of twice any angle in terms of the sine and cosine of the angle. If the angle is denoted by $\frac{1}{2} A$, twice the angle will be A, and formulas **1** to **4** take the forms of formulas **5** to **8**.

OBLIQUE TRIANGLES

FUNDAMENTAL PRINCIPLES

NOTE.—For the general method of marking and naming the sides and angles of a triangle, see *Plane Trigonometry*, Part 1.

18. Principle of Sines.—*In any triangle, the sides are proportional to the sines of the opposite angles.* That is,

$$\frac{a}{b} = \frac{\sin A}{\sin B}, \quad \frac{a}{c} = \frac{\sin A}{\sin C}, \quad \frac{b}{c} = \frac{\sin B}{\sin C}$$

Let ABC, Fig. 6, be any triangle and p the perpendicular from C on the opposite side. Then, in (a), the right triangles ACD and BCD give, respectively,

$$p = b \sin A, \; p = a \sin B$$

whence, putting the two values of p equal to each other,

$$a \sin B = b \sin A$$

and, therefore, dividing by $b \sin B$,

$$\frac{a}{b} = \frac{\sin A}{\sin B}$$

In (b), the right triangles $A C D$ and $B C D$ give, respectively, $p = b \sin A$, $p = a \sin CBD$

whence, $a \sin CBD = b \sin A$

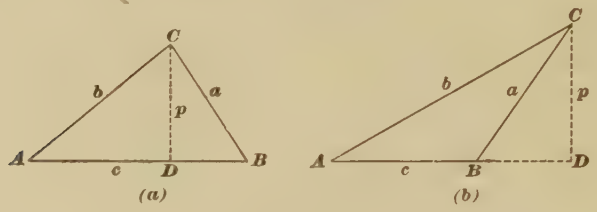

FIG. 6

But, as $CBD = 180° - B$, we may write $\sin B$ instead of $\sin CBD$ (Art. **13**), and, therefore,

$$a \sin B = b \sin A$$

whence, as before,

$$\frac{a}{b} = \frac{\sin A}{\sin B} \qquad (1)$$

By drawing a perpendicular from B on $A C$, and reasoning in the same manner, it may be shown that

$$\frac{a}{c} = \frac{\sin A}{\sin C} \qquad (2)$$

Similarly, $$\frac{b}{c} = \frac{\sin B}{\sin C}$$

By transforming equation (1), we obtain

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

and by a similar transformation of equation (2),

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

We have, therefore,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The principle of sines may, then, be stated in this form: *In every triangle, the quotient obtained by dividing the length of any side by the sine of the opposite angle is the same, whatever the side taken.*

This quotient is called the **modulus** of the triangle, and will here be denoted by M. The modulus can be found when any of the sides and the opposite angle are known.

The principle of sines is one of the most important in trigonometry, and both forms in which it is stated in this article should be committed to memory.

19. The Cosine Principle.—*In any triangle, the square of one side is equal to the sum of the squares of the other two sides minus twice the product of these two sides and the cosine of their included angle.* That is (Fig. 6),

$$a^2 = b^2 + c^2 - 2\,b\,c\cos A$$
$$b^2 = a^2 + c^2 - 2\,a\,c\cos B$$
$$c^2 = a^2 + b^2 - 2\,a\,b\cos C$$

These formulas are derived in Appendix II.

20. Principle of Tangents.—*The sum of any two sides of a triangle is to their difference as the tangent of half the sum of the opposite angles is to the tangent of half their difference.* That is (Fig. 6),

$$\frac{a+b}{a-b} = \frac{\tan\frac{1}{2}(A+B)}{\tan\frac{1}{2}(A-B)}$$

The derivation of this formula is given in Appendix III. The student should have no difficulty in committing the formula to memory, as its symmetry makes it very easy to remember.

SOLUTION OF OBLIQUE TRIANGLES

21. The solution of oblique triangles is treated under four cases:

Case I: Given Two Sides and the Included Angle. Let a, b, and C, Fig. 6, be given and A, B, and c be required. Of the two methods given below, the first is preferable in most cases.

First Method.—From the formula in Art. **20,** the following is readily derived:

$$\tan\tfrac{1}{2}(A-B) = \frac{a-b}{a+b}\tan\tfrac{1}{2}(A+B) \qquad (1)$$

Now, since $A + B + C = 180°$, we have also,

$$A + B = 180° - C; \text{ and } \tfrac{1}{2}(A + B) = \tfrac{1}{2}(180° - C)$$
$$= 90° - \tfrac{1}{2}C$$

Therefore, $\frac{1}{2}C$ is the complement of $\frac{1}{2}(A + B)$, and hence, $\tan \frac{1}{2}(A + B) = \cot \frac{1}{2}C$. Substituting this value in equation (1), the following formula is derived:

$$\tan \tfrac{1}{2}(A - B) = \frac{a - b}{a + b} \cot \tfrac{1}{2}C \qquad (1)$$

If the student remembers the formula in Art. **20,** or the principle of tangents, he will have no difficulty in remembering this formula, which is derived from the formula in Art. **20,** by simply writing $\cot \frac{1}{2}C$ instead of $\tan \frac{1}{2}(A + B)$.

From this formula $\frac{1}{2}(A - B)$ can be found. Let this value of $\frac{1}{2}(A - B)$ be denoted by D. We have also, as explained above, $\frac{1}{2}(A + B) = \frac{1}{2}(180° - C) = 90° - \frac{1}{2}C$.

$$\tfrac{1}{2}(A + B) = 90° - \tfrac{1}{2}C \qquad (2)$$
$$\tfrac{1}{2}(A - B) = D \qquad (3)$$

Adding equations (2) and (3) gives

$$A = (90° - \tfrac{1}{2}C) + D$$

Subtracting equation (3) from (2) gives

$$B = (90° - \tfrac{1}{2}C) - D$$

Knowing A and B, the side c may be found from the relation (Art. **18**),

$$\frac{c}{\sin C} = \frac{a}{\sin A}, \text{ which gives } c = \frac{a \sin C}{\sin A}$$

It is, however, more convenient to find c from the following formula, the derivation of which is given in Appendix IV:

$$c = \frac{(a - b) \cos \frac{1}{2}C}{\sin \frac{1}{2}(A - B)} \qquad (2)$$

It will be noticed that, for calculating $\tan \frac{1}{2}(A - B)$, the logarithms of $(a - b)$ and $\cot \frac{1}{2}C$ have to be found. The logarithm of $\cos \frac{1}{2}C$ may be taken out of the table at the same time as that of $\cot \frac{1}{2}C$. Also, when the angle $\frac{1}{2}(A - B)$ is taken from the table, its logarithmic sine should be taken at the same time. This greatly simplifies the application of formula **2.**

Second Method.—The third side c can be found directly from the formula in Art. **19**, which gives

$$c = \sqrt{a^2 + b^2 - 2\,a\,b\cos C}$$

Then, by the principle of sines,

$$\sin A = \frac{a \sin C}{c},\ \sin B = \frac{b \sin C}{c}$$

This method is of value when the only required part is the side c, especially if a and b are convenient numbers to square.

EXAMPLE 1.—In a triangle, $a = 17$ feet, $b = 12$ feet, and the included angle $C = 59° 23'$. To find the other parts of the triangle.

SOLUTION.—Here $\frac{1}{2}\ C = 29° 41' 30''$; $a + b = 17 + 12 = 29$, and $a - b = 17 - 12 = 5$. Then, by the first method,

$$\tan \tfrac{1}{2}\,(A - B) = \frac{5}{29} \times \cot 29° 41' 30''$$

log 5 =	.69897	log 5 =	.69897
log 29 =	1.46240	log cos 29° 41′ 30″ =	$\bar{1}$.93887
	$\bar{1}$.23657		.63784
log cot 29° 41′ 30″ =	.24397	log sin D =	$\bar{1}$.46154
log tan $\frac{1}{2}(A - B)$ =	$\bar{1}$.48054	log c =	1.17630
$D = \frac{1}{2}(A - B)$ =	16° 49′ 25″;	c =	15.007. Ans.

$$A = (90° - 29° 41' 30'') + 16° 49' 25'' = 77° 7' 55''.\ \text{Ans.}$$
$$B = (90° - 29° 41' 30'') - 16° 49' 25'' = 43° 29' 5''.\ \text{Ans.}$$

EXAMPLE 2.—Given $a = 10$, $b = 15$, and $C = 60°$; to find c.

SOLUTION.—By the second method,

$$c = \sqrt{10^2 + 15^2 - 2 \times 10 \times 15 \cos 60°}$$
$$= \sqrt{325 - 300 \times .5} = \sqrt{175} = 13.229 \text{ ft.}\ \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Given $a = 37.46$ feet, $b = 59.17$ feet, and $C = 69° 13'$; find A, B, and c.

Ans. $A = 37° 21' 30''$; $B = 73° 25' 30''$; $c = 57.72$ ft.

2. Two sides of a triangle are, respectively, 687.64 and 319.58 feet long, and their included angle is 47° 15′ 8″; find the other two angles and the third side.

Ans. Angles, 106° 14′ 56″ and 26° 29′ 56″; Third side = 525.97

3. Given $c = 4$ chains, $a = 6$ chains, and $B = 45° 18'$; find b.

Ans. $b = 4.271$ ch.

4. Given $b = 43.16$ chains, $c = 51.29$ chains, and $A = 35° 8' 10''$; find B, C, and a.

Ans. $\begin{cases} B = 57° 13' 20'' \\ C = 87° 38' 30'' \\ a = 29.544 \text{ ch.} \end{cases}$

---

22. Case II: Given a Side and Two Angles.—Let c, A, and B be known, to find a, b, and C. The angle $C = 180° - A - B$. By the principle of sines,

$$\frac{a}{\sin A} = \frac{c}{\sin C}, \text{ whence } a = \frac{c}{\sin C} \sin A$$

Similarly, $$b = \frac{c}{\sin C} \sin B$$

Since $\frac{c}{\sin C}$ is the modulus of the triangle (Art. **18**), these formulas may be thus stated: *Any side of a triangle is equal to the modulus of the triangle multiplied by the sine of the angle opposite that side.*

EXAMPLE.—Given $a = 98.48$, $B = 60° 45'$, and $C = 39° 15'$; to find b, c, and A.

SOLUTION.— $A = 180° - (60° 45' + 39° 15') = 80°$. Ans.

$$M = \frac{98.48}{\sin 80°}; \; b = \frac{98.48}{\sin 80°} \sin 60° 45'; \; c = \frac{98.48}{\sin 80°} \sin 39° 15'$$

$\log 98.48 = 1.99335$
$\log \sin 80° = \overline{1}.99335$
$\log M = 2.00000$

$\log b = 1.94076$; $b = 87.248$. Ans.
$\log \sin 60° 45' = \overline{1}.94076$
$\log M = 2.00000$
$\log \sin 39° 15' = \overline{1}.80120$
$\log c = \overline{1}.80120$; $c = 63.27$. Ans.

NOTE.—Attention is called to the convenient way in which the work is here arranged. Having determined $\log M$, this logarithm is copied, and then one of the logarithms to be added to it is written above it, the other under it, the addition being performed upwards in one case, and downwards in the other.

EXAMPLES FOR PRACTICE

1. Given $a = 45.39$ feet, $B = 38° 12'$, and $C = 11° 11' 34''$; find A, b, and c.

Ans. $\begin{cases} A = 130° 36' 26'' \\ b = 36.973 \text{ ft.} \\ c = 11.605 \text{ ft.} \end{cases}$

2. Given $c = 101.11$ chains, $C = 55° 55' 55''$, and $A = 10° 10' 10''$; find B, a, and b.

Ans. $\begin{cases} B = 113° 53' 55'' \\ a = 21.551 \text{ ch.} \\ b = 111.59 \text{ ch.} \end{cases}$

23. Case III: Given Three Sides.—Let a, b, and c be given, to find A, B, and C.

First Method.—The angles can be found directly from the cosine formulas (Art. **19**), which, being solved for cos A, cos B, and cos C, respectively, give

$$\left.\begin{aligned} \cos A &= \frac{b^2 + c^2 - a^2}{2\,bc} \\ \cos B &= \frac{a^2 + c^2 - b^2}{2\,ac} \\ \cos C &= \frac{a^2 + b^2 - c^2}{2\,ab} \end{aligned}\right\} \qquad (1)$$

These formulas are to be used when the numbers a, b, c are convenient to square; otherwise, they are too cumbersome, and those given below for the functions of half the angles should be employed. It is necessary to apply the formulas in determining only two of the angles, as the third follows from the relation $A + B + C = 180°$. As a check, however, the formulas should be applied to the third angle also.

It should be borne in mind that, if the cosine of an angle is found to be negative, this implies that the angle is obtuse (Art. **13**). In such case, the cosine is treated as positive, and the corresponding angle taken from the table is subtracted from 180° to obtain the required angle. Thus, if cos $A = -.97030$, we look for the angle whose cosine is $+.97030$, which is 14°. Then, $A = 180° - 14° = 166°$.

EXAMPLE.—Given $a = 4$ inches, $b = 5$ inches, and $c = 7$ inches; to find A, B, and C.

SOLUTION.— $\cos A = \dfrac{b^2 + c^2 - a^2}{2\,bc} = \dfrac{5^2 + 7^2 - 4^2}{2 \times 5 \times 7} = \dfrac{58}{70} = .82857,$

and, therefore, $A = 34° 2' 53''$. Ans.

$$\cos B = \frac{a^2 + c^2 - b^2}{2\,ac} = \frac{4^2 + 7^2 - 5^2}{2 \times 4 \times 7} = \frac{40}{56} = .71429$$

and, therefore, $B = 44° 24' 54''$. Ans.

$$C = 180° - A - B = 101° 32' 13''. \text{ Ans.}$$

As a check, we have

$$\cos C = \frac{a^2 + b^2 - c^2}{2\,ab} = \frac{4^2 + 5^2 - 7^2}{2 \times 4 \times 5} = -\frac{8}{40} = -.20000$$

The angle whose cosine is .20000 is 78° 27′ 47″. Therefore, $C = 180° - 78° 27′ 47″ = 101° 32′ 13″$.

Second Method.—As said before, this method is to be applied when the operations required by formula **1** involve too much labor, which happens when the lengths of the given sides consist of three or more significant figures—the usual case. If the sum of the sides is denoted by $2s$, or half their sum by s, the angles A, B, C may be found by the following formulas, which are derived in Appendix V:

$$\left.\begin{aligned} \tan \tfrac{1}{2} A &= \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \\ \tan \tfrac{1}{2} B &= \sqrt{\frac{(s-a)(s-c)}{s(s-b)}} \\ \tan \tfrac{1}{2} C &= \sqrt{\frac{(s-a)(s-b)}{s(s-c)}} \end{aligned}\right\} \qquad (2)$$

$$\left.\begin{aligned} \cos \tfrac{1}{2} A &= \sqrt{\frac{s(s-a)}{bc}} \\ \cos \tfrac{1}{2} B &= \sqrt{\frac{s(s-b)}{ac}} \\ \cos \tfrac{1}{2} C &= \sqrt{\frac{s(s-c)}{ab}} \end{aligned}\right\} \qquad (3)$$

For angles differing but little from 90° (say between 85° and 90°), use the cosine formulas **3**; in all other cases, the tangent formulas **2**.

We have also,

$$\sin \tfrac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{bc}} \qquad (4)$$

with similar formulas for $\sin \frac{1}{2} B$ and $\sin \frac{1}{2} C$. These formulas are of value for deriving the tangent formulas **2**, as well as for deriving an expression for the area of a triangle when the sides are given. They may also be used instead of the tangent formulas **2** for the determination of the angles, but the latter are preferable.

EXAMPLE.—In the triangle ABC, $a = 567$ feet, $b = 736$ feet, and $c = 264$ feet; to find the angles A, B, and C.

SOLUTION.—The tangent formulas will be used.

To find A

$a = 567$
$b = 736$
$c = 264$
$2s = 1{,}567$
$s = 783.5$
$s - a = 216.5$
$s - b = 47.5$
$s - c = 519.5$

$\log (s - c) = 2.71559$
$\log (s - b) = 1.67669$
4.39228
$\log s = 2.89404$
$\log (s - a) = 2.33546$
5.22950
$2)\bar{1}.16278$
$\log \tan \frac{1}{2} A = \bar{1}.58139$
$\frac{1}{2} A = 20° 52' 38''$, $A = 41° 45' 16''$. Ans.

To find B

$\log (s - a) = 2.33546$
$\log (s - c) = 2.71559$
5.05105
$\log s = 2.89404$
$\log (s - b) = 1.67669$
4.57073
$2)0.48032$
$\log \tan \frac{1}{2} B = 0.24016$
$\frac{1}{2} B = 60° 5' 29''$; $B = 120° 10' 58''$ Ans.

To find C

$\log (s - a) = 2.33546$
$\log (s - b) = 1.67669$
4.01215
$\log s = 2.89404$
$\log (s - c) = 2.71559$
5.60963
$2)\bar{2}.40252$
$\log \tan \frac{1}{2} C = \bar{1}.20126$
$\frac{1}{2} C = 9° 1' 54''$; $C = 18° 3' 48''$ Ans.

To check, add the angles:

$41° 45' 16''$
$120 \quad 10 \quad 58$
$18 \quad 3 \quad 48$
$180° 00' 2''$

The triangle closes within 2 sec. This error is due to the use of five-place tables, and to the fact that the angle in each case was taken out to the nearest second.

EXAMPLES FOR PRACTICE

1. Given $a = 1$ mile, $b = 2$ miles, and $c = 1.5$ miles; find A, B, and C. (Use first method.)

Ans. $\begin{cases} A = 28° 57' 17'' \\ B = 104° 28' 39'' \\ C = 46° 34' 4'' \end{cases}$

2. Given $a = 50$ chains, $b = 30$ chains, and $c = 45$ chains; find A, B, and C. (Use first method.)

Ans. $\begin{cases} A = 80° 56' 36'' \\ B = 36° 20' 7'' \\ C = 62° 43' 17'' \end{cases}$

3. Given $a = 63.47$ feet, $b = 89.36$ feet, and $c = 109.83$ feet; find A, B, and C. (Use second method.)

Ans. $\begin{cases} A = 35° 18' 10'' \\ B = 54° 27' 2'' \\ C = 90° 14' 50'' \end{cases}$

4. Given $a = 2{,}354$ feet, $b = 3{,}115$ feet, and $c = 836.6$ feet; find A, B, and C. (Use second method.)

Ans. $\begin{cases} A = 21° 7' 24'' \\ B = 151° 31' 8'' \\ C = 7° 21' 30'' \end{cases}$

---

24. Case IV: Given Two Sides and the Angle Opposite One of Them.—In the triangle ABC, let a, b, and A be given, to find B, C, and c. The angle B or C is found by means of the principle of sines; thus,

$$\frac{a}{\sin A} = \frac{b}{\sin B}, \text{ whence } \sin B = \frac{b \sin A}{a}$$

Then, $$C = 180° - A - B, \text{ and } c = \frac{a}{\sin A} \sin C$$

When the data are given as above, without any further restrictions, there may be two triangles that will answer the given conditions; and the problem is said to have two solutions. For here the angle B is determined from its sine, and as every sine corresponds to two supplementary angles, either of these angles may be taken. Thus, if $\sin B$ is found to be .64746, the corresponding angle may be either 40° 21′ or 180° − 40° 21′ = 139° 39′, since these angles both have the same sine (Art. **13**).

The same result is obtained from geometrical considerations. On any line AX, Fig. 7, construct an angle equal to the given angle A, and on its side AC take AC equal to one of the given sides b. From C as a center, with a radius equal to the side a, describe an arc. This arc will generally cut AX at two points, B_1 and B_2, and either of the triangles ACB_1 or ACB_2 will answer the conditions of the problem, for they both contain the given sides b and a, and the angle A opposite a.

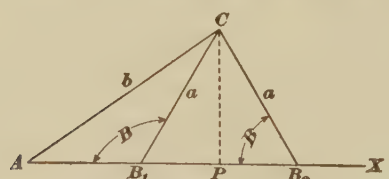

FIG. 7

The problem will have but one solution in the following cases:

1. If $a = b \sin A$. For in this case a will be equal to the perpendicular CP, Fig. 7, and the arc described from C will touch AX at P only.

2. If $a = b$. For in this case the angles A and B must be equal, and therefore both acute, since a triangle cannot have two obtuse angles. In this case B_1 coincides with A in Fig. 7, since $CB_2 = CA$.

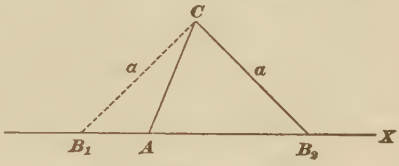

FIG. 8

3. When a is greater than b. For in this case A must be greater than B, and the latter angle must therefore be acute. This is shown by Fig. 8; the arc described from C cuts AX produced at B_1, and CB_1, although equal to a, is not opposite A ($= CAB_2$).

When a is less than $b \sin A$, the problem is impossible. For then a is less than CP, Fig. 7, and the arc does not cut AX at all. This is also shown by the formula $\sin B = \dfrac{b \sin A}{a}$, which would give $\sin B$ a value greater than 1, which is an impossible value, for no sine can be greater than 1.

EXAMPLE.—Given $a = 273$ feet, $b = 392$ feet, and $A = 37° 14'$; to find B, C, and c.

SOLUTION.—Here a is less than b, and, unless $\sin B$ is found to be greater than 1 (in which case the problem is impossible), there are two solutions.

$$\sin B = \frac{b \sin A}{a} = \frac{392 \times \sin 37° 14'}{273}; \quad B = \begin{cases} 60° 19' 17'', \text{ or} \\ 180° - 60° 19' 17'' \\ = 119° 40' 43''. \quad \text{Ans.} \end{cases}$$

$$C = \begin{cases} 180° - 37° 14' - 60° 19' 17'' = 82° 26' 43'', \text{ or} \\ 180° - 37° 14' - 119° 40' 43'' = 23° 5' 17''. \quad \text{Ans.} \end{cases}$$

$$c = \frac{a}{\sin A} \sin C = \frac{273}{\sin 37° 14'} \sin \begin{cases} 82° 26' 43'', \text{ or} \\ 23° 5' 17'' \end{cases} = \begin{cases} 447.27 \text{ ft., or} \\ 176.93 \text{ ft.} \quad \text{Ans.} \end{cases}$$

PRACTICAL EXAMPLES

EXAMPLE 1.—The distance between two points A and B, Fig. 9, is 360.38 feet; the angles from A and B to a station C are found, with a transit, to be, respectively, 62° 17′ and 39° 51′. What are the distances of C from A and B?

SOLUTION.— $C = 180° - 62° 17' - 39° 51' = 77° 52'$. Modulus ($M$) of triangle $= \dfrac{360.38}{\sin 77° 52'}$. Then (Art. **18**),

$$a = \frac{360.38}{\sin 77° 52'} \sin 62° 17' = 326.32 \text{ ft.} \quad \text{Ans.}$$

$$b = \frac{360.38}{\sin 77° 52'} \sin 39° 51' = 236.2 \text{ ft.} \quad \text{Ans.}$$

EXAMPLE 2.—The distances of a fort C from two other forts A and B are as marked in Fig. 10; the lines of sight from C to A and B make an angle of 53° 8′ 16″. What is the distance between the two forts A and B?

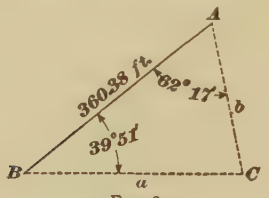

FIG. 9

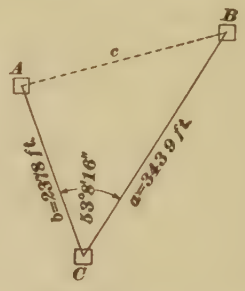

FIG. 10

SOLUTION.—The two sides and the included angle are given, and formulas **1** and **2**, Art. **21**, will be applied. It is not necessary to determine the angles A and B, for they are not required. Formula **1**, Art. **21**,

$$\tan \tfrac{1}{2}(A - B) = \frac{a - b}{a + b} \cot \tfrac{1}{2} C$$

$$= \frac{3{,}439 - 2{,}378}{3{,}439 + 2{,}378} \cot \frac{53° 8' 16''}{2}$$

$$= \frac{1{,}061}{5{,}817} \cot 26° 34' 8''$$

$$\tfrac{1}{2}(A - B) = 20° 2' 20''$$

Formula **2**, Art. **21**,

$$c = AB = \frac{(a - b) \cos \tfrac{1}{2} C}{\sin \tfrac{1}{2}(A - B)}$$

$$= \frac{1{,}061 \cos 26° 34' 8''}{\sin 20° 2' 20''}$$

$$= 2{,}769.4 \text{ ft.} \quad \text{Ans.}$$

FIG. 11

EXAMPLE 3.—A weight W, Fig. 11, is to be hung from a pulley sliding freely on the rope OQP. The length of the rope is l, and its ends are fastened at two points O and P, whose horizontal distance is d and whose vertical distance is h, as shown. It being proved in mechanics that the pulley will

rest in equilibrium when the vertical line WQ bisects the angle OQP, what are the lengths x $(= OQ)$ and y $(= PQ)$ of the two segments of the rope for which that condition obtains?

NOTE.—This problem is given here as an illustration of the many problems that occur in practice requiring the exercise of some ingenuity and the performance of some transformations, both algebraical and trigonometrical, before the required results are obtained.

SOLUTION.—Let QD be a vertical line through Q. According to the data, this line makes equal angles with OQ and QP. These angles are denoted by Z. The angles made by OQ and PQ with OP are denoted by Y and X, respectively. The line OR is horizontal, and PR vertical

As OR and RP are known, the right triangle OPR gives

$$\tan M = \frac{h}{d}$$

Also, in the triangle OED, $N = 90° - M$.

The angles M and N may, therefore, be assumed to be known.

Drawing PF parallel to RO, we have

$$d\,(= OR) = OD + DR = OD + PF$$

or, substituting the values of OD and PF from the triangles ODQ and PFQ,

$$d = x \sin Z + y \sin Z = (x + y) \sin Z = l \sin Z$$

whence,

$$\sin Z = \frac{d}{l}$$

Having found Z, we have

$$X = 180° - (N + Z) \text{ (triangle } PEQ)$$
$$Y = N - Z \text{ (triangle } OEQ)$$

The modulus of the triangle OPQ is

$$\frac{OP}{\sin OQP} = \frac{OP}{\sin 2Z} = \frac{d \div \cos M}{\sin 2Z} = \frac{d}{\cos M \sin 2Z} = \frac{d}{\sin N \sin 2Z}$$

Therefore (Art. **18**), $x = \dfrac{d}{\sin N \sin 2Z} \sin X$

or, substituting the value of X, and noticing that $\sin[180° - (N + Z)] = \sin(N + Z)$,

$$x = \frac{d}{\sin N \sin 2Z} \sin(N + Z). \quad \text{Ans.}$$

Likewise,

$$y = \frac{d}{\sin N \sin 2Z} \sin(N - Z). \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. Find the distance MN across the lake from the data shown in Fig. 12.

Ans. $MN = 669.51$ ft.

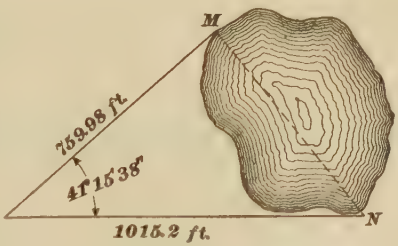

FIG. 12

2. The angles from two stations M and N, Fig. 13, to two inaccessible

points P and Q being as shown, and the distance MN being 550 feet, find the distance PQ.

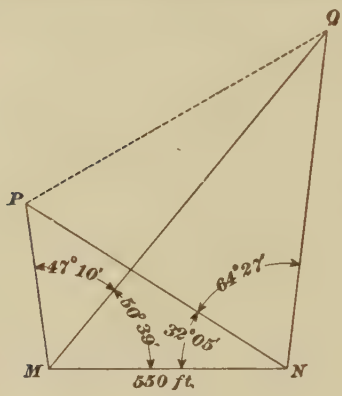

FIG. 13

HINT.—First calculate MP, then MQ, and finally PQ.

Ans. $PQ = 799.7$ ft.

3. In Fig. 14, the sides AB and DE were measured and the angles were turned as marked. Find the lengths of the sides BC, CA, CF, AF, CD, FD, EF.

Ans. $\begin{cases} BC = 677.92 \text{ ft.} \\ CA = 1{,}065.8 \text{ ft.} \\ CF = 905.46 \text{ ft.} \\ AF = 703.1 \text{ ft.} \\ CD = 696.83 \text{ ft.} \\ FD = 1{,}019.7 \text{ ft.} \\ EF = 687.97 \text{ ft.} \end{cases}$

4. Two observers on the same side of a steeple, and in the same vertical plane with it, are 100 feet apart, and find that the angles of elevation are 26° 28′ and 49° 14′. What is the height of the steeple?

Ans. 87.225 ft.

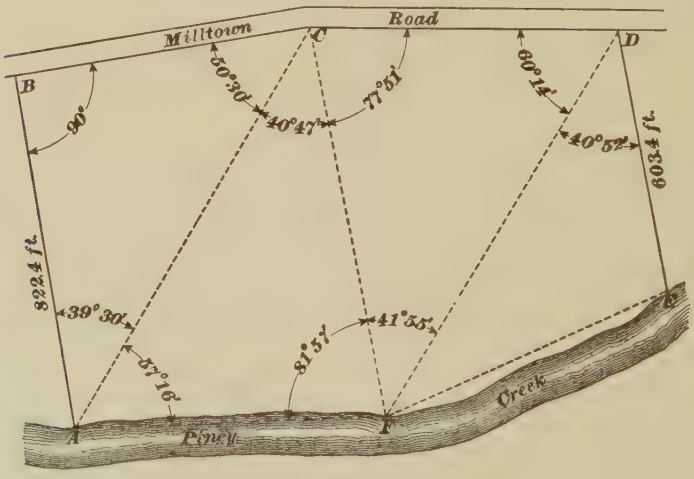

FIG. 14

5. Find the altitude h and the lengths of the sides AB and CD of the trapezoid $ABCD$, Fig. 15.

Ans. $\begin{cases} h = 62.22 \text{ ft.} \\ AB = 87.56 \text{ ft.} \\ CD = 64.579 \text{ ft.} \end{cases}$

6. The connecting-rod AB, Fig. 16, of an engine is 9 feet 3 inches, and the crank-arm CB is $10\frac{1}{2}$ inches; the figure shows the crank after

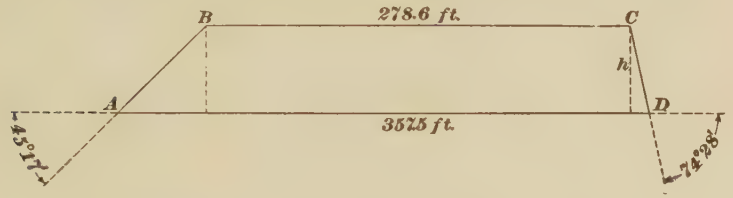

Fig. 15

it has performed one-eighth of a revolution, starting from the position CB'. Find: (a) the inclination M of the connecting-rod to the axis

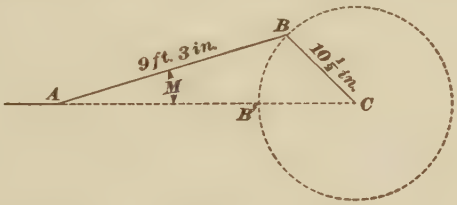

Fig. 16

of the piston rod, which is in line with CA; (b) the distance AC of the joint A from the center of the crank-circle.

Ans. $\begin{cases} (a)\ M = 3^\circ\ 50'\ 7'' \\ (b)\ AC = 9 \text{ ft. } 10\frac{1}{2} \text{ in., nearly} \end{cases}$

AREAS

LAND MEASURE

25. In surveying the public lands of the United States and Canada, all linear measurements are made with the surveyors' chain, also known as Gunter's chain, from the name of the inventor. This chain is 66 feet in length and contains 100 links, each 7.92 inches long. In private surveys, the foot is commonly taken as the unit of linear measure, and small land areas are expressed in square feet.

Land areas of considerable extent in the countries mentioned are generally expressed in acres. Fractional parts of an acre, which formerly were expressed in roods, square rods or perches, and square links, are now expressed decimally by nearly all surveyors. Thus, 40.35 acres is written instead of 40 acres, 1 rood, and 16 square rods.

Tables of linear and square measure are given in *Arithmetic*, and to those tables the student is referred for detailed information regarding the subject. The following table gives the relative values of the units of area used in land surveying in the countries referred to above. As already stated, the square foot and acre are now the units most commonly employed.

TABLE OF LAND MEASURE

1 square yard (sq. yd.)	=	9 square feet (sq. ft.)
1 square rod* (sq. rd)	=	$30\frac{1}{4}$ square yards = $272\frac{1}{4}$ square feet
1 square chain (sq. ch.)	=	16 square rods = 4,356 square feet
1 acre (A.)	=	10 square chains = 43,560 square feet
1 rood (R.)	=	40 square rods = 10,890 square feet
1 acre	=	4 roods = 160 square rods
1 square mile (sq. mi.)	=	640 acres = 6,400 square chains
1 township (Tp.)	=	36 square miles = 23,040 acres (app.)

*Sometimes called a perch or pole, and designated by the abbreviation P.

As will be observed, there are 10 square chains in an acre. In order, therefore, to reduce to acres any number of square chains, it is sufficient to move the decimal point one place toward the left, which is equivalent to dividing by 10. It must also be borne in mind that, since there are 100 links in 1 chain, links are usually expressed decimally as hundredths of a chain. Thus, 6.72 chains is written instead of 6 chains 72 links.

EXAMPLE 1.—A rectangular piece of land is 1,060 feet in length by 820 feet in breadth; what is its area: (*a*) in acres and decimals? (*b*) in acres, roods, and perches?

SOLUTION.— (*a*) $1{,}060 \times 820 = 869{,}200$ sq. ft.; $869{,}200 \div 43{,}560 = 19.954$ A. Ans.

(*b*) .954 A. $= .954 \times 4 = 3.816$ R.; .816 R. is equal to $.816 \times 40 = 32.64$ P. Hence, the area is 19 A. 3 R. 32.64 P. Ans.

EXAMPLE 2.—A rectangular piece of land is 12 chains and 6 links (12.06 chains) in length by 8 chains and 55 links (8.55 chains) in breadth; what is its area: (*a*) in acres and decimals? (*b*) in acres, roods, and perches?

SOLUTION.— (*a*) $12.06 \times 8.55 = 103.11$ sq. ch.; $103.11 \div 10 = 10.311$ A. Ans.

(*b*) .311 A. $= .311 \times 4 = 1.244$ R.; .244 R. is equal to $.244 \times 40 = 9.76$ P. Hence, the area is 10 A. 1 R. 9.76 P. Ans.

EXAMPLES FOR PRACTICE

1. A rectangular piece of land is 1,190 feet in length by 700 feet in breadth; what is its area: (*a*) in acres and decimals? (*b*) in acres, roods, and perches?

Ans. { (*a*) 19.123 A.
(*b*) 19 A. 0 R. 19.7 P.

2. A rectangular piece of land is 525 feet long by 250 feet wide, what is its area: (*a*) in acres and decimals? (*b*) in acres, roods, and perches?

Ans. { (*a*) 3.013 A.
(*b*) 3 A. 0 R. 2.08 P.

3. A rectangular piece of land is 15 chains and 65 links in length by 8 chains and 16 links in breadth; what is its area: (*a*) in acres and decimals? (*b*) in acres, roods, and perches?

Ans. { (*a*) 12.77 A.
(*b*) 12 A. 3 R. 3.2 P.

AREAS OF POLYGONS

THE TRIANGLE

NOTE.—In all that follows, the area of any figure under consideration will be designated by S, unless otherwise stated.

26. Given the Base and Altitude.—Any of the sides of a triangle may be taken as the base, the altitude being the length of the perpendicular drawn on the base from the vertex of the opposite angle. In Fig. 17, b is taken as the base, and the perpendicular BH, denoted by h, is the altitude.

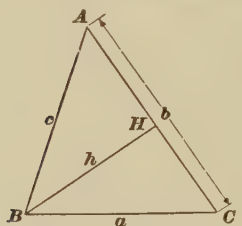

FIG. 17

It was shown in *Geometry*, Part 2, that the area of a triangle, when the base b and altitude h are known, is given by the formula

$$S = \tfrac{1}{2}\, b\, h$$

27. Given Two Sides and the Included Angle.—Let b, c, and A, Fig. 17, be given. In the right triangle ABH, we have $h = c \sin A$. The substitution of this value of h in the formula in Art. **26** gives

$$S = \tfrac{1}{2}\, b\, c \sin A$$

In words, *the area of a triangle is equal to one-half the product of any two sides and the sine of their included angle.*

EXAMPLE.—Two of the sides of a triangular field are 39.47 and 59.23 chains, respectively, and their included angle is 65° 10′ 40″. To find the contents of the field, in acres.

SOLUTION.—By the formula, S (square chains) $= \tfrac{1}{2} \times 39.47 \times 59.23$ sin 65° 10′ 40″ = 1,060.9 sq. ch.; whence, dividing by 10 (Art. **25**),

S (acres) = 106.09 A. Ans.

28. Given One Side and Two Angles.—The other angle may be at once found by subtracting the sum of the two given angles from 180°. It may, therefore, be assumed that the three angles are known. Let b, Fig. 17, be the given side. From Art. **22**, the value of c is equal to the modulus of the triangle multiplied by sin C, or,

$$c = \frac{b}{\sin B} \sin C$$

Substituting this value in the formula in Art. **27**, we obtain

$$S = \frac{b^2 \sin A \sin C}{2 \sin B}$$

29. The formula in Art. **28** is convenient when logarithmic functions are employed. For the use of natural functions, the following is preferable:

In the right triangles ABH and CBH, Fig. 17, we have,

$$AH = h \cot A, \; CH = h \cot C$$

whence, adding these two equations,

$$AH + CH = h \cot A + h \cot C$$

that is,

$$b = h(\cot A + \cot C)$$

and, therefore,

$$h = \frac{b}{\cot A + \cot C} \qquad (1)$$

This formula is useful and should be committed to memory. It may be stated in words thus: *The altitude of a triangle is equal to the base divided by the sum of the cotangents of the adjacent angles.*

By substituting, in the formula in Art. **26**, the value of h given in formula **1**, we obtain

$$S = \frac{b^2}{2(\cot A + \cot C)} \qquad (2)$$

In words, *the area of a triangle is equal to the square of any side divided by twice the sum of the cotangents of the angles adjacent to that side.*

EXAMPLE.—One side of a triangular field is 127.64 chains, and the adjacent angles are 46° 15′ and 60° 41′. To find the area.

SOLUTION BY LOGARITHMIC FUNCTIONS.—Here, $b = 127.64$, $A = 46° 15'$, $C = 60° 41'$, and $B = 180° - 46° 15' - 60° 41' = 73° 4'$. Formula of Art. **28**,

$$S = \frac{127.64^2 \sin 46° 15' \sin 60° 41'}{2 \sin 73° 4'}$$

$$= 5{,}363.4 \text{ sq. ch.} = 536.34 \text{ A.} \quad \text{Ans.}$$

SOLUTION BY NATURAL FUNCTIONS.—By formula **2**,

$$S = \frac{127.64^2}{2(\cot 46° 15' + \cot 60° 41')} = \frac{127.64^2}{2(.95729 + .56156)}$$

$$= \frac{127.64^2}{3.0377} = 5{,}363.4 \text{ sq. ch.} = 536.34 \text{ A.} \quad \text{Ans.}$$

NOTE.—Even if natural functions are used, the division is advantageously performed by means of logarithms.

EXAMPLES FOR PRACTICE

1. Two sides of a triangular field are 3,760 and 2,757 feet, respectively, and their included angle is 54° 13′ 13″. What is the area of the field, in acres? Ans. $S = 96.534$ A.

2. One side of a triangle is 96.34 chains; the opposite angle is 49° 10′, and one of the adjacent angles, 69° 45′ 30″. What is the area of the triangle, in acres? Ans. $S = 503.69$ A.

3. One side of a triangle is 8.93 inches, and the adjacent angles are 34° 16′ and 17° 37′ 18″. What is the area of the triangle?
Ans. $S = 8.638$ sq. in.

4. Two sides of a triangle are 17 and 25 feet, respectively, and the included angle is 76° 13′. What is the area of the triangle?
Ans. $S = 206.38$ sq. ft.

---

30. Given the Three Sides.—Let a, b, and c, Fig. 17, be given, and denote $\frac{1}{2}(a + b + c)$ by s. The area S of the triangle is given by the following formula, which is derived in Appendix VI:

$$S = \sqrt{s(s-a)(s-b)(s-c)}$$

EXAMPLE.—The sides of a triangular tract are 1,634.6 (= a, say), 978.28 (= b, say), and 2,176.4 (= c, say) feet, respectively; to find the area, in acres.

SOLUTION.—The work may be conveniently arranged as shown below. The numbers in marks of parenthesis indicate the order in which the several quantities are set down. In (6), s is placed above a, b, c in order to facilitate the subtractions. The differences $s - a$, $s - b$, $s - c$ are written, as the subtractions are performed, horizontally opposite a, b, and c, respectively.

(6)	$s = 2{,}394.64$		
(1)	$a = 1{,}634.60$	(7) $s - a =$	760.04
(2)	$b = 978.28$	(8) $s - b =$	1,416.36
(3)	$c = 2{,}176.40$	(9) $s - c =$	218.24
(4)	$2\,s = 4{,}789.28$		
(5)	$s = 2{,}394.64$		
(10)	$\log s = 3.37924$		
(11)	$\log (s-a) = 2.88083$		
(12)	$\log (s-b) = 3.15117$		
(13)	$\log (s-c) = 2.33893$		
	$2)\,11.75017$		

$\log S = 5.87509$; $S = 750{,}050$ sq. ft. $= 17.22$ A. Ans.

EXAMPLES FOR PRACTICE

1. Find the area of a triangular tract whose sides are 54.36, 73.19, and 101.76 chains, respectively. Ans. $S = 192.26$ A.

2. Find the area of a triangular plate whose sides are 17.12, 12.75, and 8.95 inches, respectively. Ans. $S = 55.646$ sq. in.

THE TRAPEZOID

31. Notation.—In Fig. 18, the bases, or parallel sides, of the trapezoid $A\,B\,C\,D$ are denoted by b_1 and b_2; the altitude, by h; and the sides $A\,D$ and $B\,C$, by a and c, respectively. The angles will be designated by the letters A, B, C, D at the vertexes. The line $D\,B'$ is drawn through D parallel to $C\,B$, thus forming a parallelogram in which $B'\,B = D\,C = b_2$, and $D\,B' = C\,B = c$. Also, angle $D\,B'\,A = B$, and $A\,B' = A\,B - B'\,B = b_1 - b_2$. For some purposes, it is convenient to represent this difference by a single letter d, as shown in the figure.

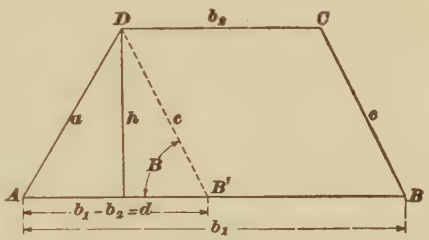

FIG. 18

32. Given the Bases and the Altitude.—As shown in *Geometry*, Part 2, the area of a trapezoid is equal to one-half the product of the altitude by the sum of the bases; that is,

$$S = \tfrac{1}{2}(b_1 + b_2)h$$

33. Given the Bases and the Angles Adjacent to One of Them.—Let b_1, b_2, A, and B, Fig. 18, be given. In the triangle $A\,D\,B'$ we have (formula **1**, Art. **29**),

$$h = \frac{b_1 - b_2}{\cot A + \cot B}$$

If this value of h is substituted in the formula of Art. **32**, the result is,

$$S = \frac{(b_1 - b_2)(b_1 + b_2)}{2(\cot A + \cot B)} \qquad (1)$$

As the product of the sum of two quantities by their difference is equal to the difference between the squares of the

quantities, $(b_1 - b_2)(b_1 + b_2)$ is equal to $b_1^2 - b_2^2$; and, therefore, formula **1** may also be written:

$$S = \frac{b_1^2 - b_2^2}{2(\cot A + \cot B)} \qquad (2)$$

For the use of logarithmic functions, formula **1** may be transformed into the following (see Appendix VII):

$$S = \frac{(b_1 - b_2)(b_1 + b_2)\sin A \sin B}{2 \sin (A + B)} \qquad (3)$$

In the application of these formulas, the student should bear in mind that the cotangent of an angle greater than 90° is negative, and numerically equal to the cotangent of the supplement of the angle; also, that the sine of an angle greater than 90° is equal to the sine of its supplement. Thus, $\cot 105° = -\cot (180° - 105°) = -\cot 75° = -.26795$; and $\sin 105° = \sin 75° = .96593$.

EXAMPLE 1.—The two bases of a trapezoid are 350 and 137 chains, respectively; the angles adjacent to the longer base are 75° 10′ and 63° 54′. What is the area of the trapezoid?

SOLUTION BY NATURAL FUNCTIONS.—Let $350 = b_1$, $137 = b_2$, $A = 75° 10'$, $B = 63° 54'$. As b_1 and b_2 are not convenient numbers to square, formula **1**, which is better adapted to logarithmic work, will be used.

$$S = \frac{(350 - 137)(350 + 137)}{2(\cot 75° 10' + \cot 63° 54')} = \frac{213 \times 487}{2(.26483 + .48989)} = 68{,}721 \text{ sq. ch.}$$
$$= 6{,}872.1 \text{ A. Ans.}$$

SOLUTION BY LOGARITHMIC FUNCTIONS.—By formula **3**,

$$S = \frac{(350 - 137)(350 + 137)\sin 75° 10' \sin 63° 54'}{2 \sin 139° 4'}$$

or, replacing $\sin 139° 4'$ by $\sin (180° - 139° 4') = \sin 40° 56'$,

$$S = \frac{213 \times 487 \sin 75° 10' \sin 63° 54'}{2 \sin 40° 56'} = 68{,}721 \text{ sq. ch.} = 6{,}872.1 \text{ A. Ans.}$$

EXAMPLE 2.—The bases of a trapezoid are 100 and 70 feet, the angles adjacent to the shorter base being 52° 47′ and 143° 14′. What is the area of the trapezoid?

SOLUTION.—Since the bases are parallel, the two angles adjacent to each of the non-parallel sides are supplementary. Thus, in Fig. 18, $A + D = 180°$, $B + C = 180°$; and, therefore, $A = 180° - D$; $B = 180° - C$. Let $52° 47' = D$, $143° 14' = C$. Then,

$$A = 180° - 52° 47' = 127° 13'$$
$$B = 180° - 143° 14' = 36° 46'$$
$$\cot A = -\cot (180° - 127° 13') = -\cot 52° 47' = -.75950$$
$$\cot B = \cot 36° 46' = 1.33835$$

Formula 2,

$$S = \frac{100^2 - 70^2}{2(+1.33835 - .7595)} = \frac{5{,}100}{1.1577} = 4{,}405.3 \text{ sq. ft.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE

1. The bases of a trapezoidal tract are 78.63 and 54.71 chains, respectively; the angles adjacent to the longer base are 55° 18′ and 62° 53′. Find the area, in acres. Ans. $S = 132.4$ A.

2. Find the number of square feet in a trapezoidal cross-section of a canal 40 feet wide at the bottom, 65 feet wide at the top, and whose non-parallel sides are inclined to the horizontal at an angle of 50°. (The dimensions across the top and bottom are measured horizontally.) Ans. $S = 782.09$ sq. ft.

3. The two bases of a trapezoid are 10.25 and 18.76 inches, respectively; one of the angles adjacent to the shorter base is 76° 45′ 10″, and the angle diagonally opposite is 66° 8′ 9″; find the area of the trapezoid. (Use logarithmic functions.) Ans. $S = 596.4$ sq. in.

---

34. Given the Four Sides.—If the difference between the two bases added to the sum of the non-parallel sides is denoted by $2s$; that is, if the expression $\frac{1}{2}(a + c + d)$, Fig. 18, is denoted by s, the area of the trapezoid is given by the following formula (see Appendix VIII):

$$S = \frac{b_1 + b_2}{d}\sqrt{s(s-a)(s-c)(s-d)}$$

EXAMPLE FOR PRACTICE

The bases of a trapezoidal field are 136.43 and 210.18 chains, respectively; one of the non-parallel sides is 96.73 chains, and the other 164.37 chains. Find the area of the tract, in acres.

Ans. $S = 864.97$ A.

THE REGULAR POLYGON

35. Given the Number of Sides and the Radius. Let MN, Fig. 19, be one of the sides of a regular polygon of n sides; O, the center, and r the radius, of the circumscribed circle (called also the center and radius, respectively, of the polygon); and A, the angle at the center subtended by a side of the polygon. The length of the side MN will be denoted by l.

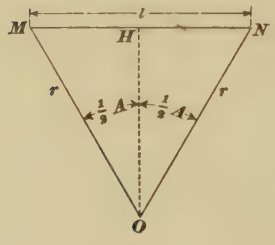

FIG. 19

Let n and r be given, to find the area S of the polygon and the length l of each of its sides. From *Geometry*, Part 2, the angle MON, or A, is found by dividing 360° by the number of sides in the polygon; that is,

$$A = \frac{360°}{n}$$

The area of the triangle MON is (Art. **27**) $\frac{1}{2} OM \times ON \sin MON$, or $\frac{1}{2} r \times r \sin A = \frac{1}{2} r^2 \sin A = \frac{1}{2} r^2 \sin \frac{360°}{n}$.

Since the polygon consists of n triangles equal to MON, its area S is equal to n times the area of MON; that is,

$$S = n \times \tfrac{1}{2} r^2 \sin \frac{360°}{n}$$

or

$$S = \tfrac{1}{2} n r^2 \sin \frac{360°}{n} \qquad (1)$$

In the right triangle MOH, we have,

$$MH = r \sin \frac{A}{2}$$

or, since MH is one-half of MN, or of l,

$$\frac{l}{2} = r \sin \frac{A}{2}$$

whence, multiplying by 2,

$$l = 2r \sin \frac{A}{2}$$

Finally, $\frac{A}{2} = \frac{1}{2} \frac{360°}{n} = \frac{180°}{n}$. By the substitution of this

value in the expression for l just found, we get, finally,

$$l = 2r \sin \frac{180°}{n} \qquad (2)$$

36. When the Number of Sides and Their Common Length Are Given.—Let n and l, Fig. 19, be given, to find the radius r and the area S. The radius is found by solving formula **2**, Art. **35**, for r, which gives,

$$r = \frac{l}{2 \sin \frac{180°}{n}} \qquad (1)$$

In the triangle MOH, we have,

$$OH = MH \cot \tfrac{1}{2}A = \frac{MN}{2} \cot \tfrac{1}{2}A$$

The area of MON is $\frac{1}{2} MN \times OH$. Writing instead of OH the value just found,

$$\tfrac{1}{2} MN \times \frac{MN}{2} \cot \tfrac{1}{2}A = \frac{MN^2}{4} \cot \tfrac{1}{2}A$$

$$= \frac{l^2}{4} \cot \tfrac{1}{2}A = \frac{l^2}{4} \cot \frac{180°}{n}$$

Multiplying this by n, we obtain, for the area of the polygon,

$$S = \frac{n l^2}{4} \cot \frac{180°}{n} \qquad (2)$$

EXAMPLE 1.—Find the area, and also the length of the side, of a regular decagon inscribed in a 15-inch circle.

SOLUTION.—In practice, it is usual to refer to a circle by its diameter, and so a 15-in. circle is a circle whose diameter is 15 in. We have, therefore, $r = \frac{15}{2} = 7.5$, $n = 10$, $\frac{360°}{n} = \frac{360°}{10} = 36°$, $\frac{180°}{n} = 18°$, and formulas **1** and **2**, Art. **35**, give

$$S = \tfrac{1}{2} \times 10 \times 7.5^2 \sin 36° = 165.32 \text{ sq. in.} \quad \text{Ans.}$$

$$l = 2 \times 7.5 \sin 18° = 4.635 \text{ in.} \quad \text{Ans.}$$

EXAMPLE 2.—Each of the sides of an octagonal park is 150 feet; what is the area of the park, in acres?

SOLUTION.—Here $l = 150$ ft., $n = 8$, $\frac{180°}{n} = \frac{180°}{8} = 22\frac{1}{2}° = 22° 30'$, and formula **2**, Art. **36**, gives,

$S = \frac{1}{4} \times 8 \times 150^2 \cot 22° 30' = 2 \times 22{,}500 \cot 22° 30' = (45{,}000 \cot 22° 30')$ sq. ft. $= \frac{45{,}000 \cot 22° 30'}{43{,}560}$ A. $= 2.494$ A. Ans.

EXAMPLES FOR PRACTICE

1. Find the side and area of an equilateral triangle inscribed in a 20-inch circle.

Ans. $\begin{cases} l = 17.321 \text{ in.} \\ S = 129.9 \text{ sq. in.} \end{cases}$

2. What must be the length of the side and the radius of a regular pentagon, that its area may be 46.97 square feet?

Ans. $\begin{cases} l = 5.225 \text{ ft.} \\ r = 4.445 \text{ ft.} \end{cases}$

3. An eight-sided drive is to be built around a circular park 1,500 feet in diameter, the drive to be 15 feet wide, with its outer corners on the circumference of the park. Find: (*a*) the length of each of the sides of the outer boundary of the drive; (*b*) the length of each of the sides of the inner boundary; (*c*) the cost of paving the drive with asphalt, at \$2.25 per square yard; (*d*) the difference between the exact area of the drive and the approximate area found by assuming the polygonal boundaries to coincide with the circumferences of their respective circumscribed circles.

Ans. $\begin{cases} (a)\ 574.02 \text{ ft.} \\ (b)\ 561.60 \text{ ft.} \\ (c)\ \$17{,}025 \\ (d)\ 844 \text{ sq. yd.} \end{cases}$

OTHER POLYGONS

37. The area of any polygon can be determined by dividing the polygon into triangles, and measuring in each triangle whatever parts are necessary for the determination of its area. The parts to be measured depend on special conditions and on the instruments used. The polygon may be divided into triangles either by diagonals or by lines drawn from a convenient interior point to the different vertexes. Illustrations of these methods of division will be given in connection with surveying. When the area is to be determined from a plat, the base and altitude of each triangle are usually the most convenient parts to measure.

AREAS BOUNDED BY IRREGULAR OUTLINES

AREA INCLUDED BETWEEN A STRAIGHT LINE AND AN IRREGULAR CURVE

38. By Selected Ordinates.—Let it be required to determine the area between the curve DC and the straight line AB, Fig. 20. A very convenient method is to draw perpendiculars on AB from the points of the curve at which its direction changes appreciably, and to consider the portion of the curve between two consecutive perpendiculars to be a straight line. The figure is then treated as if divided into a number of trapezoids, whose areas can be computed by the rules of geometry. The perpendiculars are called **ordinates.** Both the lengths of the ordinates and the distances between every two consecutive ordinates should be measured. The area of any of the (approximate) trapezoids into which the figure is thus divided is equal to one-half the sum of the two ordinates enclosing it multiplied by the distance between them. It should be understood that both this rule and those given further on relating to the same subject are only approximate. Since the bounding curve is irregular, that is, does not follow any mathematical law, no exact formula can be found for the area.

FIG. 20

EXAMPLE.—Referring to Fig. 20, suppose that, beginning at the left of the figure, the successive ordinates measure 15, 13, 12, 13.5, 20, 21.5, 22, 20, and 16 feet, respectively, and that the successive distances between the offsets, from left to right, measure 7.5, 10, 15, 41, 10.5, 11.5, 11.5, and 21 feet, respectively; what is the area of the surface?

SOLUTION.—The area of the figure is approximately equal to the sum of the areas of the trapezoids into which it is divided, and the area of each trapezoid is equal to one-half the sum of its parallel sides multiplied by the perpendicular distance between them. Therefore, the area of the figure is equal to

$$\frac{15+13}{2}\times 7.5+\frac{13+12}{2}\times 10+\frac{12+13.5}{2}\times 15+\frac{13.5+20}{2}\times 41$$
$$+\frac{20+21.5}{2}\times 10.5+\frac{21.5+22}{2}\times 11.5+\frac{22+20}{2}\times 11.5+\frac{20+16}{2}\times 21$$
$$= 2{,}195.5 \text{ sq. ft. Ans.}$$

39. Trapezoidal Rule: Sigma Notation.—In order to facilitate the calculations, the ordinates are often measured at regular intervals along the straight line, as shown in Fig. 21. The area $A\,B\,C\,D$ included between the straight line and the irregular boundary can then be more easily calculated by what is commonly known as the **trapezoidal rule.** This

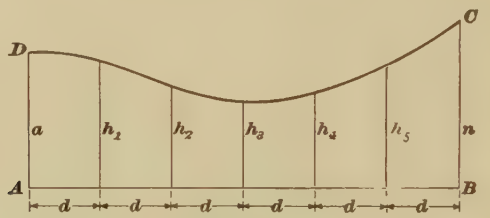

FIG. 21

is merely a rule for calculating the combined area of a series of trapezoids that have the same altitude, the areas being combined for convenience of calculation. The result given by this rule is closer the smaller the distance between the ordinates. The rule is as follows:

Rule.—*Add together one-half the two end ordinates and all the intermediate ordinates, and multiply the sum by the common distance between the ordinates.*

Let a = first ordinate;
n = last ordinate;
h_1, h_2, h_3 = intermediate ordinates;
d = common distance between ordinates;
S = area of surface.

Then, $S = [\frac{1}{2}(a+n)+h_1+h_2+h_3+\ldots]d$

This expression may be put in a simpler form by using the **sigma notation,** which is as follows: As will be noticed, all the intermediate ordinates are denoted by h,

different subscripts being used to indicate different values of h. We may, therefore, write the value of S thus,

$$S = [\tfrac{1}{2}(a + n) + \text{sum of all values of } h]d$$

Instead of the phrase *sum of all values of* h, the expression Σh, read *sigma* h, is used. The symbol Σ is the Greek letter sigma, corresponding to English S, and is very commonly used, as here, to indicate the addition of several quantities of the same character, denoted by a single symbol; hence, the name **sign of summation,** which also is often given to that letter.

By using the sigma notation, the value of S may be written

$$S = \left(\frac{a + n}{2} + \Sigma h\right)d$$

EXAMPLE.—If the ordinates from the straight line $A\,B$ to the curved boundary $D\,C$, Fig. 21, are 19, 18, 14, 12, 13, 17, and 23 links, respectively, and are at equal distances of 50 links, what is the area included between the curved boundary and the straight line?

SOLUTION.—Area $A\,B\,C\,D = \left(\frac{19 + 23}{2} + 18 + 14 + 12 + 13 + 17\right) \times 50 = 4{,}750$ sq. li. Ans.

40. Simpson's Rule.—The foregoing rule assumes that all the small figures into which the area is divided are perfect trapezoids, which assumption always involves more or less error, since the irregular boundary is in nearly all cases an irregular curve. When the offsets are taken at

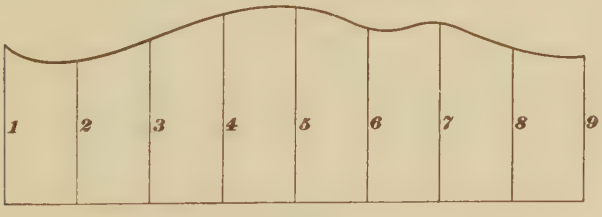

FIG. 22

regular intervals, the following rule, known as **Simpson's one-third rule,** gives a closer approximation. In applying this rule, the base line must be divided into an even number of equal parts; the ordinates measured at the points of division are numbered consecutively, as shown in Fig. 22.

Rule.—*Divide the base line into an even number of equal parts, and at the points of division erect ordinates terminating in the curve. Number the ordinates 1, 2, 3, etc., from left to right, including those at the ends of the base. Add together the end ordinates, four times the sum of all intermediate even-numbered ordinates, and twice the sum of all intermediate odd-numbered ordinates; multiply the total sum by one-third the common distance between adjacent ordinates.*

This rule has been used extensively; it can be expressed by a formula as follows:

Let h_2 = any intermediate even-numbered ordinate;
h_3 = any intermediate odd-numbered ordinate;
and let all other quantities be represented by the same letters as in the preceding article. Then,

$$S = (a + n + 4\,\Sigma\,h_2 + 2\,\Sigma\,h_3)\,\frac{d}{3}$$

The notation will be readily understood by reference to Fig. 23. The expression $4\,\Sigma\,h_2$ means four times the sum of all the ordinates h_2, or, in other words, four times the sum of all the even-numbered ordinates.

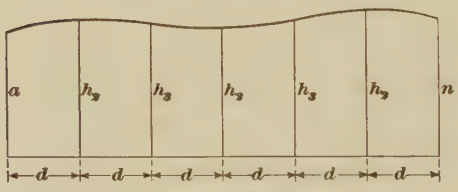

FIG. 23

EXAMPLE.—What is the area $A\,B\,C\,D$, Fig. 21, by Simpson's rule, using the same values as in the example in Art. **39**?

SOLUTION.— $S = [19 + 23 + 4\,(18 + 12 + 17) + 2\,(14 + 13)] \times \frac{50}{3}$ = 4,733 sq. li. Ans.

EXAMPLES FOR PRACTICE

1. A figure included between a straight base line, a curve, and two perpendiculars to the base at the ends has nine ordinates, including the two end perpendiculars, whose lengths are 43, 48, 39, 50, 41, 32, 37, 31, and 22 feet, respectively; the common distance between the ordinates is 60 feet. Find the area: (*a*) by the trapezoidal rule; (*b*) by Simpson's rule.

Ans. { (*a*) 18,630 sq. ft.
(*b*) 18,860 sq. ft.

2. In order to determine the area included between an irregular boundary, a straight base line, and two perpendiculars to the base at the ends, eight ordinates, including the two end perpendiculars, are measured from the straight line to the boundary. The ordinates are found to measure 16, 18, 12, 13, 15, 17, 19, and 20.5 feet, and the successive distances between them are found to measure 7.8, 10, 15, 20, 12, 40, and 5 feet, respectively. What is the area of the surface?

Ans. 1,760.9 sq. ft.

3. A surface lying between a straight base line and a curve is limited by two perpendiculars to the base line at the ends; the base line is divided into eight parts 50 feet each, and at the points of division ordinates are measured. The lengths of the successive ordinates, including the two end perpendiculars, are 10, 25, 38, 49, 58, 65, 70, 73, and 74 feet, respectively. Find the area of the surface: (*a*) by the trapezoidal rule; (*b*) by Simpson's rule.

Ans. { (*a*) 21,000 sq. ft.
(*b*) 21,067 sq. ft.

AREA BOUNDED BY AN IRREGULAR CURVE

41. By Ordinates.—Suppose that it is required to find the area enclosed by the heavy irregular curve shown in Fig. 24. A broken line *AEFMGHIA* is drawn around

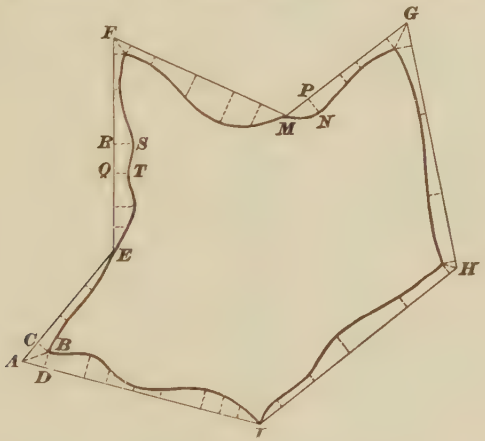

FIG. 24

the curved boundary line, and as close to it as convenient. Ordinates to the straight lines thus drawn are measured from the points where the direction of the curved boundary changes materially, as shown. The area of the polygon

$AEFMGHIA$ is calculated by one of the methods explained in preceding articles, and from it is subtracted the sum of the areas included between the curved boundary and the broken line, calculated as in Art. **39.**

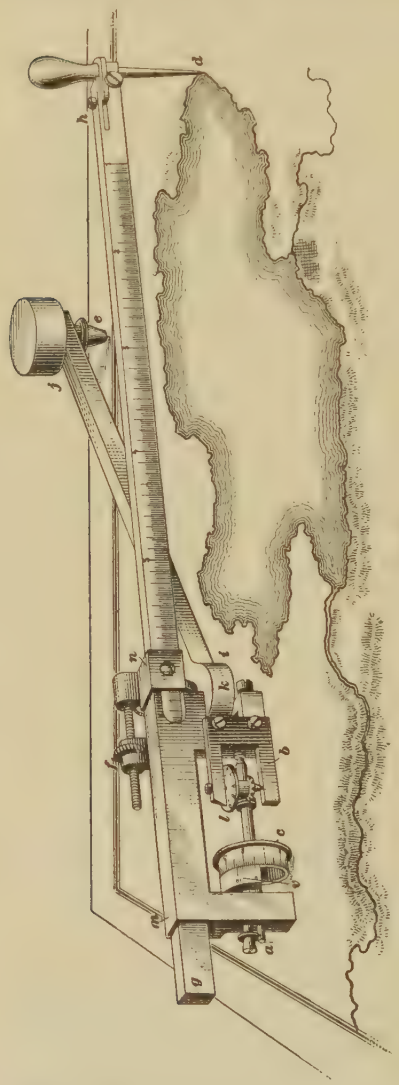

FIG. 25

At such corners as A, the triangles ABC and ABD are computed from the measured bases AC and AD and the altitudes BC and BD. All the quadrilaterals, as $QRST$, are treated as trapezoids; and such three-sided figures as MPN, as triangles. The process is so simple that it does not require any further explanation.

42. By the Planimeter.—The most convenient way to find the area of a plane surface having an irregular boundary is by the **planimeter.** There are several forms of planimeters; the one most commonly used is the **polar planimeter** (see Fig. 25). As will be seen from the illustration, this instrument has two arms ij and gh connected by a hinge joint. The point e at the end of the bar ij is called the **anchor**

point; it remains stationary while the point d, called the **pointer** or **tracer,** at the end of the bar $g\,h$ is moved over the outline of the figure whose area is to be determined. The movement of the pointer d causes the wheel c on the opposite end of the bar to roll on the paper; this wheel is called the **measuring wheel** or **counter wheel.** The graduated bar $g\,h$ can be adjusted by sliding it in or out through the socket m in the top of the frame. This bar is clamped by means of a clamp screw, a part of which is shown back of the small movable socket n, and is set at the exact length required by means of the thumbscrew f. The bar $i\,j$ is of fixed length; it is pivoted at k, the junction of the two bars. The measuring wheel c is mounted on the main axis $a\,b$, which is parallel with the bar $g\,h$. The complete revolutions of the wheel c are read on the disk l, and the fractional parts of revolutions are read on the wheel c and the vernier v, the tenths and hundredths being read on the wheel itself, and the thousandths on the vernier.

To use the planimeter, the anchor point e is fixed on the paper or drawing board, preferably outside the figure to be measured, the pointer d is placed on some point in the periphery of the figure, and a reading of the wheel c is taken. The point d is then moved carefully around the periphery of the figure, in a clockwise direction, or from left to right, to the point of beginning. A second reading of the wheel c is then taken, and the difference between the two readings is the number of revolutions of the wheel. If the wheel is set to read zero, the number of revolutions is given directly by the second reading.

If the anchor point is outside the area to be measured, the distance traversed by the wheel, or the product of the number of revolutions by the circumference of the wheel, in inches, multiplied by the length of the bar $n\,h$, in inches, is the area, in square inches, bounded by the path of the pointer d.

If the anchor point is inside the area, the product just referred to must be added to the area of the **zero circle,**

whose radius is equal to $\sqrt{p^2 + q^2 + 2\,p\,r}$, p being the length of the arm $n\,h$; r, the distance from the center of the wheel c to the center of the joint k; and q, the length of the bar $k\,j$. The bar $g\,h$ is generally set at such a length that ten times the number of revolutions of the wheel c is the area measured. This area is the actual area of the figure measured, and the area *represented* by the figure is determined from the scale of the plat. The area given by the planimeter, in square inches, must be multiplied by the square of the scale of the plat, in order to get the area sought. Thus, if the plat has been drawn to a scale of 50 feet to an inch, each square inch of the plat is equivalent to $50 \times 50 = 2{,}500$ square feet of area.

Suppose that the area bounded by the irregular line in Fig. 25, as measured by the planimeter, is 2.535 square inches, and that the scale of the plat is 100 feet to an inch; then the area represented by a square inch of the plat is $100 \times 100 = 10{,}000$ square feet, and the area represented by the closed figure is $10{,}000 \times 2.535 = 25{,}350$ square feet.

Full directions for using the planimeter are usually furnished by the maker.

APPENDIX: DERIVATION OF FORMULAS

I—FORMULAS 1 TO 4 OF ART. 16

Let $R\,O\,Q$, Fig. 26, be any angle A, and $Q\,O\,S$ any angle B. Then, $A + B = R\,O\,S$. From any point P on $O\,S$, draw $P\,N$ and $P\,M$, perpendicular, respectively, to $O\,R$ and $O\,Q$. Draw $M\,K$ parallel to $O\,R$ and therefore perpendicular to $P\,N$; also, $M\,L$ perpendicular to $O\,R$. The angles $M\,P\,K$ and $R\,O\,Q$, having their sides perpendicular each to each, are equal. Now,

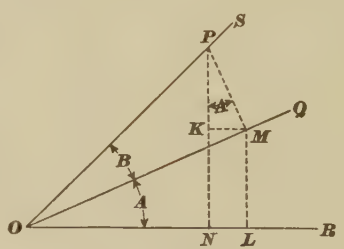

FIG. 26

$$\sin(A+B) = \frac{N\,P}{O\,P} = \frac{N\,K + K\,P}{O\,P} = \frac{M\,L}{O\,P} + \frac{K\,P}{O\,P} = \frac{O\,M \sin A}{O\,P} + \frac{P\,M \cos A}{O\,P}$$

(triangles $M L O$ and $P M K$) $= \sin A \dfrac{O M}{O P} + \cos A \dfrac{P M}{O P} = \sin A \cos B + \cos A \sin B$ (triangle $O P M$)

This is formula **1.**

Also,

$$\cos (A - B) = \sin [90° - (A - B)] = \sin [(90° - A) + B]$$

or, by formula **1**,

$$\cos (A - B) = \sin (90° - A) \cos B + \cos (90° - A) \sin B$$
$$= \cos A \cos B + \sin A \sin B$$

which is formula **4.**

Formula **3** follows from this; for

$$\sin (A - B) = \cos [90° - (A - B)] = \cos [(90° + B) - A]$$
$$= \cos (90° + B) \cos A + \sin (90° + B) \sin A$$

or, because $\cos (90° + B) = -\sin B$, and $\sin (90° + B) = \cos B$ (Art. **14**), $\sin (A - B) = -\sin B \cos A + \cos B \sin A = \sin A \cos B - \cos A \sin B$.

Finally, applying this formula,

$$\cos (A + B) = \sin [90° - (A + B)] = \sin [(90° - A) - B]$$
$$= \sin (90° - A) \cos B - \cos (90° - A) \sin B$$
$$= \cos A \cos B - \sin A \sin B$$

which is formula **2.**

II—FORMULAS OF ART. 19

Referring to Fig. 6 (a) and (b), Art. **18**,

$$a^2 = p^2 + \overline{B D}^2 \qquad (1)$$

In (a), $B D = c - A D$, whence $\overline{B D}^2 = c^2 - 2c \times A D + \overline{A D}^2$.

In (b), $B D = A D - c$, whence $\overline{B D}^2 = \overline{A D}^2 - 2c \times A D + c^2$.

Substituting this value of $B D$ in equation (1),

$$a^2 = p^2 + \overline{A D}^2 + c^2 - 2c \times A D \qquad (2)$$

But $p^2 + \overline{A D}^2 = b^2$, and $A D = b \cos A$; therefore,

$$a^2 = b^2 + c^2 - 2bc \cos A$$

When the angle opposite the side is obtuse, as B in Fig. 6 (b), the same reasoning leads to the relation,

$$b^2 = a^2 + c^2 + 2ac \times \cos C B D$$

the second member of which becomes $a^2 + c^2 - 2ac \cos B$, when $\cos C B D$ is replaced by its equal $-\cos B$ (Art. **13**).

III—FORMULAS OF ART. 20

Let $A\,B\,C$, Fig. 27, be any triangle. As usual, the angles of the triangle will be denoted by A, B, C, and the opposite sides by a, b, c, respectively; that is, angle $C\,A\,B = A$, $B\,C = a$, etc. Produce $A\,C$ to A', making $C\,A' = B\,C = a$. Draw $B\,A'$, and $A\,P$ perpendicular to it, meeting $B\,C$ at Q.

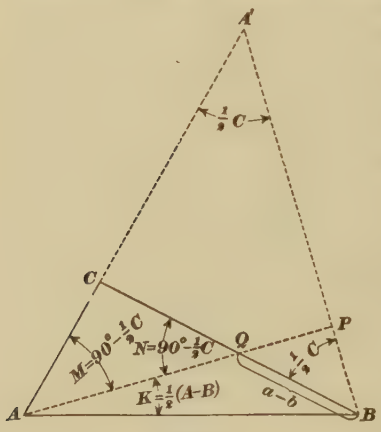

FIG. 27

Since $B\,C = C\,A'$, the triangle $B\,C\,A'$ is isosceles, and, therefore, the angles $C\,A'\,B$ and $C\,B\,A'$ are equal. The sum of these two angles, or twice either of them, is equal to the external angle $B\,C\,A$, or C, and therefore each of these two angles is equal to $\frac{1}{2}\,C$. In the right triangle $A\,P\,A'$, the angle M, being the complement of A', is equal to $90° - \frac{1}{2}\,C$.

We have also,

$$K = A - M = A - (90° - \tfrac{1}{2}\,C),$$

or, since $C = 180° - (A + B) = 180° - A - B$,

$$K = A - [90° - \tfrac{1}{2}(180° - A - B)] = \tfrac{1}{2}(A - B)$$

The angle N being external to the triangle $A\,Q\,B$, we have

$$N = K + B = \tfrac{1}{2}(A - B) + B = \tfrac{1}{2}(A + B)$$
$$= \tfrac{1}{2}(180° - C) = 90° - \tfrac{1}{2}\,C = M$$

Therefore, the triangle $A\,Q\,C$ is isosceles, and $Q\,C = A\,C = b$; and, consequently, $B\,Q = a - b$.

The right triangle $A\,B\,P$ gives,

$$\tan \tfrac{1}{2}(A - B) = \frac{B\,P}{A\,P}$$

or, writing the values of $B\,P$ and $A\,P$ from the triangles $B\,Q\,P$ and $A\,P\,A'$,

$$\tan \tfrac{1}{2}(A - B) = \frac{B\,Q \cos \frac{1}{2}\,C}{A\,A' \sin \frac{1}{2}\,C} = \frac{B\,Q}{A\,A'} \cot \tfrac{1}{2}\,C$$

that is,

$$\tan \tfrac{1}{2}(A - B) = \frac{a - b}{a + b} \cot \tfrac{1}{2}\,C \qquad (1)$$

Now, $\frac{1}{2}\,C = \frac{1}{2}[180° - (A + B)] = 90° - \frac{1}{2}(A + B)$, and therefore, $\cot \frac{1}{2}\,C = \tan \frac{1}{2}(A + B)$. By substituting this value in equation (1), and transforming, the formula in Art. **20** is obtained.

IV—FORMULA 2 OF ART. 21

This formula is derived from Fig. 27 as follows: In the triangle BPQ,

$$BP = BQ \cos \tfrac{1}{2} C = (a - b) \cos \tfrac{1}{2} C \qquad (1)$$

and, in the triangle ABP,

$$c\,(= AB) = \frac{BP}{\sin \tfrac{1}{2}(A - B)}$$

which becomes formula **2** when BP is replaced by its value (1).

V—FORMULAS 2 TO 4 OF ART. 23

We have (formula **8**, Art. **17**),

$$2 \cos^2 \tfrac{1}{2} A = 1 + \cos A$$

or, substituting the value of cos A from formula **1**, Art. **23**,

$$2 \cos^2 \tfrac{1}{2} A = 1 + \frac{b^2 + c^2 - a^2}{2bc} = \frac{2bc + b^2 + c^2 - a^2}{2bc} = \frac{(b + c)^2 - a^2}{2bc}$$

or, remembering that the difference between the squares of two numbers is equal to their sum multiplied by their difference,

$$2 \cos^2 \tfrac{1}{2} A = \frac{(b + c + a)(b + c - a)}{2bc} \qquad (1)$$

Now, since $a + b + c = 2s$, we have, subtracting $2a$ from both members, $b + c - a = 2s - 2a = 2(s - a)$. Likewise, $a + b - c = 2(s - c)$, and $a + c - b = 2(s - b)$. Substituting these values in equation (1),

$$2 \cos^2 \tfrac{1}{2} A = \frac{2s \times 2(s - a)}{2bc} = \frac{2s(s - a)}{bc}$$

whence,

$$\cos \tfrac{1}{2} A = \sqrt{\frac{s(s - a)}{bc}} \qquad (2)$$

which is formula **3**, Art. **23**.

Likewise (formula **7**, Art. **17**),

$$2 \sin^2 \tfrac{1}{2} A = 1 - \cos A = 1 - \frac{b^2 + c^2 - a^2}{2bc}$$

$$= \frac{2bc - b^2 - c^2 + a^2}{2bc} = \frac{a^2 - (b^2 - 2bc + c^2)}{2bc} = \frac{a^2 - (b - c)^2}{2bc}$$

$$= \frac{(a + b - c)(a - b + c)}{2bc} = \frac{2(s - c) \times 2(s - b)}{2bc} = \frac{2(s - b)(s - c)}{bc}$$

whence,

$$\sin \tfrac{1}{2} A = \sqrt{\frac{(s - b)(s - c)}{bc}} \qquad (3)$$

which is formula **4**, Art. **23**.

Formula **2** is obtained by dividing equation (3) by equation (2).

VI—FORMULA OF ART. 30

Formulas **3** and **4** of Art. **23** are:

$$\sin \tfrac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{bc}} \qquad (1)$$

$$\cos \tfrac{1}{2} A = \sqrt{\frac{s(s-a)}{bc}} \qquad (2)$$

Also (formula **5**, Art. **17**),

$$\sin A = 2 \sin \tfrac{1}{2} A \cos \tfrac{1}{2} A \qquad (3)$$

Substituting in equation (3) the values of $\sin \frac{1}{2} A$ and $\cos \frac{1}{2} A$ from equations (1) and (2),

$$\sin A = 2 \sqrt{\frac{(s-b)(s-c)}{bc}} \sqrt{\frac{s(s-a)}{bc}} = 2 \sqrt{\frac{s(s-a)(s-b)(s-c)}{b^2 c^2}}$$

$$= 2 \frac{\sqrt{s(s-a)(s-b)(s-c)}}{bc}$$

Substituting this value in formula of Art. **27**,

$$S = \sqrt{s(s-a)(s-b)(s-c)}$$

VII—FORMULA 3 OF ART. 33

We have, since $\cot = \frac{\cos}{\sin}$,

$$\frac{1}{\cot A + \cot B} = \frac{1}{\frac{\cos A}{\sin A} + \frac{\cos B}{\sin B}} = \frac{\sin A \sin B}{\sin B \cos A + \cos B \sin A}$$

$$= \frac{\sin A \sin B}{\sin (A+B)}$$

By substituting this value in formula **1**, we obtain

$$S = \frac{(b_1 - b_2)(b_1 + b_2) \sin A \sin B}{2 \sin (A+B)}$$

VIII—FORMULA OF ART. 34

Let the area of the triangle ADB', Fig. 18, be denoted by T, and that of the parallelogram $BCDB'$ by P. Then,

$$S = P + T \qquad (1)$$

Now, $P = b_2 h$, $T = \frac{1}{2} d h$

Dividing the first of these equations by the second,

$$\frac{P}{T} = \frac{b_2}{\frac{1}{2} d} = \frac{2 b_2}{d} = \frac{2 b_2}{b_1 - b_2}$$

whence, $$P = \frac{2 b_2}{b_1 - b_2} T$$

Substituting this value of P in equation (1),

$$S = \frac{2b_2}{b_1 - b_2} T + T = \left(\frac{2b_2}{b_1 - b_2} + 1\right) T = \frac{b_1 + b_2}{b_1 - b_2} T = \frac{b_1 + b_2}{d} T \qquad (2)$$

Let $\frac{1}{2}(a + c + d) = s$. Then (formula of Art. **30**),

$$T = \sqrt{s(s-a)(s-c)(s-d)}$$

and, substituting this value in equation (2),

$$S = \frac{b_1 + b_2}{d} \sqrt{s(s-a)(s-c)(s-d)}$$

TABLE OF TRIGONOMETRIC FORMULAS

The principal formulas occurring in the text, and others that can be readily derived from these, are tabulated in the following pages for convenient reference. As these formulas, which include those for the solution of triangles, are here systematically classified and arranged, the student will find this table useful in the solution of all kinds of problems requiring the application of trigonometry. He is advised to refer to it often, so as to become familiar with its contents and use.

FORMULAS DEFINING THE TRIGONOMETRIC FUNCTIONS

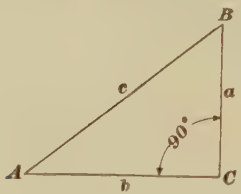

FIG. 28

1. $\sin A = \frac{a}{c}$
2. $\tan A = \frac{a}{b}$
3. $\cos A = \sin (90° - A) = \frac{b}{c}$
4. $\cot A = \tan (90° - A) = \frac{b}{a}$
5. $\sec A = \frac{c}{b}$
6. $\csc A = \sec (90° - A) = \frac{c}{a}$
7. $\text{vers } A = 1 - \cos A = 1 - \frac{b}{c}$
8. $\text{covers } A = \text{vers } (90° - A) = 1 - \sin A = 1 - \frac{a}{c}$

FUNCTIONS OF 0° AND 90°

9. $\sin 0° = 0$
10. $\tan 0° = 0$
11. $\cos 0° = 1$
12. $\cot 0° = \infty$
13. $\sec 0° = 1$
14. $\csc 0° = \infty$
15. $\sin 90° = 1$
16. $\tan 90° = \infty$
17. $\cos 90° = 0$
18. $\cot 90° = 0$
19. $\sec 90° = \infty$
20. $\csc 90° = 1$

FUNCTIONS OF NEGATIVE ANGLES

21. $\sin (-A) = -\sin A$
22. $\tan (-A) = -\tan A$
23. $\cos (-A) = \cos A$
24. $\cot (-A) = -\cot A$
25. $\sec (-A) = \sec A$
26. $\csc (-A) = -\csc A$

FUNCTIONS OF $90° + A$

27. $\sin(90° + A) = \cos A$
28. $\tan(90° + A) = -\cot A$
29. $\cos(90° + A) = -\sin A$
30. $\cot(90° + A) = -\tan A$
31. $\sec(90° + A) = -\csc A$
32. $\csc(90° + A) = \sec A$

FUNCTIONS OF $180° - A$ AND OF $180° + A$

33. $\sin(180° - A) = \sin A$
34. $\tan(180° - A) = -\tan A$
35. $\cos(180° - A) = -\cos A$
36. $\cot(180° - A) = -\cot A$
37. $\sec(180° - A) = -\sec A$
38. $\csc(180° - A) = \csc A$
39. $\sin(180° + A) = -\sin A$
40. $\tan(180° + A) = \tan A$
41. $\cos(180° + A) = -\cos A$
42. $\cot(180° + A) = \cot A$
43. $\sec(180° + A) = -\sec A$
44. $\csc(180° + A) = -\csc A$

FUNCTIONS OF $360° - A$ AND OF $360° + A$

45. $\sin(360° - A) = -\sin A$
46. $\tan(360° - A) = -\tan A$
47. $\cos(360° - A) = \cos A$
48. $\cot(360° - A) = -\cot A$
49. $\sec(360° - A) = \sec A$
50. $\csc(360° - A) = -\csc A$
51. $\sin(360° + A) = \sin A$
52. $\tan(360° + A) = \tan A$
53. $\cos(360° + A) = \cos A$
54. $\cot(360° + A) = \cot A$
55. $\sec(360° + A) = \sec A$
56. $\csc(360° + A) = \csc A$

FUNCTIONS OF $(A + B)$ AND OF $(A - B)$

57. $\sin(A + B) = \sin A \cos B + \cos A \sin B$
58. $\sin(A - B) = \sin A \cos B - \cos A \sin B$
59. $\cos(A + B) = \cos A \cos B - \sin A \sin B$
60. $\cos(A - B) = \cos A \cos B + \sin A \sin B$
61. $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$
62. $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

FUNCTIONS OF $2A$ AND OF $\frac{1}{2}A$

63. $\sin 2A = 2 \sin A \cos A$

64. $\cos 2A = \cos^2 A - \sin^2 A$

65. $\cos 2A = 2 \cos^2 A - 1$

66. $\cos 2A = 1 - 2 \sin^2 A$

67. $\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

68. $\sin \frac{1}{2}A = \sqrt{\dfrac{1 - \cos A}{2}}$

69. $\cos \frac{1}{2}A = \sqrt{\dfrac{1 + \cos A}{2}}$

70. $\tan \frac{1}{2}A = \sqrt{\dfrac{1 - \cos A}{1 + \cos A}}$

71. $\tan \frac{1}{2}A = \dfrac{1 - \cos A}{\sin A}$

SUMS AND DIFFERENCES OF FUNCTIONS

72. $\sin A + \sin B = 2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$

73. $\sin A - \sin B = 2 \sin \frac{1}{2}(A - B) \cos \frac{1}{2}(A + B)$

74. $\cos A + \cos B = 2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$

75. $\cos A - \cos B = 2 \sin \frac{1}{2}(A + B) \sin \frac{1}{2}(B - A)$

76. $\tan A + \tan B = \dfrac{\sin (A + B)}{\cos A \cos B}$

77. $\tan A - \tan B = \dfrac{\sin (A - B)}{\cos A \cos B}$

78. $\sin^2 A - \sin^2 B = \sin (A + B) \sin (A - B)$

79. $\cos^2 A - \cos^2 B = \sin (A + B) \sin (B - A)$

80. $\cos^2 A - \sin^2 B = \cos (A + B) \cos (A - B)$

RELATIONS AMONG THE FUNCTIONS OF AN ANGLE

$\sin A =$	$\tan A =$	$\cos A =$	$\cot A =$	$\sec A =$	$\csc A =$
81. $\dfrac{\tan A}{\sqrt{1+\tan^2 A}}$	86. $\dfrac{\sin A}{\sqrt{1-\sin^2 A}}$	91. $\sqrt{1-\sin^2 A}$	96. $\dfrac{\sqrt{1-\sin^2 A}}{\sin A}$	101. $\dfrac{1}{\sqrt{1-\sin^2 A}}$	106. $\dfrac{1}{\sin A}$
82. $\sqrt{1-\cos^2 A}$	87. $\dfrac{\sqrt{1-\cos^2 A}}{\cos A}$	92. $\dfrac{1}{\sqrt{1+\tan^2 A}}$	97. $\dfrac{1}{\tan A}$	102. $\sqrt{1+\tan^2 A}$	107. $\dfrac{\sqrt{1+\tan^2 A}}{\tan A}$
83. $\dfrac{1}{\sqrt{1+\cot^2 A}}$	88. $\dfrac{1}{\cot A}$	93. $\dfrac{\cot A}{\sqrt{1+\cot^2 A}}$	98. $\dfrac{\cos A}{\sqrt{1-\cos^2 A}}$	103. $\dfrac{1}{\cos A}$	108. $\dfrac{1}{\sqrt{1-\cos^2 A}}$
84. $\dfrac{\sqrt{\sec^2 A-1}}{\sec A}$	89. $\sqrt{\sec^2 A-1}$	94. $\dfrac{1}{\sec A}$	99. $\dfrac{1}{\sqrt{\sec^2 A-1}}$	104. $\dfrac{\sqrt{1+\cot^2 A}}{\cot A}$	109. $\sqrt{1+\cot^2 A}$
85. $\dfrac{1}{\csc A}$	90. $\dfrac{1}{\sqrt{\csc^2 A-1}}$	95. $\dfrac{\sqrt{\csc^2 A-1}}{\csc A}$	100. $\sqrt{\csc^2 A-1}$	105. $\dfrac{\csc A}{\sqrt{\csc^2 A-1}}$	110. $\dfrac{\sec A}{\sqrt{\sec^2 A-1}}$

FORMULAS FOR THE SOLUTION OF RIGHT TRIANGLES

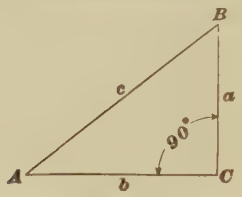

FIG. 29

Given	Required	Formula
a, A	B, b, c	111. $B = 90° - A$ 112. $b = a \cot A$ 113. $c = \dfrac{a}{\sin A} = a \csc A$
a, B	A, b, c	114. $A = 90° - B$ 115. $b = a \tan B$ 116. $c = \dfrac{a}{\cos B} = a \sec B$
c, A	B, a, b	117. $B = 90° - A$ 118. $a = c \sin A$ 119. $b = c \cos A$
a, b	A, B, c	120. $\tan A = \dfrac{a}{b}$ 121. $\tan B = \dfrac{b}{a}$, or $B = 90° - A$ 122. $c = \sqrt{a^2 + b^2}$ 123. $c = \dfrac{a}{\sin A} = a \csc A$
a, c	A, B, b	124. $\sin A = \dfrac{a}{c}$ 125. $\cos B = \dfrac{a}{c}$, or $B = 90° - A$ 126. $b = \sqrt{c^2 - a^2} = \sqrt{(c + a)(c - a)}$ 127. $b = a \cot A$

FORMULAS FOR THE SOLUTION OF OBLIQUE TRIANGLES

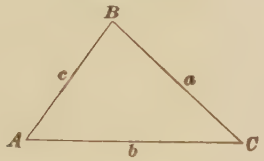

FIG. 30

Given	Required	Formulas
a, b, C	A, B, c	128. $\begin{cases} \tan \frac{1}{2}(A-B) = \frac{a-b}{a+b} \cot \frac{1}{2} C \\ A = (90^\circ - \frac{1}{2} C) + \frac{1}{2}(A-B) \\ B = (90^\circ - \frac{1}{2} C) - \frac{1}{2}(A-B) \end{cases}$ 129. $c = \frac{(a-b)\cos \frac{1}{2} C}{\sin \frac{1}{2}(A-B)}$ 130. $c = \sqrt{a^2 + b^2 - 2ab \cos C}$
c, A, B	C, a, b	131. $C = 180^\circ - (A+B)$ 132. $a = \frac{c}{\sin C} \sin A$ 133. $b = \frac{c}{\sin C} \sin B$
a, b, A	B, C, c	134. $\sin B = \frac{b}{a} \sin A$ 135. $C = 180^\circ - A - B$ 136. $c = \frac{a}{\sin A} \sin C$
a, b, c $\frac{1}{2}(a+b+c) = s$	A	137. $\tan \frac{1}{2} A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$ 138. $\cos \frac{1}{2} A = \sqrt{\frac{s(s-a)}{bc}}$ 139. $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$

NATURAL TRIGONOMETRIC FUNCTIONS

′	0°		1°		2°		3°		4°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.00000	1.	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	60
1	.00029	1.	.01774	.99984	.03519	.99938	.05263	.99861	.07005	.99754	59
2	.00058	1.	.01803	.99984	.03548	.99937	.05292	.99860	.07034	.99752	58
3	.00087	1.	.01832	.99983	.03577	.99936	.05321	.99858	.07063	.99750	57
4	.00116	1.	.01862	.99983	.03606	.99935	.05350	.99857	.07092	.99748	56
5	.00145	1.	.01891	.99982	.03635	.99934	.05379	.99855	.07121	.99746	55
6	.00175	1.	.01920	.99982	.03664	.99933	.05408	.99854	.07150	.99744	54
7	.00204	1.	.01949	.99981	.03693	.99932	.05437	.99852	.07179	.99742	53
8	.00233	1.	.01978	.99980	.03723	.99931	.05466	.99851	.07208	.99740	52
9	.00262	1.	.02007	.99980	.03752	.99930	.05495	.99849	.07237	.99738	51
10	.00291	1.	.02036	.99979	.03781	.99929	.05524	.99847	.07266	.99736	50
11	.00320	.99999	.02065	.99979	.03810	.99927	.05553	.99846	.07295	.99734	49
12	.00349	.99999	.02094	.99978	.03839	.99926	.05582	.99844	.07324	.99731	48
13	.00378	.99999	.02123	.99977	.03868	.99925	.05611	.99842	.07353	.99729	47
14	.00407	.99999	.02152	.99977	.03897	.99924	.05640	.99841	.07382	.99727	46
15	.00436	.99999	.02181	.99976	.03926	.99923	.05669	.99839	.07411	.99725	45
16	.00465	.99999	.02211	.99976	.03955	.99922	.05698	.99838	.07440	.99723	44
17	.00495	.99999	.02240	.99975	.03984	.99921	.05727	.99836	.07469	.99721	43
18	.00524	.99999	.02269	.99974	.04013	.99919	.05756	.99834	.07498	.99719	42
19	.00553	.99998	.02298	.99974	.04042	.99918	.05785	.99833	.07527	.99716	41
20	.00582	.99998	.02327	.99973	.04071	.99917	.05814	.99831	.07556	.99714	40
21	.00611	.99998	.02356	.99972	.04100	.99916	.05844	.99829	.07585	.99712	39
22	.00640	.99998	.02385	.99972	.04129	.99915	.05873	.99827	.07614	.99710	38
23	.00669	.99998	.02414	.99971	.04159	.99913	.05902	.99826	.07643	.99708	37
24	.00698	.99998	.02443	.99970	.04188	.99912	.05931	.99824	.07672	.99705	36
25	.00727	.99997	.02472	.99969	.04217	.99911	.05960	.99822	.07701	.99703	35
26	.00756	.99997	.02501	.99969	.04246	.99910	.05989	.99821	.07730	.99701	34
27	.00785	.99997	.02530	.99968	.04275	.99909	.06018	.99819	.07759	.99699	33
28	.00814	.99997	.02560	.99967	.04304	.99907	.06047	.99817	.07788	.99696	32
29	.00844	.99996	.02589	.99966	.04333	.99906	.06076	.99815	.07817	.99694	31
30	.00873	.99996	.02618	.99966	.04362	.99905	.06105	.99813	.07846	.99692	30
31	.00902	.99996	.02647	.99965	.04391	.99904	.06134	.99812	.07875	.99689	29
32	.00931	.99996	.02676	.99964	.04420	.99902	.06163	.99810	.07904	.99687	28
33	.00960	.99995	.02705	.99963	.04449	.99901	.06192	.99808	.07933	.99685	27
34	.00989	.99995	.02734	.99963	.04478	.99900	.06221	.99806	.07962	.99683	26
35	.01018	.99995	.02763	.99962	.04507	.99898	.06250	.99804	.07991	.99680	25
36	.01047	.99995	.02792	.99961	.04536	.99897	.06279	.99803	.08020	.99678	24
37	.01076	.99994	.02821	.99960	.04565	.99896	.06308	.99801	.08049	.99676	23
38	.01105	.99994	.02850	.99959	.04594	.99894	.06337	.99799	.08078	.99673	22
39	.01134	.99994	.02879	.99959	.04623	.99893	.06366	.99797	.08107	.99671	21
40	.01164	.99993	.02908	.99958	.04653	.99892	.06395	.99795	.08136	.99668	20
41	.01193	.99993	.02938	.99957	.04682	.99890	.06424	.99793	.08165	.99666	19
42	.01222	.99993	.02967	.99956	.04711	.99889	.06453	.99792	.08194	.99664	18
43	.01251	.99992	.02996	.99955	.04740	.99888	.06482	.99790	.08223	.99661	17
44	.01280	.99992	.03025	.99954	.04769	.99886	.06511	.99788	.08252	.99659	16
45	.01309	.99991	.03054	.99953	.04798	.99885	.06540	.99786	.08281	.99657	15
46	.01338	.99991	.03083	.99952	.04827	.99883	.06569	.99784	.08310	.99654	14
47	.01367	.99991	.03112	.99952	.04856	.99882	.06598	.99782	.08339	.99652	13
48	.01396	.99990	.03141	.99951	.04885	.99881	.06627	.99780	.08368	.99649	12
49	.01425	.99990	.03170	.99950	.04914	.99879	.06656	.99778	.08397	.99647	11
50	.01454	.99989	.03199	.99949	.04943	.99878	.06685	.99776	.08426	.99644	10
51	.01483	.99989	.03228	.99948	.04972	.99876	.06714	.99774	.08455	.99642	9
52	.01513	.99989	.03257	.99947	.05001	.99875	.06743	.99772	.08484	.99639	8
53	.01542	.99988	.03286	.99946	.05030	.99873	.06773	.99770	.08513	.99637	7
54	.01571	.99988	.03316	.99945	.05059	.99872	.06802	.99768	.08542	.99635	6
55	.01600	.99987	.03345	.99944	.05088	.99870	.06831	.99766	.08571	.99632	5
56	.01629	.99987	.03374	.99943	.05117	.99869	.06860	.99764	.08600	.99630	4
57	.01658	.99986	.03403	.99942	.05146	.99867	.06889	.99762	.08629	.99627	3
58	.01687	.99986	.03432	.99941	.05175	.99866	.06918	.99760	.08658	.99625	2
59	.01716	.99985	.03461	.99940	.05205	.99864	.06947	.99758	.08687	.99622	1
60	.01745	.99985	.03490	.99939	.05234	.99863	.06976	.99756	.08716	.99619	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	89°		88°		87°		86°		85°		

′	5°		6°		7°		8°		9°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.08716	.99619	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	60
1	.08745	.99617	.10482	.99449	.12216	.99251	.13946	.99023	.15672	.98764	59
2	.08774	.99614	.10511	.99446	.12245	.99248	.13975	.99019	.15701	.98760	58
3	.08803	.99612	.10540	.99443	.12274	.99244	.14004	.99015	.15730	.98755	57
4	.08831	.99609	.10569	.99440	.12302	.99240	.14033	.99011	.15758	.98751	56
5	.08860	.99607	.10597	.99437	.12331	.99237	.14061	.99006	.15787	.98746	55
6	.08889	.99604	.10626	.99434	.12360	.99233	.14090	.99002	.15816	.98741	54
7	.08918	.99602	.10655	.99431	.12389	.99230	.14119	.98998	.15845	.98737	53
8	.08947	.99599	.10684	.99428	.12418	.99226	.14148	.98994	.15873	.98732	52
9	.08976	.99596	.10713	.99424	.12447	.99222	.14177	.98990	.15902	.98728	51
10	.09005	.99594	.10742	.99421	.12476	.99219	.14205	.98986	.15931	.98723	50
11	.09034	.99591	.10771	.99418	.12504	.99215	.14234	.98982	.15959	.98718	49
12	.09063	.99588	.10800	.99415	.12533	.99211	.14263	.98978	.15988	.98714	48
13	.09092	.99586	.10829	.99412	.12562	.99208	.14292	.98973	.16017	.98709	47
14	.09121	.99583	.10858	.99409	.12591	.99204	.14320	.98969	.16046	.98704	46
15	.09150	.99580	.10887	.99406	.12620	.99200	.14349	.98965	.16074	.98700	45
16	.09179	.99578	.10916	.99402	.12649	.99197	.14378	.98961	.16103	.98695	44
17	.09208	.99575	.10945	.99399	.12678	.99193	.14407	.98957	.16132	.98690	43
18	.09237	.99572	.10973	.99396	.12706	.99189	.14436	.98953	.16160	.98686	42
19	.09266	.99570	.11002	.99393	.12735	.99186	.14464	.98948	.16189	.98681	41
20	.09295	.99567	.11031	.99390	.12764	.99182	.14493	.98944	.16218	.98676	40
21	.09324	.99564	.11060	.99386	.12793	.99178	.14522	.98940	.16246	.98671	39
22	.09353	.99562	.11089	.99383	.12822	.99175	.14551	.98936	.16275	.98667	38
23	.09382	.99559	.11118	.99380	.12851	.99171	.14580	.98931	.16304	.98662	37
24	.09411	.99556	.11147	.99377	.12880	.99167	.14608	.98927	.16333	.98657	36
25	.09440	.99553	.11176	.99374	.12908	.99163	.14637	.98923	.16361	.98652	35
26	.09469	.99551	.11205	.99370	.12937	.99160	.14666	.98919	.16390	.98648	34
27	.09498	.99548	.11234	.99367	.12966	.99156	.14695	.98914	.16419	.98643	33
28	.09527	.99545	.11263	.99364	.12995	.99152	.14723	.98910	.16447	.98638	32
29	.09556	.99542	.11291	.99360	.13024	.99148	.14752	.98906	.16476	.98633	31
30	.09585	.99540	.11320	.99357	.13053	.99144	.14781	.98902	.16505	.98629	30
31	.09614	.99537	.11349	.99354	.13081	.99141	14810	.98897	.16533	.98624	29
32	.09642	.99534	.11378	.99351	.13110	.99137	.14838	.98893	.16562	.98619	28
33	09671	99531	.11407	99347	.13139	.99133	.14867	.98889	.16591	.98614	27
34	.09700	.99528	.11436	.99344	.13168	.99129	.14896	.98884	.16620	.98609	26
35	.09729	.99526	.11465	.99341	.13197	.99125	.14925	.98880	.16648	.98604	25
36	.09758	.99523	.11494	.99337	.13226	.99122	.14954	.98876	.16677	.98600	24
37	.09787	.99520	.11523	.99334	.13254	.99118	.14982	.98871	.16706	.98595	23
38	.09816	.99517	.11552	.99331	.13283	.99114	.15011	.98867	.16734	.98590	22
39	.09845	.99514	.11580	.99327	.13312	.99110	.15040	.98863	.16763	.98585	21
40	.09874	.99511	.11609	.99324	.13341	.99106	.15069	.98858	.16792	.98580	20
41	.09903	.99508	.11638	.99320	.13370	.99102	.15097	.98854	.16820	.98575	19
42	.09932	.99506	.11667	.99317	.13399	.99098	.15126	.98849	.16849	.98570	18
43	.09961	.99503	.11696	.99314	.13427	.99094	.15155	.98845	.16878	.98565	17
44	.09990	.99500	.11725	.99310	.13456	.99091	.15184	.98841	.16906	.98561	16
45	.10019	.99497	.11754	.99307	.13485	.99087	.15212	.98836	.16935	.98556	15
46	.10048	.99494	.11783	.99303	.13514	.99083	.15241	.98832	.16964	.98551	14
47	.10077	.99491	.11812	.99300	.13543	.99079	.15270	.98827	.16992	.98546	13
48	.10106	.99488	.11840	.99297	.13572	.99075	.15299	.98823	.17021	.98541	12
49	.10135	.99485	.11869	.99293	.13600	.99071	.15327	.98818	.17050	.98536	11
50	.10164	.99482	.11898	.99290	.13629	.99067	.15356	.98814	.17078	.98531	10
51	.10192	.99479	.11927	.99286	.13658	.99063	.15385	.98809	.17107	.98526	9
52	.10221	.99476	.11956	.99283	.13687	.99059	.15414	.98805	.17136	.98521	8
53	.10250	.99473	.11985	.99279	.13716	.99055	.15442	.98800	.17164	.98516	7
54	.10279	.99470	.12014	.99276	.13744	.99051	.15471	.98796	.17193	.98511	6
55	.10308	.99467	.12043	.99272	.13773	.99047	.15500	.98791	.17222	.98506	5
56	.10337	.99464	.12071	.99269	.13802	.99043	.15529	.98787	.17250	.98501	4
57	.10366	.99461	.12100	.99265	.13831	.99039	.15557	.98782	.17279	.98496	3
58	.10395	.99458	.12129	.99262	.13860	.99035	.15586	.98778	.17308	.98491	2
59	.10424	.99455	.12158	.99258	.13889	.99031	.15615	.98773	.17336	.98486	1
60	.10453	.99452	.12187	.99255	.13917	.99027	.15643	.98769	.17365	.98481	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	84°		83°		82°		81°		80°		

′	10°		11°		12°		13°		14°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.17365	.98481	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	60
1	.17393	.98476	.19109	.98157	.20820	.97809	.22523	.97430	.24220	.97023	59
2	.17422	.98471	.19138	.98152	.20848	.97803	.22552	.97424	.24249	.97015	58
3	.17451	.98466	.19167	.98146	.20877	.97797	.22580	.97417	.24277	.97008	57
4	.17479	.98461	.19195	.98140	.20905	.97791	.22608	.97411	.24305	.97001	56
5	.17508	.98455	.19224	.98135	.20933	.97784	.22637	.97404	.24333	.96994	55
6	.17537	.98450	.19252	.98129	.20962	.97778	.22665	.97398	.24362	.96987	54
7	.17565	.98445	.19281	.98124	.20990	.97772	.22693	.97391	.24390	.96980	53
8	.17594	.98440	.19309	.98118	.21019	.97766	.22722	.97384	.24418	.96973	52
9	.17623	.98435	.19338	.98112	.21047	.97760	.22750	.97378	.24446	.96966	51
10	.17651	.98430	.19366	.98107	.21076	.97754	.22778	.97371	.24474	.96959	50
11	.17680	.98425	.19395	.98101	.21104	.97748	.22807	.97365	.24503	.96952	49
12	.17708	.98420	.19423	.98096	.21132	.97742	.22835	.97358	.24531	.96945	48
13	.17737	.98414	.19452	.98090	.21161	.97735	.22863	.97351	.24559	.96937	47
14	.17766	.98409	.19481	.98084	.21189	.97729	.22892	.97345	.24587	.96930	46
15	.17794	.98404	.19509	.98079	.21218	.97723	.22920	.97338	.24615	.96923	45
16	.17823	.98399	.19538	.98073	.21246	.97717	.22948	.97331	.24644	.96916	44
17	.17852	.98394	.19566	.98067	.21275	.97711	.22977	.97325	.24672	.96909	43
18	.17880	.98389	.19595	.98061	.21303	.97705	.23005	.97318	.24700	.96902	42
19	.17909	.98383	.19623	.98056	.21331	.97698	.23033	.97311	.24728	.96894	41
20	.17937	.98378	.19652	.98050	.21360	.97692	.23062	.97304	.24756	.96887	40
21	.17966	.98373	.19680	.98044	.21388	.97686	.23090	.97298	.24784	.96880	39
22	.17995	.98368	.19709	.98039	.21417	.97680	.23118	.97291	.24813	.96873	38
23	.18023	.98362	.19737	.98033	.21445	.97673	.23146	.97284	.24841	.96866	37
24	.18052	.98357	.19766	.98027	.21474	.97667	.23175	.97278	.24869	.96858	36
25	.18081	.98352	.19794	.98021	.21502	.97661	.23203	.97271	.24897	.96851	35
26	.18109	.98347	.19823	.98016	.21530	.97655	.23231	.97264	.24925	.96844	34
27	.18138	.98341	.19851	.98010	.21559	.97648	.23260	.97257	.24954	.96837	33
28	.18166	.98336	.19880	.98004	.21587	.97642	.23288	.97251	.24982	.96829	32
29	.18195	.98331	.19908	.97998	.21616	.97636	.23316	.97244	.25010	.96822	31
30	.18224	.98325	.19937	.97992	.21644	.97630	.23345	.97237	.25038	.96815	30
31	.18252	.98320	.19965	.97987	.21672	.97623	.23373	.97230	.25066	.96807	29
32	.18281	.98315	.19994	.97981	.21701	.97617	.23401	.97223	.25094	.96800	28
33	.18309	.98310	.20022	.97975	.21729	.97611	.23429	.97217	.25122	.96793	27
34	.18338	.98304	.20051	.97969	.21758	.97604	.23458	.97210	.25151	.96786	26
35	.18367	.98299	.20079	.97963	.21786	.97598	.23486	.97203	.25179	.96778	25
36	.18395	.98294	.20108	.97958	.21814	.97592	.23514	.97196	.25207	.96771	24
37	.18424	.98288	.20136	.97952	.21843	.97585	.23542	.97189	.25235	.96764	23
38	.18452	.98283	.20165	.97946	.21871	.97579	.23571	.97182	.25263	.96756	22
39	.18481	.98277	.20193	.97940	.21899	.97573	.23599	.97176	.25291	.96749	21
40	.18509	.98272	.20222	.97934	.21928	.97566	.23627	.97169	.25320	.96742	20
41	.18538	.98267	.20250	.97928	.21956	.97560	.23656	.97162	.25348	.96734	19
42	.18567	.98261	.20279	.97922	.21985	.97553	.23684	.97155	.25376	.96727	18
43	.18595	.98256	.20307	.97916	.22013	.97547	.23712	.97148	.25404	.96719	17
44	.18624	.98250	.20336	.97910	.22041	.97541	.23740	.97141	.25432	.96712	16
45	.18652	.98245	.20364	.97905	.22070	.97534	.23769	.97134	.25460	.96705	15
46	.18681	.98240	.20393	.97899	.22098	.97528	.23797	.97127	.25488	.96697	14
47	.18710	.98234	.20421	.97893	.22126	.97521	.23825	.97120	.25516	.96690	13
48	.18738	.98229	.20450	.97887	.22155	.97515	.23853	.97113	.25545	.96682	12
49	.18767	.98223	.20478	.97881	.22183	.97508	.23882	.97106	.25573	.96675	11
50	.18795	.98218	.20507	.97875	.22212	.97502	.23910	.97100	.25601	.96667	10
51	.18824	.98212	.20535	.97869	.22240	.97496	.23938	.97093	.25629	.96660	9
52	.18852	.98207	.20563	.97863	.22268	.97489	.23966	.97086	.25657	.96653	8
53	.18881	.98201	.20592	.97857	.22297	.97483	.23995	.97079	.25685	.96645	7
54	.18910	.98196	.20620	.97851	.22325	.97476	.24023	.97072	.25713	.96638	6
55	.18938	.98190	.20649	.97845	.22353	.97470	.24051	.97065	.25741	.96630	5
56	.18967	.98185	.20677	.97839	.22382	.97463	.24079	.97058	.25769	.96623	4
57	.18995	.98179	.20706	.97833	.22410	.97457	.24108	.97051	.25798	.96615	3
58	.19024	.98174	.20734	.97827	.22438	.97450	.24136	.97044	.25826	.96608	2
59	.19052	.98168	.20763	.97821	.22467	.97444	.24164	.97037	.25854	.96600	1
60	.19081	.98163	.20791	.97815	.22495	.97437	.24192	.97030	.25882	.96593	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	79°		78°		77°		76°		75°		

′	15°		16°		17°		18°		19°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.25882	.96593	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	60
1	.25910	.96585	.27592	.96118	.29265	.95622	.30929	.95097	.32584	.94542	59
2	.25938	.96578	.27620	.96110	.29293	.95613	.30957	.95088	.32612	.94533	58
3	.25966	.96570	.27648	.96102	.29321	.95605	.30985	.95079	.32639	.94523	57
4	.25994	.96562	.27676	.96094	.29348	.95596	.31012	.95070	.32667	.94514	56
5	.26022	.96555	.27704	.96086	.29376	.95588	.31040	.95061	.32694	.94504	55
6	.26050	.96547	.27731	.96078	.29404	.95579	.31068	.95052	.32722	.94495	54
7	.26079	.96540	.27759	.96070	.29432	.95571	.31095	.95043	.32749	.94485	53
8	.26107	.96532	.27787	.96062	.29460	.95562	.31123	.95033	.32777	.94476	52
9	.26135	.96524	.27815	.96054	.29487	.95554	.31151	.95024	.32804	.94466	51
10	.26163	.96517	.27843	.96046	.29515	.95545	.31178	.95015	.32832	.94457	50
11	.26191	.96509	.27871	.96037	.29543	.95536	.31206	.95006	.32859	.94447	49
12	.26219	.96502	.27899	.96029	.29571	.95528	.31233	.94997	.32887	.94438	48
13	.26247	.96494	.27927	.96021	.29599	.95519	.31261	.94988	.32914	.94428	47
14	.26275	.96486	.27955	.96013	.29626	.95511	.31289	.94979	.32942	.94418	46
15	.26303	.96479	.27983	.96005	.29654	.95502	.31316	.94970	.32969	.94409	45
16	.26331	.96471	.28011	.95997	.29682	.95493	.31344	.94961	.32997	.94399	44
17	.26359	.96463	.28039	.95989	.29710	.95485	.31372	.94952	.33024	.94390	43
18	.26387	.96456	.28067	.95981	.29737	.95476	.31399	.94943	.33051	.94380	42
19	.26415	.96448	.28095	.95972	.29765	.95467	.31427	.94933	.33079	.94370	41
20	.26443	.96440	.28123	.95964	.29793	.95459	.31454	.94924	.33106	.94361	40
21	.26471	.96433	.28150	.95956	.29821	.95450	.31482	.94915	.33134	.94351	39
22	.26500	.96425	.28178	.95948	.29849	.95441	.31510	.94906	.33161	.94342	38
23	.26528	.96417	.28206	.95940	.29876	.95433	.31537	.94897	.33189	.94332	37
24	.26556	.96410	.28234	.95931	.29904	.95424	.31565	.94888	.33216	.94322	36
25	.26584	.96402	.28262	.95923	.29932	.95415	.31593	.94878	.33244	.94313	35
26	.26612	.96394	.28290	.95915	.29960	.95407	.31620	.94869	.33271	.94303	34
27	.26640	.96386	.28318	.95907	.29987	.95398	.31648	.94860	.33298	.94293	33
28	.26668	.96379	.28346	.95898	.30015	.95389	.31675	.94851	.33326	.94284	32
29	.26696	.96371	.28374	.95890	.30043	.95380	.31703	.94842	.33353	.94274	31
30	.26724	.96363	.28402	.95882	.30071	.95372	.31730	.94832	.33381	.94264	30
31	.26752	.96355	.28429	.95874	.30098	.95363	.31758	.94823	.33408	.94254	29
32	.26780	.96347	.28457	.95865	.30126	.95354	.31786	.94814	.33436	.94245	28
33	.26808	.96340	.28485	.95857	.30154	.95345	.31813	.94805	.33463	.94235	27
34	.26836	.96332	.28513	.95849	.30182	.95337	.31841	.94795	.33490	.94225	26
35	.26864	.96324	.28541	.95841	.30209	.95328	.31868	.94786	.33518	.94215	25
36	.26892	.96316	.28569	.95832	.30237	.95319	.31896	.94777	.33545	.94206	24
37	.26920	.96308	.28597	.95824	.30265	.95310	.31923	.94768	.33573	.94196	23
38	.26948	.96301	.28625	.95816	.30292	.95301	.31951	.94758	.33600	.94186	22
39	.26976	.96293	.28652	.95807	.30320	.95293	.31979	.94749	.33627	.94176	21
40	.27004	.96285	.28680	.95799	.30348	.95284	.32006	.94740	.33655	.94167	20
41	.27032	.96277	.28708	.95791	.30376	.95275	.32034	.94730	.33682	.94157	19
42	.27060	.96269	.28736	.95782	.30403	.95266	.32061	.94721	.33710	.94147	18
43	.27088	.96261	.28764	.95774	.30431	.95257	.32089	.94712	.33737	.94137	17
44	.27116	.96253	.28792	.95766	.30459	.95248	.32116	.94702	.33764	.94127	16
45	.27144	.96246	.28820	.95757	.30486	.95240	.32144	.94693	.33792	.94118	15
46	.27172	.96238	.28847	.95749	.30514	.95231	.32171	.94684	.33819	.94108	14
47	.27200	.96230	.28875	.95740	.30542	.95222	.32199	.94674	.33846	.94098	13
48	.27228	.96222	.28903	.95732	.30570	.95213	.32227	.94665	.33874	.94088	12
49	.27256	.96214	.28931	.95724	.30597	.95204	.32254	.94656	.33901	.94078	11
50	.27284	.96206	.28959	.95715	.30625	.95195	.32282	.94646	.33929	.94068	10
51	.27312	.96198	.28987	.95707	.30653	.95186	.32309	.94637	.33956	.94058	9
52	.27340	.96190	.29015	.95698	.30680	.95177	.32337	.94627	.33983	.94049	8
53	.27368	.96182	.29042	.95690	.30708	.95168	.32364	.94618	.34011	.94039	7
54	.27396	.96174	.29070	.95681	.30736	.95159	.32392	.94609	.34038	.94029	6
55	.27424	.96166	.29098	.95673	.30763	.95150	.32419	.94599	.34065	.94019	5
56	.27452	.96158	.29126	.95664	.30791	.95142	.32447	.94590	.34093	.94009	4
57	.27480	.96150	.29154	.95656	.30819	.95133	.32474	.94580	.34120	.93999	3
58	.27508	.96142	.29182	.95647	.30846	.95124	.32502	.94571	.34147	.93989	2
59	.27536	.96134	.29209	.95639	.30874	.95115	.32529	.94561	.34175	.93979	1
60	.27564	.96126	.29237	.95630	.30902	.95106	.32557	.94552	.34202	.93969	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	74°		73°		72°		71°		70°		

′	20°		21°		22°		23°		24°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.34202	.93969	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	60
1	.34229	.93959	.35864	.93348	.37488	.92707	.39100	.92039	.40700	.91343	59
2	.34257	.93949	.35891	.93337	.37515	.92697	.39127	.92028	.40727	.91331	58
3	.34284	.93939	.35918	.93327	.37542	.92686	.39153	.92016	.40753	.91319	57
4	.34311	.93929	.35945	.93316	.37569	.92675	.39180	.92005	.40780	.91307	56
5	.34339	.93919	.35973	.93306	.37595	.92664	.39207	.91994	.40806	.91295	55
6	.34366	.93909	.36000	.93295	.37622	.92653	.39234	.91982	.40833	.91283	54
7	.34393	.93899	.36027	.93285	.37649	.92642	.39260	.91971	.40860	.91272	53
8	.34421	.93889	.36054	.93274	.37676	.92631	.39287	.91959	.40886	.91260	52
9	.34448	.93879	.36081	.93264	.37703	.92620	.39314	.91948	.40913	.91248	51
10	.34475	.93869	.36108	.93253	.37730	.92609	.39341	.91936	.40939	.91236	50
11	.34503	.93859	.36135	.93243	.37757	.92598	.39367	.91925	.40966	.91224	49
12	.34530	.93849	.36162	.93232	.37784	.92587	.39394	.91914	.40992	.91212	48
13	.34557	.93839	.36190	.93222	.37811	.92576	.39421	.91902	.41019	.91200	47
14	.34584	.93829	.36217	.93211	.37838	.92565	.39448	.91891	.41045	.91188	46
15	.34612	.93819	.36244	.93201	.37865	.92554	.39474	.91879	.41072	.91176	45
16	.34639	.93809	.36271	.93190	.37892	.92543	.39501	.91868	.41098	.91164	44
17	.34666	.93799	.36298	.93180	.37919	.92532	.39528	.91856	.41125	.91152	43
18	.34694	.93789	.36325	.93169	.37946	.92521	.39555	.91845	.41151	.91140	42
19	.34721	.93779	.36352	.93159	.37973	.92510	.39581	.91833	.41178	.91128	41
20	.34748	.93769	.36379	.93148	.37999	.92499	.39608	.91822	.41204	.91116	40
21	.34775	.93759	.36406	.93137	.38026	.92488	.39635	.91810	.41231	.91104	39
22	.34803	.93748	.36434	.93127	.38053	.92477	.39661	.91799	.41257	.91092	38
23	.34830	.93738	.36461	.93116	.38080	.92466	.39688	.91787	.41284	.91080	37
24	.34857	.93728	.36488	.93106	.38107	.92455	.39715	.91775	.41310	.91068	36
25	.34884	.93718	.36515	.93095	.38134	.92444	.39741	.91764	.41337	.91056	35
26	.34912	.93708	.36542	.93084	.38161	.92432	.39768	.91752	.41363	.91044	34
27	.34939	.93698	.36569	.93074	.38188	.92421	.39795	.91741	.41390	.91032	33
28	.34966	.93688	.36596	.93063	.38215	.92410	.39822	.91729	.41416	.91020	32
29	.34993	.93677	.36623	.93052	.38241	.92399	.39848	.91718	.41443	.91008	31
30	.35021	.93667	.36650	.93042	.38268	.92388	.39875	.91706	.41469	.90996	30
31	.35048	.93657	.36677	.93031	.38295	.92377	.39902	.91694	.41496	.90984	29
32	.35075	.93647	.36704	.93020	.38322	.92366	.39928	.91683	.41522	.90972	28
33	.35102	.93637	.36731	.93010	.38349	.92355	.39955	.91671	.41549	.90960	27
34	.35130	.93626	.36758	.92999	.38376	.92343	.39982	.91660	.41575	.90948	26
35	.35157	.93616	.36785	.92988	.38403	.92332	.40008	.91648	.41602	.90936	25
36	.35184	.93606	.36812	.92978	.38430	.92321	.40035	.91636	.41628	.90924	24
37	.35211	.93596	.36839	.92967	.38456	.92310	.40062	.91625	.41655	.90911	23
38	.35239	.93585	.36867	.92956	.38483	.92299	.40088	.91613	.41681	.90899	22
39	.35266	.93575	.36894	.92945	.38510	.92287	.40115	.91601	.41707	.90887	21
40	.35293	.93565	.36921	.92935	.38537	.92276	.40141	.91590	.41734	.90875	20
41	.35320	.93555	.36948	.92924	.38564	.92265	.40168	.91578	.41760	.90863	19
42	.35347	.93544	.36975	.92913	.38591	.92254	.40195	.91566	.41787	.90851	18
43	.35375	.93534	.37002	.92902	.38617	.92243	.40221	.91555	.41813	.90839	17
44	.35402	.93524	.37029	.92892	.38644	.92231	.40248	.91543	.41840	.90826	16
45	.35429	.93514	.37056	.92881	.38671	.92220	.40275	.91531	.41866	.90814	15
46	.35456	.93503	.37083	.92870	.38698	.92209	.40301	.91519	.41892	.90802	14
47	.35484	.93493	.37110	.92859	.38725	.92198	.40328	.91508	.41919	.90790	13
48	.35511	.93483	.37137	.92849	.38752	.92186	.40355	.91496	.41945	.90778	12
49	.35538	.93472	.37164	.92838	.38778	.92175	.40381	.91484	.41972	.90766	11
50	.35565	.93462	.37191	.92827	.38805	.92164	.40408	.91472	.41998	.90753	10
51	.35592	.93452	.37218	.92816	.38832	.92152	.40434	.91461	.42024	.90741	9
52	.35619	.93441	.37245	.92805	.38859	.92141	.40461	.91449	.42051	.90729	8
53	.35647	.93431	.37272	.92794	.38886	.92130	.40488	.91437	.42077	.90717	7
54	.35674	.93420	.37299	.92784	.38912	.92119	.40514	.91425	.42104	.90704	6
55	.35701	.93410	.37326	.92773	.38939	.92107	.40541	.91414	.42130	.90692	5
56	.35728	.93400	.37353	.92762	.38966	.92096	.40567	.91402	.42156	.90680	4
57	.35755	.93389	.37380	.92751	.38993	.92085	.40594	.91390	.42183	.90668	3
58	.35782	.93370	.37407	.92740	.39020	.92073	.40621	.91378	.42209	.90655	2
59	.35810	.93368	.37434	.92729	.39046	.92062	.40647	.91366	.42235	.90643	1
60	.35837	.93358	.37461	.92718	.39073	.92050	.40674	.91355	.42262	.90631	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	69°		68°		67°		66°		65°		

′	25°		26°		27°		28°		29°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.42262	.90631	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	60
1	.42288	.90618	.43863	.89867	.45425	.89087	.46973	.88281	.48506	.87448	59
2	.42315	.90606	.43889	.89854	.45451	.89074	.46999	.88267	.48532	.87434	58
3	.42341	.90594	.43916	.89841	.45477	.89061	.47024	.88254	.48557	.87420	57
4	.42367	.90582	.43942	.89828	.45503	.89048	.47050	.88240	.48583	.87406	56
5	.42394	.90569	.43968	.89816	.45529	.89035	.47076	.88226	.48608	.87391	55
6	.42420	.90557	.43994	.89803	.45554	.89021	.47101	.88213	.48634	.87377	54
7	.42446	.90545	.44020	.89790	.45580	.89008	.47127	.88199	.48659	.87363	53
8	.42473	.90532	.44046	.89777	.45606	.88995	.47153	.88185	.48684	.87349	52
9	.42499	.90520	.44072	.89764	.45632	.88981	.47178	.88172	.48710	.87335	51
10	.42525	.90507	.44098	.89752	.45658	.88968	.47204	.88158	.48735	.87321	50
11	.42552	.90495	.44124	.89739	.45684	.88955	.47229	.88144	.48761	.87306	49
12	.42578	.90483	.44151	.89726	.45710	.88942	.47255	.88130	.48786	.87292	48
13	.42604	.90470	.44177	.89713	.45736	.88928	.47281	.88117	.48811	.87278	47
14	.42631	.90458	.44203	.89700	.45762	.88915	.47306	.88103	.48837	.87264	46
15	.42657	.90446	.44229	.89687	.45787	.88902	.47332	.88089	.48862	.87250	45
16	.42683	.90433	.44255	.89674	.45813	.88888	.47358	.88075	.48888	.87235	44
17	.42709	.90421	.44281	.89662	.45839	.88875	.47383	.88062	.48913	.87221	43
18	.42736	.90408	.44307	.89649	.45865	.88862	.47409	.88048	.48938	.87207	42
19	.42762	.90396	.44333	.89636	.45891	.88848	.47434	.88034	.48964	.87193	41
20	.42788	.90383	.44359	.89623	.45917	.88835	.47460	.88020	.48989	.87178	40
21	.42815	.90371	.44385	.89610	.45942	.88822	.47486	.88006	.49014	.87164	39
22	.42841	.90358	.44411	.89597	.45968	.88808	.47511	.87993	.49040	.87150	38
23	.42867	.90346	.44437	.89584	.45994	.88795	.47537	.87979	.49065	.87136	37
24	.42894	.90334	.44464	.89571	.46020	.88782	.47562	.87965	.49090	.87121	36
25	.42920	.90321	.44490	.89558	.46046	.88768	.47588	.87951	.49116	.87107	35
26	.42946	.90309	.44516	.89545	.46072	.88755	.47614	.87937	.49141	.87093	34
27	.42972	.90296	.44542	.89532	.46097	.88741	.47639	.87923	.49166	.87079	33
28	.42999	.90284	.44568	.89519	.46123	.88728	.47665	.87909	.49192	.87064	32
29	.43025	.90271	.44594	.89506	.46149	.88715	.47690	.87896	.49217	.87050	31
30	.43051	.90259	.44620	.89493	.46175	.88701	.47716	.87882	.49242	.87036	30
31	.43077	.90246	.44646	.89480	.46201	.88688	.47741	.87868	.49268	.87021	29
32	.43104	.90233	.44672	.89467	.46226	.88674	.47767	.87854	.49293	.87007	28
33	.43130	.90221	.44698	.89454	.46252	.88661	.47793	.87840	.49318	.86993	27
34	.43156	.90208	.44724	.89441	.46278	.88647	.47818	.87826	.49344	.86978	26
35	.43182	.90196	.44750	.89428	.46304	.88634	.47844	.87812	.49369	.86964	25
36	.43209	.90183	.44776	.89415	.46330	.88620	.47869	.87798	.49394	.86949	24
37	.43235	.90171	.44802	.89402	.46355	.88607	.47895	.87784	.49419	.86935	23
38	.43261	.90158	.44828	.89389	.46381	.88593	.47920	.87770	.49445	.86921	22
39	.43287	.90146	.44854	.89376	.46407	.88580	.47946	.87756	.49470	.86906	21
40	.43313	.90133	.44880	.89363	.46433	.88566	.47971	.87743	.49495	.86892	20
41	.43340	.90120	.44906	.89350	.46458	.88553	.47997	.87729	.49521	.86878	19
42	.43366	.90108	.44932	.89337	.46484	.88539	.48022	.87715	.49546	.86863	18
43	.43392	.90095	.44958	.89324	.46510	.88526	.48048	.87701	.49571	.86849	17
44	.43418	.90082	.44984	.89311	.46536	.88512	.48073	.87687	.49596	.86834	16
45	.43445	.90070	.45010	.89298	.46561	.88499	.48099	.87673	.49622	.86820	15
46	.43471	.90057	.45036	.89285	.46587	.88485	.48124	.87659	.49647	.86805	14
47	.43497	.90045	.45062	.89272	.46613	.88472	.48150	.87645	.49672	.86791	13
48	.43523	.90032	.45088	.89259	.46639	.88458	.48175	.87631	.49697	.86777	12
49	.43549	.90019	.45114	.89245	.46664	.88445	.48201	.87617	.49723	.86762	11
50	.43575	.90007	.45140	.89232	.46690	.88431	.48226	.87603	.49748	.86748	10
51	.43602	.89994	.45166	.89219	.46716	.88417	.48252	.87589	.49773	.86733	9
52	.43628	.89981	.45192	.89206	.46742	.88404	.48277	.87575	.49798	.86719	8
53	.43654	.89968	.45218	.89193	.46767	.88390	.48303	.87561	.49824	.86704	7
54	.43680	.89956	.45243	.89180	.46793	.88377	.48328	.87546	.49849	.86690	6
55	.43706	.89943	.45269	.89167	.46819	.88363	.48354	.87532	.49874	.86675	5
56	.43733	.89930	.45295	.89153	.46844	.88349	.48379	.87518	.49899	.86661	4
57	.43759	.89918	.45321	.89140	.46870	.88336	.48405	.87504	.49924	.86646	3
58	.43785	.89905	.45347	.89127	.46896	.88322	.48430	.87490	.49950	.86632	2
59	.43811	.89892	.45373	.89114	.46921	.88308	.48456	.87476	.49975	.86617	1
60	.43837	.89879	.45399	.89101	.46947	.88295	.48481	.87462	.50000	.86603	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	64°		63°		62°		61°		60°		

′	30°		31°		32°		33°		34°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.50000	.86603	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	60
1	.50025	.86588	.51529	.85702	.53017	.84789	.54488	.83851	.55943	.82887	59
2	.50050	.86573	.51554	.85687	.53041	.84774	.54513	.83835	.55968	.82871	58
3	.50076	.86559	.51579	.85672	.53066	.84759	.54537	.83819	.55992	.82855	57
4	.50101	.86544	.51604	.85657	.53091	.84743	.54561	.83804	.56016	.82839	56
5	.50126	.86530	.51628	.85642	.53115	.84728	.54586	.83788	.56040	.82822	55
6	.50151	.86515	.51653	.85627	.53140	.84712	.54610	.83772	.56064	.82806	54
7	.50176	.86501	.51678	.85612	.53164	.84697	.54635	.83756	.56088	.82790	53
8	.50201	.86486	.51703	.85597	.53189	.84681	.54659	.83740	.56112	.82773	52
9	.50227	.86471	.51728	.85582	.53214	.84666	.54683	.83724	.56136	.82757	51
10	.50252	.86457	.51753	.85567	.53238	.84650	.54708	.83708	.56160	.82741	50
11	.50277	.86442	.51778	.85551	.53263	.84635	.54732	.83692	.56184	.82724	49
12	.50302	.86427	.51803	.85536	.53288	.84619	.54756	.83676	.56208	.82708	48
13	.50327	.86413	.51828	.85521	.53312	.84604	.54781	.83660	.56232	.82692	47
14	.50352	.86398	.51852	.85506	.53337	.84588	.54805	.83645	.56256	.82675	46
15	.50377	.86384	.51877	.85491	.53361	.84573	.54829	.83629	.56280	.82659	45
16	.50403	.86369	.51902	.85476	.53386	.84557	.54854	.83613	.56305	.82643	44
17	.50428	.86354	.51927	.85461	.53411	.84542	.54878	.83597	.56329	.82626	43
18	.50453	.86340	.51952	.85446	.53435	.84526	.54902	.83581	.56353	.82610	42
19	.50478	.86325	.51977	.85431	.53460	.84511	.54927	.83565	.56377	.82593	41
20	.50503	.86310	.52002	.85416	.53484	.84495	.54951	.83549	.56401	.82577	40
21	.50528	.86295	.52026	.85401	.53509	.84480	.54975	.83533	.56425	.82561	39
22	.50553	.86281	.52051	.85385	.53534	.84464	.54999	.83517	.56449	.82544	38
23	.50578	.86266	.52076	.85370	.53558	.84448	.55024	.83501	.56473	.82528	37
24	.50603	.86251	.52101	.85355	.53583	.84433	.55048	.83485	.56497	.82511	36
25	.50628	.86237	.52126	.85340	.53607	.84417	.55072	.83469	.56521	.82495	35
26	.50654	.86222	.52151	.85325	.53632	.84402	.55097	.83453	.56545	.82478	34
27	.50679	.86207	.52175	.85310	.53656	.84386	.55121	.83437	.56569	.82462	33
28	.50704	.86192	.52200	.85294	.53681	.84370	.55145	.83421	.56593	.82446	32
29	.50729	.86178	.52225	.85279	.53705	.84355	.55169	.83405	.56617	.82429	31
30	.50754	.86163	.52250	.85264	.53730	.84339	.55194	.83389	.56641	.82413	30
31	.50779	.86148	.52275	.85249	.53754	.84324	.55218	.83373	.56665	.82396	29
32	.50804	.86133	.52299	.85234	.53779	.84308	.55242	.83356	.56689	.82380	28
33	.50829	.86119	.52324	.85218	.53804	.84292	.55266	.83340	.56713	.82363	27
34	.50854	.86104	.52349	.85203	.53828	.84277	.55291	.83324	.56736	.82347	26
35	.50879	.86089	.52374	.85188	.53853	.84261	.55315	.83308	.56760	.82330	25
36	.50904	.86074	.52399	.85173	.53877	.84245	.55339	.83292	.56784	.82314	24
37	.50929	.86059	.52423	.85157	.53902	.84230	.55363	.83276	.56808	.82297	23
38	.50954	.86045	.52448	.85142	.53926	.84214	.55388	.83260	.56832	.82281	22
39	.50979	.86030	.52473	.85127	.53951	.84198	.55412	.83244	.56856	.82264	21
40	.51004	.86015	.52498	.85112	.53975	.84182	.55436	.83228	.56880	.82248	20
41	.51029	.86000	.52522	.85096	.54000	.84167	.55460	.83212	.56904	.82231	19
42	.51054	.85985	.52547	.85081	.54024	.84151	.55484	.83195	.56928	.82214	18
43	.51079	.85970	.52572	.85066	.54049	.84135	.55509	.83179	56952	.82198	17
44	.51104	.85956	.52597	.85051	.54073	.84120	.55533	.83163	.56976	.82181	16
45	.51129	.85941	.52621	.85035	.54097	.84104	.55557	.83147	.57000	.82165	15
46	.51154	.85926	.52646	.85020	.54122	.84088	.55581	.83131	.57024	.82148	14
47	.51179	.85911	.52671	.85005	.54146	.84072	.55605	.83115	.57047	.82132	13
48	.51204	.85896	.52696	.84989	.54171	.84057	.55630	.83098	.57071	.82115	12
49	.51229	.85881	.52720	.84974	.54195	.84041	.55654	.83082	.57095	.82098	11
50	.51254	.85866	.52745	.84959	.54220	.84025	.55678	.83066	.57119	.82082	10
51	.51279	.85851	.52770	.84943	.54244	.84009	.55702	.83050	.57143	.82065	9
52	.51304	.85836	.52794	.84928	.54269	.83994	.55726	.83034	.57167	.82048	8
53	.51329	.85821	.52819	.84913	.54293	.83978	.55750	.83017	.57191	.82032	7
54	.51354	.85806	.52844	.84897	.54317	.83962	.55775	.83001	.57215	.82015	6
55	.51379	.85792	.52869	.84882	.54342	.83946	.55799	.82985	.57238	.81999	5
56	.51404	.85777	.52893	.84866	.54366	.83930	.55823	.82969	.57262	.81982	4
57	.51429	.85762	.52918	.84851	.54391	.83915	.55847	.82953	.57286	.81965	3
58	.51454	.85747	.52943	.84836	.54415	.83899	.55871	.82936	.57310	.81949	2
59	.51479	.85732	.52967	.84820	.54440	.83883	.55895	.82920	.57334	.81932	1
60	.51504	.85717	.52992	.84805	.54464	.83867	.55919	.82904	.57358	.81915	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	59°		58°		57°		56°		55°		

′	35°		36°		37°		38°		39°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.57358	.81915	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	60
1	.57381	.81899	.58802	.80885	.60205	.79846	.61589	.78783	.62955	.77696	59
2	.57405	.81882	.58826	.80867	.60228	.79829	.61612	.78765	.62977	.77678	58
3	.57429	.81865	.58849	.80850	.60251	.79811	.61635	.78747	.63000	.77660	57
4	.57453	.81848	.58873	.80833	.60274	.79793	.61658	.78729	.63022	.77641	56
5	.57477	.81832	.58896	.80816	.60298	.79776	.61681	.78711	.63045	.77623	55
6	.57501	.81815	.58920	.80799	.60321	.79758	.61704	.78694	.63068	.77605	54
7	.57524	.81798	.58943	.80782	.60344	.79741	.61726	.78676	.63090	.77586	53
8	.57548	.81782	.58967	.80765	.60367	.79723	.61749	.78658	.63113	.77568	52
9	.57572	.81765	.58990	.80748	.60390	.79706	.61772	.78640	.63135	.77550	51
10	.57596	.81748	.59014	.80730	.60414	.79688	.61795	.78622	.63158	.77531	50
11	.57619	.81731	.59037	.80713	.60437	.79671	.61818	.78604	.63180	.77513	49
12	.57643	.81714	.59061	.80696	.60460	.79653	.61841	.78586	.63203	.77494	48
13	.57667	.81698	.59084	.80679	.60483	.79635	.61864	.78568	.63225	.77476	47
14	.57691	.81681	.59108	.80662	.60506	.79618	.61887	.78550	.63248	.77458	46
15	.57715	.81664	.59131	.80644	.60529	.79600	.61909	.78532	.63271	.77439	45
16	.57738	.81647	.59154	.80627	.60553	.79583	.61932	.78514	.63293	.77421	44
17	.57762	.81631	.59178	.80610	.60576	.79565	.61955	.78496	.63316	.77402	43
18	.57786	.81614	.59201	.80593	.60599	.79547	.61978	.78478	.63338	.77384	42
19	.57810	.81597	.59225	.80576	.60622	.79530	.62001	.78460	.63361	.77366	41
20	.57833	.81580	.59248	.80558	.60645	.79512	.62024	.78442	.63383	.77347	40
21	.57857	.81563	.59272	.80541	.60668	.79494	.62046	.78424	.63406	.77329	39
22	.57881	.81546	.59295	.80524	.60691	.79477	.62069	.78405	.63428	.77310	38
23	.57904	.81530	.59318	.80507	.60714	.79459	.62092	.78387	.63451	.77292	37
24	.57928	.81513	.59342	.80489	.60738	.79441	.62115	.78369	.63473	.77273	36
25	.57952	.81496	.59365	.80472	.60761	.79424	.62138	.78351	.63496	.77255	35
26	.57976	.81479	.59389	.80455	.60784	.79406	.62160	.78333	.63518	.77236	34
27	.57999	.81462	.59412	.80438	.60807	.79388	.62183	.78315	.63540	.77218	33
28	.58023	.81445	.59436	.80420	.60830	.79371	.62206	.78297	.63563	.77199	32
29	.58047	.81428	.59459	.80403	.60853	.79353	.62229	.78279	.63585	.77181	31
30	.58070	.81412	.59482	.80386	.60876	.79335	.62251	.78261	.63608	.77162	30
31	.58094	.81395	.59506	.80368	.60899	.79318	.62274	.78243	.63630	.77144	29
32	.58118	.81378	.59529	.80351	.60922	.79300	.62297	.78225	.63653	.77125	28
33	.58141	.81361	.59552	.80334	.60945	.79282	.62320	.78206	.63675	.77107	27
34	.58165	.81344	.59576	.80316	.60968	.79264	.62342	.78188	.63698	.77088	26
35	.58189	.81327	.59599	.80299	.60991	.79247	.62365	.78170	.63720	.77070	25
36	.58212	.81310	.59622	.80282	.61015	.79229	.62388	.78152	.63742	.77051	24
37	.58236	.81293	.59646	.80264	.61038	.79211	.62411	.78134	.63765	.77033	23
38	.58260	.81276	.59669	.80247	.61061	.79193	.62433	.78116	.63787	.77014	22
39	.58283	.81259	.59693	.80230	.61084	.79176	.62456	.78098	.63810	.76996	21
40	.58307	.81242	.59716	.80212	.61107	.79158	.62479	.78079	.63832	.76977	20
41	.58330	.81225	.59739	.80195	.61130	.79140	.62502	.78061	.63854	.76959	19
42	.58354	.81208	.59763	.80178	.61153	.79122	.62524	.78043	.63877	.76940	18
43	.58378	.81191	.59786	.80160	.61176	.79105	.62547	.78025	.63899	.76921	17
44	.58401	.81174	.59809	.80143	.61199	.79087	.62570	.78007	.63922	.76903	16
45	.58425	.81157	.59832	.80125	.61222	.79069	.62592	.77988	.63944	.76884	15
46	.58449	.81140	.59856	.80108	.61245	.79051	.62615	.77970	.63966	.76866	14
47	.58472	.81123	.59879	.80091	.61268	.79033	.62638	.77952	.63989	.76847	13
48	.58496	.81106	.59902	.80073	.61291	.79016	.62660	.77934	.64011	.76828	12
49	.58519	.81089	.59926	.80056	.61314	.78998	.62683	.77916	.64033	.76810	11
50	.58543	.81072	.59949	.80038	.61337	.78980	.62706	.77897	.64056	.76791	10
51	.58567	.81055	.59972	.80021	.61360	.78962	.62728	.77879	.64078	.76772	9
52	.58590	.81038	.59995	.80003	.61383	.78944	.62751	.77861	.64100	.76754	8
53	.58614	.81021	.60019	.79986	.61406	.78926	.62774	.77843	.64123	.76735	7
54	.58637	.81004	.60042	.79968	.61429	.78908	.62796	.77824	.64145	.76717	6
55	.58661	.80987	.60065	.79951	.61451	.78891	.62819	.77806	.64167	.76698	5
56	.58684	.80970	.60089	.79934	.61474	.78873	.62842	.77788	.64190	.76679	4
57	.58708	.80953	.60112	.79916	.61497	.78855	.62864	.77769	.64212	.76661	3
58	.58731	.80936	.60135	.79899	.61520	.78837	.62887	.77751	.64234	.76642	2
59	.58755	.80919	.60158	.79881	.61543	.78819	.62909	.77733	.64256	.76623	1
60	.58779	.80902	.60182	.79864	.61566	.78801	.62932	.77715	.64279	.76604	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	54°		53°		52°		51°		50°		

′	40°		41°		42°		43°		44°		′
	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	
0	.64279	.76604	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	60
1	.64301	.76586	.65628	.75452	.66935	.74295	.68221	.73116	.69487	.71914	59
2	.64323	.76567	.65650	.75433	.66956	.74276	.68242	.73096	.69508	.71894	58
3	.64346	.76548	.65672	.75414	.66978	.74256	.68264	.73076	.69529	.71873	57
4	.64368	.76530	.65694	.75395	.66999	.74237	.68285	.73056	.69549	.71853	56
5	.64390	.76511	.65716	.75375	.67021	.74217	.68306	.73036	.69570	.71833	55
6	.64412	.76492	.65738	.75356	.67043	.74198	.68327	.73016	.69591	.71813	54
7	.64435	.76473	.65759	.75337	.67064	.74178	.68349	.72996	.69612	.71792	53
8	.64457	.76455	.65781	.75318	.67086	.74159	.68370	.72976	.69633	.71772	52
9	.64479	.76436	.65803	.75299	.67107	.74139	.68391	.72957	.69654	.71752	51
10	.64501	.76417	.65825	.75280	.67129	.74120	.68412	.72937	.69675	.71732	50
11	.64524	.76398	.65847	.75261	.67151	.74100	.68434	.72917	.69696	.71711	49
12	.64546	.76380	.65869	.75241	.67172	.74080	.68455	.72897	.69717	.71691	48
13	.64568	.76361	.65891	.75222	.67194	.74061	.68476	.72877	.69737	.71671	47
14	.64590	.76342	.65913	.75203	.67215	.74041	.68497	.72857	.69758	.71650	46
15	.64612	.76323	.65935	.75184	.67237	.74022	.68518	.72837	.69779	.71630	45
16	.64635	.76304	.65956	.75165	.67258	.74002	.68539	.72817	.69800	.71610	44
17	.64657	.76286	.65978	.75146	.67280	.73983	.68561	.72797	.69821	.71590	43
18	.64679	.76267	.66000	.75126	.67301	.73963	.68582	.72777	.69842	.71569	42
19	.64701	.76248	.66022	.75107	.67323	.73944	.68603	.72757	.69862	.71549	41
20	.64723	.76229	.66044	.75088	.67344	.73924	.68624	.72737	.69883	.71529	40
21	.64746	.76210	.66066	.75069	.67366	.73904	.68645	.72717	.69904	.71508	39
22	.64768	.76192	.66088	.75050	.67387	.73885	.68666	.72697	.69925	.71488	38
23	.64790	.76173	.66109	.75030	.67409	.73865	.68688	.72677	.69946	.71468	37
24	.64812	.76154	.66131	.75011	.67430	.73846	.68709	.72657	.69966	.71447	36
25	.64834	.76135	.66153	.74992	.67452	.73826	.68730	.72637	.69987	.71427	35
26	.64856	.76116	.66175	.74973	.67473	.73806	.68751	.72617	.70008	.71407	34
27	.64878	.76097	.66197	.74953	.67495	.73787	.68772	.72597	.70029	.71386	33
28	.64901	.76078	.66218	.74934	.67516	.73767	.68793	.72577	.70049	.71366	32
29	.64923	.76059	.66240	.74915	.67538	.73747	.68814	.72557	.70070	.71345	31
30	.64945	.76041	.66262	.74896	.67559	.73728	.68835	.72537	.70091	.71325	30
31	.64967	.76022	.66284	.74876	.67580	.73708	.68857	.72517	.70112	.71305	29
32	.64989	.76003	.66306	.74857	.67602	.73688	.68878	.72497	.70132	.71284	28
33	.65011	.75984	.66327	.74838	.67623	.73669	.68899	.72477	.70153	.71264	27
34	.65033	.75965	.66349	.74818	.67645	.73649	.68920	.72457	.70174	.71243	26
35	.65055	.75946	.66371	.74799	.67666	.73629	.68941	.72437	.70195	.71223	25
36	.65077	.75927	.66393	.74780	.67688	.73610	.68962	.72417	.70215	.71203	24
37	.65100	.75908	.66414	.74760	.67709	.73590	.68983	.72397	.70236	.71182	23
38	.65122	.75889	.66436	.74741	.67730	.73570	.69004	.72377	.70257	.71162	22
39	.65144	.75870	.66458	.74722	.67752	.73551	.69025	.72357	.70277	.71141	21
40	.65166	.75851	.66480	.74703	.67773	.73531	.69046	.72337	.70298	.71121	20
41	.65188	.75832	.66501	.74683	.67795	.73511	.69067	.72317	.70319	.71100	19
42	.65210	.75813	.66523	.74664	.67816	.73491	.69088	.72297	.70339	.71080	18
43	.65232	.75794	.66545	.74644	.67837	.73472	.69109	.72277	.70360	.71059	17
44	.65254	.75775	.66566	.74625	.67859	.73452	.69130	.72257	.70381	.71039	16
45	.65276	.75756	.66588	.74606	.67880	.73432	.69151	.72236	.70401	.71019	15
46	.65298	.75738	66610	.74586	.67901	.73413	.69172	.72216	.70422	.70998	14
47	.65320	.75719	.66632	.74567	.67923	.73393	.69193	.72196	.70443	.70978	13
48	.65342	.75700	.66653	.74548	.67944	.73373	.69214	.72176	.70463	.70957	12
49	.65364	.75680	.66675	.74528	.67965	.73353	.69235	.72156	.70484	.70937	11
50	.65386	.75661	.66697	.74509	.67987	.73333	.69256	.72136	.70505	.70916	10
51	.65408	.75642	.66718	.74489	.68008	.73314	.69277	./2116	.70525	.70896	9
52	.65430	.75623	.66740	.74470	.68029	.73294	.69298	.72095	.70546	.70875	8
53	.65452	.75604	.66762	.74451	.68051	.73274	.69319	.72075	.70567	.70855	7
54	.65474	.75585	.66783	.74431	.68072	.73254	.69340	.72055	.70587	.70834	6
55	.65496	.75566	.66805	.74412	.68093	.73234	.69361	.72035	.70608	.70813	5
56	.65518	.75547	.66827	.74392	.68115	.73215	.69382	.72015	.70628	.70793	4
57	.65540	.75528	.66848	.74373	.68136	.73195	.69403	.71995	.70649	.70772	3
58	.65562	.75509	.66870	.74353	.68157	.73175	.69424	.71974	.70670	.70752	2
59	.65584	.75490	.66891	.74334	.68179	.73155	.69445	.71954	.70690	.70731	1
60	.65606	.75471	.66913	.74314	.68200	.73135	.69466	.71934	.70711	.70711	0
′	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	Cosine	Sine	′
	49°		48°		47°		46°		45°		

′	0°		1°		2°		3°		4°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.00000	Infinite	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	60
1	.00029	3437.75	.01775	56.3506	.03521	28.3994	.05270	18.9755	.07022	14.2411	59
2	.00058	1718.87	.01804	55.4415	.03550	28.1664	.05299	18.8711	.07051	14.1821	58
3	.00087	1145.92	.01833	54.5613	.03579	27.9372	.05328	18.7678	.07080	14.1235	57
4	.00116	859.436	.01862	53.7086	.03609	27.7117	.05357	18.6656	.07110	14.0655	56
5	.00145	687.549	.01891	52.8821	.03638	27.4899	.05387	18.5645	.07139	14.0079	55
6	.00175	572.957	.01920	52.0807	.03667	27.2715	.05416	18.4645	.07168	13.9507	54
7	.00204	491.106	.01949	51.3032	.03696	27.0566	.05445	18.3655	.07197	13.8940	53
8	.00233	429.718	.01978	50.5485	.03725	26.8450	.05474	18.2677	.07227	13.8378	52
9	.00262	381.971	.02007	49.8157	.03754	26.6367	.05503	18.1708	.07256	13.7821	51
10	.00291	343.774	.02036	49.1039	.03783	26.4316	.05533	18.0750	.07285	13.7267	50
11	.00320	312.521	.02066	48.4121	.03812	26.2296	.05562	17.9802	.07314	13.6719	49
12	.00349	286.478	.02095	47.7395	.03842	26.0307	.05591	17.8863	.07344	13.6174	48
13	.00378	264.441	.02124	47.0853	.03871	25.8348	.05620	17.7934	.07373	13.5634	47
14	.00407	245.552	.02153	46.4489	.03900	25.6418	.05649	17.7015	.07402	13.5098	46
15	.00436	229.182	.02182	45.8294	.03929	25.4517	.05678	17.6106	.07431	13.4566	45
16	.00465	214.858	.02211	45.2261	.03958	25.2644	.05708	17.5205	.07461	13.4039	44
17	.00495	202.219	.02240	44.6386	.03987	25.0798	.05737	17.4314	.07490	13.3515	43
18	.00524	190.984	.02269	44.0661	.04016	24.8978	.05766	17.3432	.07519	13.2996	42
19	.00553	180.932	.02298	43.5081	.04046	24.7185	.05795	17.2558	.07548	13.2480	41
20	.00582	171.885	.02328	42.9641	.04075	24.5418	.05824	17.1693	.07578	13.1969	40
21	.00611	163.700	.02357	42.4335	.04104	24.3675	.05854	17.0837	.07607	13.1461	39
22	.00640	156.259	.02386	41.9158	.04133	24.1957	.05883	16.9990	.07636	13.0958	38
23	.00669	149.465	.02415	41.4106	.04162	24.0263	.05912	16.9150	.07665	13.0458	37
24	.00698	143.237	.02444	40.9174	.04191	23.8593	.05941	16.8319	.07695	12.9962	36
25	.00727	137.507	.02473	40.4358	.04220	23.6945	.05970	16.7496	.07724	12.9469	35
26	.00756	132.219	.02502	39.9655	.04250	23.5321	.05999	16.6681	.07753	12.8981	34
27	.00785	127.321	.02531	39.5059	.04279	23.3718	.06029	16.5874	.07782	12.8496	33
28	.00815	122.774	.02560	39.0568	.04308	23.2137	.06058	16.5075	.07812	12.8014	32
29	.00844	118.540	.02589	38.6177	.04337	23.0577	.06087	16.4283	.07841	12.7536	31
30	.00873	114.589	.02619	38.1885	.04366	22.9038	.06116	16.3499	.07870	12.7062	30
31	.00902	110.892	.02648	37.7686	.04395	22.7519	.06145	16.2722	.07899	12.6591	29
32	.00931	107.426	.02677	37.3579	.04424	22.6020	.06175	16.1952	.07929	12.6124	28
33	.00960	104.171	.02706	36.9560	.04454	22.4541	.06204	16.1190	.07958	12.5660	27
34	.00989	101.107	.02735	36.5627	.04483	22.3081	.06233	16.0435	.07987	12.5199	26
35	.01018	98.2179	.02764	36.1776	.04512	22.1640	.06262	15.9687	.08017	12.4742	25
36	.01047	95.4895	.02793	35.8006	.04541	22.0217	.06291	15.8945	.08046	12.4288	24
37	.01076	92.9085	.02822	35.4313	.04570	21.8813	.06321	15.8211	.08075	12.3838	23
38	.01105	90.4633	.02851	35.0695	.04599	21.7426	.06350	15.7483	.08104	12.3390	22
39	.01135	88.1436	.02881	34.7151	.04628	21.6056	.06379	15.6762	.08134	12.2946	21
40	.01164	85.9398	.02910	34.3678	.04658	21.4704	.06408	15.6048	.08163	12.2505	20
41	.01193	83.8435	.02939	34.0273	.04687	21.3369	.06437	15.5340	.08192	12.2067	19
42	.01222	81.8470	.02968	33.6935	.04716	21.2049	.06467	15.4638	.08221	12.1632	18
43	.01251	79.9434	.02997	33.3662	.04745	21.0747	.06496	15.3943	.08251	12.1201	17
44	.01280	78.1263	.03026	33.0452	.04774	20.9460	.06525	15.3254	.08280	12.0772	16
45	.01309	76.3900	.03055	32.7303	.04803	20.8188	.06554	15.2571	.08309	12.0346	15
46	.01338	74.7292	.03084	32.4213	.04833	20.6932	.06584	15.1893	.08339	11.9923	14
47	.01367	73.1390	.03114	32.1181	.04862	20.5691	.06613	15.1222	.08368	11.9504	13
48	.01396	71.6151	.03143	31.8205	.04891	20.4465	.06642	15.0557	.08397	11.9087	12
49	.01425	70.1533	.03172	31.5284	.04920	20.3253	.06671	14.9898	.08427	11.8673	11
50	.01455	68.7501	.03201	31.2416	.04949	20.2056	.06700	14.9244	.08456	11.8262	10
51	.01484	67.4019	.03230	30.9599	.04978	20.0872	.06730	14.8596	.08485	11.7853	9
52	.01513	66.1055	.03259	30.6833	.05007	19.9702	.06759	14.7954	.08514	11.7448	8
53	.01542	64.8580	.03288	30.4116	.05037	19.8546	.06788	14.7317	.08544	11.7045	7
54	.01571	63.6567	.03317	30.1446	.05066	19.7403	.06817	14.6685	.08573	11.6645	6
55	.01600	62.4992	.03346	29.8823	.05095	19.6273	.06847	14.6059	.08602	11.6248	5
56	.01629	61.3829	.03376	29.6245	.05124	19.5156	.06876	14.5438	.08632	11.5853	4
57	.01658	60.3058	.03405	29.3711	.05153	19.4051	.06905	14.4823	.08661	11.5461	3
58	.01687	59.2659	.03434	29.1220	.05182	19.2959	.06934	14.4212	.08690	11.5072	2
59	.01716	58.2612	.03463	28.8771	.05212	19.1879	.06963	14.3607	.08720	11.4685	1
60	.01746	57.2900	.03492	28.6363	.05241	19.0811	.06993	14.3007	.08749	11.4301	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	89°		88°		87°		86°		85°		

′	5°		6°		7°		8°		9°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.08749	11.4301	.10510	9.51436	.12278	8.14435	.14054	7.11537	.15838	6.31375	60
1	.08778	11.3919	.10540	9.48781	.12308	8.12481	.14084	7.10038	.15868	6.30189	59
2	.08807	11.3540	.10569	9.46141	.12338	8.10536	.14113	7.08546	.15898	6.29007	58
3	.08837	11.3163	.10599	9.43515	.12367	8.08600	.14143	7.07059	.15928	6.27829	57
4	.08866	11.2789	.10628	9.40904	.12397	8.06674	.14173	7.05579	.15958	6.26655	56
5	.08895	11.2417	.10657	9.38307	.12426	8.04756	.14202	7.04105	.15988	6.25486	55
6	.08925	11.2048	.10687	9.35724	.12456	8.02848	.14232	7.02637	.16017	6.24321	54
7	.08954	11.1681	.10716	9.33155	.12485	8.00948	.14262	7.01174	.16047	6.23160	53
8	.08983	11.1316	.10746	9.30599	.12515	7.99058	.14291	6.99718	.16077	6.22003	52
9	.09013	11.0954	.10775	9.28058	.12544	7.97176	.14321	6.98268	.16107	6.20851	51
10	.09042	11.0594	.10805	9.25530	.12574	7.95302	.14351	6.96823	.16137	6.19703	50
11	.09071	11.0237	.10834	9.23016	.12603	7.93438	.14381	6.95385	.16167	6.18559	49
12	.09101	10.9882	.10863	9.20516	.12633	7.91582	.14410	6.93952	.16196	6.17419	48
13	.09130	10.9529	.10893	9.18028	.12662	7.89734	.14440	6.92525	.16226	6.16283	47
14	.09159	10.9178	.10922	9.15554	.12692	7.87895	.14470	6.91104	.16256	6.15151	46
15	.09189	10.8829	.10952	9.13093	.12722	7.86064	.14499	6.89688	.16286	6.14023	45
16	.09218	10.8483	.10981	9.10646	.12751	7.84242	.14529	6.88278	.16316	6.12899	44
17	.09247	10.8139	.11011	9.08211	.12781	7.82428	.14559	6.86874	.16346	6.11779	43
18	.09277	10.7797	.11040	9.05789	.12810	7.80622	.14588	6.85475	.16376	6.10664	42
19	.09306	10.7457	.11070	9.03379	.12840	7.78825	.14618	6.84082	.16405	6.09552	41
20	.09335	10.7119	.11099	9.00983	.12869	7.77035	.14648	6.82694	.16435	6.08444	40
21	.09365	10.6783	.11128	8.98598	.12899	7.75254	.14678	6.81312	.16465	6.07340	39
22	.09394	10.6450	.11158	8.96227	.12929	7.73480	.14707	6.79936	.16495	6.06240	38
23	.09423	10.6118	.11187	8.93867	.12958	7.71715	.14737	6.78564	.16525	6.05143	37
24	.09453	10.5789	.11217	8.91520	.12988	7.69957	.14767	6.77199	.16555	6.04051	36
25	.09482	10.5462	.11246	8.89185	.13017	7.68208	.14796	6.75838	.16585	6.02962	35
26	.09511	10.5136	.11276	8.86862	.13047	7.66466	.14826	6.74483	.16615	6.01878	34
27	.09541	10.4813	.11305	8.84551	.13076	7.64732	.14856	6.73133	.16645	6.00797	33
28	.09570	10.4491	.11335	8.82252	.13106	7.63005	.14886	6.71789	.16674	5.99720	32
29	.09600	10.4172	.11364	8.79964	.13136	7.61287	.14915	6.70450	.16704	5.98646	31
30	.09629	10.3854	.11394	8.77689	.13165	7.59575	.14945	6.69116	.16734	5.97576	30
31	.09658	10.3538	.11423	8.75425	.13195	7.57872	.14975	6.67787	.16764	5.96510	29
32	.09688	10.3224	.11452	8.73172	.13224	7.56176	.15005	6.66463	.16794	5.95448	28
33	.09717	10.2913	.11482	8.70931	.13254	7.54487	.15034	6.65144	.16824	5.94390	27
34	.09746	10.2602	.11511	8.68701	.13284	7.52806	.15064	6.63831	.16854	5.93335	26
35	.09776	10.2294	.11541	8.66482	.13313	7.51132	.15094	6.62523	.16884	5.92283	25
36	.09805	10.1988	.11570	8.64275	.13343	7.49465	.15124	6.61219	.16914	5.91236	24
37	.09834	10.1683	.11600	8.62078	.13372	7.47806	.15153	6.59921	.16944	5.90191	23
38	.09864	10.1381	.11629	8.59893	.13402	7.46154	.15183	6.58627	.16974	5.89151	22
39	.09893	10.1080	.11659	8.57718	.13432	7.44509	.15213	6.57339	.17004	5.88114	21
40	.09923	10.0780	.11688	8.55555	.13461	7.42871	.15243	6.56055	.17033	5.87080	20
41	.09952	10.0483	.11718	8.53402	.13491	7.41240	.15272	6.54777	.17063	5.86051	19
42	.09981	10.0187	.11747	8.51259	.13521	7.39616	.15302	6.53503	.17093	5.85024	18
43	.10011	9.98931	.11777	8.49128	.13550	7.37999	.15332	6.52234	.17123	5.84001	17
44	.10040	9.96007	.11806	8.47007	.13580	7.36389	.15362	6.50970	.17153	5.82982	16
45	.10069	9.93101	.11836	8.44896	.13609	7.34786	.15391	6.49710	.17183	5.81966	15
46	.10099	9.90211	.11865	8.42795	.13639	7.33190	.15421	6.48456	.17213	5.80953	14
47	.10128	9.87338	.11895	8.40705	.13669	7.31600	.15451	6.47206	.17243	5.79944	13
48	.10158	9.84482	.11924	8.38625	.13698	7.30018	.15481	6.45961	.17273	5.78938	12
49	.10187	9.81641	.11954	8.36555	.13728	7.28442	.15511	6.44720	.17303	5.77936	11
50	.10216	9.78817	.11983	8.34496	.13758	7.26873	.15540	6.43484	.17333	5.76937	10
51	.10246	9.76009	.12013	8.32446	.13787	7.25310	.15570	6.42253	.17363	5.75941	9
52	.10275	9.73217	.12042	8.30406	.13817	7.23754	.15600	6.41026	.17393	5.74949	8
53	.10305	9.70441	.12072	8.28376	.13846	7.22204	.15630	6.39804	.17423	5.73960	7
54	.10334	9.67680	.12101	8.26355	.13876	7.20661	.15660	6.38587	.17453	5.72974	6
55	.10363	9.64935	.12131	8.24345	.13906	7.19125	.15689	6.37374	.17483	5.71992	5
56	.10393	9.62205	.12160	8.22344	.13935	7.17594	.15719	6.36165	.17513	5.71013	4
57	.10422	9.59490	.12190	8.20352	.13965	7.16071	.15749	6.34961	.17543	5.70037	3
58	.10452	9.56791	.12219	8.18370	.13995	7.14553	.15779	6.33761	.17573	5.69064	2
59	.10481	9.54106	.12249	8.16398	.14024	7.13042	.15809	6.32566	.17603	5.68094	1
60	.10510	9.51436	.12278	8.14435	.14054	7.11537	.15838	6.31375	.17633	5.67128	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	84°		83°		82°		81°		80°		

′	10°		11°		12°		13°		14°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.17633	5.67128	.19438	5.14455	.21256	4.70463	.23087	4.33148	.24933	4.01078	60
1	.17663	5.66165	.19468	5.13658	.21286	4.69791	.23117	4.32573	.24964	4.00582	59
2	.17693	5.65205	.19498	5.12862	.21316	4.69121	.23148	4.32001	.24995	4.00086	58
3	.17723	5.64248	.19529	5.12069	.21347	4.68452	.23179	4.31430	.25026	3.99592	57
4	.17753	5.63295	.19559	5.11279	.21377	4.67786	.23209	4.30860	.25056	3.99099	56
5	.17783	5.62344	.19589	5.10490	.21408	4.67121	.23240	4.30291	.25087	3.98607	55
6	.17813	5.61397	.19619	5.09704	.21438	4.66458	.23271	4.29724	.25118	3.98117	54
7	.17843	5.60452	.19649	5.08921	.21469	4.65797	.23301	4.29159	.25149	3.97627	53
8	.17873	5.59511	.19680	5.08139	.21499	4.65138	.23332	4.28595	.25180	3.97139	52
9	.17903	5.58573	.19710	5.07360	.21529	4.64480	.23363	4.28032	.25211	3.96651	51
10	.17933	5.57638	.19740	5.06584	.21560	4.63825	.23393	4.27471	.25242	3.96165	50
11	.17963	5.56706	.19770	5.05809	.21590	4.63171	.23424	4.26911	.25273	3.95680	49
12	.17993	5.55777	.19801	5.05037	.21621	4.62518	.23455	4.26352	.25304	3.95196	48
13	.18023	5.54851	.19831	5.04267	.21651	4.61868	.23485	4.25795	.25335	3.94713	47
14	.18053	5.53927	.19861	5.03499	.21682	4.61219	.23516	4.25239	.25366	3.94232	46
15	.18083	5.53007	.19891	5.02734	.21712	4.60572	.23547	4.24685	.25397	3.93751	45
16	.18113	5.52090	.19921	5.01971	.21743	4.59927	.23578	4.24132	.25428	3.93271	44
17	.18143	5.51176	.19952	5.01210	.21773	4.59283	.23608	4.23580	.25459	3.92793	43
18	.18173	5.50264	.19982	5.00451	.21804	4.58641	.23639	4.23030	.25490	3.92316	42
19	.18203	5.49356	.20012	4.99695	.21834	4.58001	.23670	4.22481	.25521	3.91839	41
20	.18233	5.48451	.20042	4.98940	.21864	4.57363	.23700	4.21933	.25552	3.91364	40
21	.18263	5.47548	.20073	4.98188	.21895	4.56726	.23731	4.21387	.25583	3.90890	39
22	.18293	5.46648	.20103	4.97438	.21925	4.56091	.23762	4.20842	.25614	3.90417	38
23	.18323	5.45751	.20133	4.96690	.21956	4.55458	.23793	4.20298	.25645	3.89945	37
24	.18353	5.44857	.20164	4.95945	.21986	4.54826	.23823	4.19756	.25676	3.89474	36
25	.18384	5.43966	.20194	4.95201	.22017	4.54196	.23854	4.19215	.25707	3.89004	35
26	.18414	5.43077	.20224	4.94460	.22047	4.53568	.23885	4.18675	.25738	3.88536	34
27	.18444	5.42192	.20254	4.93721	.22078	4.52941	.23916	4.18137	.25769	3.88068	33
28	.18474	5.41309	.20285	4.92984	.22108	4.52316	.23946	4.17600	.25800	3.87601	32
29	.18504	5.40429	.20315	4.92249	.22139	4.51693	.23977	4.17064	.25831	3.87136	31
30	.18534	5.39552	.20345	4.91516	.22169	4.51071	.24008	4.16530	.25862	3.86671	30
31	.18564	5.38677	.20376	4.90785	.22200	4.50451	.24039	4.15997	.25893	3.86208	29
32	.18594	5.37805	.20406	4.90056	.22231	4.49832	.24069	4.15465	.25924	3.85745	28
33	.18624	5.36936	.20436	4.89330	.22261	4.49215	.24100	4.14934	.25955	3.85284	27
34	.18654	5.36070	.20466	4.88605	.22292	4.48600	.24131	4.14405	.25986	3.84824	26
35	.18684	5.35206	.20497	4.87882	.22322	4.47986	.24162	4.13877	.26017	3.84364	25
36	.18714	5.34345	.20527	4.87162	.22353	4.47374	.24193	4.13350	.26048	3.83906	24
37	.18745	5.33487	.20557	4.86444	.22383	4.46764	.24223	4.12825	.26079	3.83449	23
38	.18775	5.32631	.20588	4.85727	.22414	4.46155	.24254	4.12301	.26110	3.82992	22
39	.18805	5.31778	.20618	4.85013	.22444	4.45548	.24285	4.11778	.26141	3.82537	21
40	.18835	5.30928	.20648	4.84300	.22475	4.44942	.24316	4.11256	.26172	3.82083	20
41	.18865	5.30080	.20679	4.83590	.22505	4.44338	.24347	4.10736	.26203	3.81630	19
42	.18895	5.29235	.20709	4.82882	.22536	4.43735	.24377	4.10216	.26235	3.81177	18
43	.18925	5.28393	.20739	4.82175	.22567	4.43134	.24408	4.09699	.26266	3.80726	17
44	.18955	5.27553	.20770	4.81471	.22597	4.42534	.24439	4.09182	.26297	3.80276	16
45	.18986	5.26715	.20800	4.80769	.22628	4.41936	.24470	4.08666	.26328	3.79827	15
46	.19016	5.25880	.20830	4.80068	.22658	4.41340	.24501	4.08152	.26359	3.79378	14
47	.19046	5.25048	.20861	4.79370	.22689	4.40745	.24532	4.07639	.26390	3.78931	13
48	.19076	5.24218	.20891	4.78673	.22719	4.40152	.24562	4.07127	.26421	3.78485	12
49	.19106	5.23391	.20921	4.77978	.22750	4.39560	.24593	4.06616	.26452	3.78040	11
50	.19136	5.22566	.20952	4.77286	.22781	4.38969	.24624	4.06107	.26483	3.77595	10
51	.19166	5.21744	.20982	4.76595	.22811	4.38381	.24655	4.05599	.26515	3.77152	9
52	.19197	5.20925	.21013	4.75906	.22842	4.37793	.24686	4.05092	.26546	3.76709	8
53	.19227	5.20107	.21043	4.75219	.22872	4.37207	.24717	4.04586	.26577	3.76268	7
54	.19257	5.19293	.21073	4.74534	.22903	4.36623	.24747	4.04081	.26608	3.75828	6
55	.19287	5.18480	.21104	4.73851	.22934	4.36040	.24778	4.03578	.26639	3.75388	5
56	.19317	5.17671	.21134	4.73170	.22964	4.35459	.24809	4.03076	.26670	3.74950	4
57	.19347	5.16863	.21164	4.72490	.22995	4.34879	.24840	4.02574	.26701	3.74512	3
58	.19378	5.16058	.21195	4.71813	.23026	4.34300	.24871	4.02074	.26733	3.74075	2
59	.19408	5.15256	.21225	4.71137	.23056	4.33723	.24902	4.01576	.26764	3.73640	1
60	.19438	5.14455	.21256	4.70463	.23087	4.33148	.24933	4.01078	.26795	3.73205	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	79°		78°		77°		76°		75°		

′	15°		16°		17°		18°		19°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.26795	3.73205	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	60
1	.26826	3.72771	.28706	3.48359	.30605	3.26745	.32524	3.07464	.34465	2.90147	59
2	.26857	3.72338	.28738	3.47977	.30637	3.26406	.32556	3.07160	.34498	2.89873	58
3	.26888	3.71907	.28769	3.47596	.30669	3.26067	.32588	3.06857	.34530	2.89600	57
4	.26920	3.71476	.28800	3.47216	.30700	3.25729	.32621	3.06554	.34563	2.89327	56
5	.26951	3.71046	.28832	3.46837	.30732	3.25392	.32653	3.06252	.34596	2.89055	55
6	.26982	3.70616	.28864	3.46458	.30764	3.25055	.32685	3.05950	.34628	2.88783	54
7	.27013	3.70188	.28895	3.46080	.30796	3.24719	.32717	3.05649	.34661	2.88511	53
8	.27044	3.69761	.28927	3.45703	.30828	3.24383	.32749	3.05349	.34693	2.88240	52
9	.27076	3.69335	.28958	3.45327	.30860	3.24049	.32782	3.05049	.34726	2.87970	51
10	.27107	3.68909	.28990	3.44951	.30891	3.23714	.32814	3.04749	.34758	2.87700	50
11	.27138	3.68485	.29021	3.44576	.30923	3.23381	.32846	3.04450	.34791	2.87430	49
12	.27169	3.68061	.29053	3.44202	.30955	3.23048	.32878	3.04152	.34824	2.87161	48
13	.27201	3.67638	.29084	3.43829	.30987	3.22715	.32911	3.03854	.34856	2.86892	47
14	.27232	3.67217	.29116	3.43456	.31019	3.22384	.32943	3.03556	.34889	2.86624	46
15	.27263	3.66796	.29147	3.43084	.31051	3.22053	.32975	3.03260	.34922	2.86356	45
16	.27294	3.66376	.29179	3.42713	.31083	3.21722	.33007	3.02963	.34954	2.86089	44
17	.27326	3.65957	.29210	3.42343	.31115	3.21392	.33040	3.02667	.34987	2.85822	43
18	.27357	3.65538	.29242	3.41973	.31147	3.21063	.33072	3.02372	.35020	2.85555	42
19	.27388	3.65121	.29274	3.41604	.31178	3.20734	.33104	3.02077	.35052	2.85289	41
20	.27419	3.64705	.29305	3.41236	.31210	3.20406	.33136	3.01783	.35085	2.85023	40
21	.27451	3.64289	.29337	3.40869	.31242	3.20079	.33169	3.01489	.35118	2.84758	39
22	.27482	3.63874	.29368	3.40502	.31274	3.19752	.33201	3.01196	.35150	2.84494	38
23	.27513	3.63461	.29400	3.40136	.31306	3.19426	.33233	3.00903	.35183	2.84229	37
24	.27545	3.63048	.29432	3.39771	.31338	3.19100	.33266	3.00611	.35216	2.83965	36
25	.27576	3.62636	.29463	3.39406	.31370	3.18775	.33298	3.00319	.35248	2.83702	35
26	.27607	3.62224	.29495	3.39042	.31402	3.18451	.33330	3.00028	.35281	2.83439	34
27	.27638	3.61814	.29526	3.38679	.31434	3.18127	.33363	2.99738	.35314	2.83176	33
28	.27670	3.61405	.29558	3.38317	.31466	3.17804	.33395	2.99447	.35346	2.82914	32
29	.27701	3.60996	.29590	3.37955	.31498	3.17481	.33427	2.99158	.35379	2.82653	31
30	.27732	3.60588	.29621	3.37594	.31530	3.17159	.33460	2.98868	.35412	2.82391	30
31	.27764	3.60181	.29653	3.37234	.31562	3.16838	.33492	2.98580	.35445	2.82130	29
32	.27795	3.59775	.29685	3.36875	.31594	3.16517	.33524	2.98292	.35477	2.81870	28
33	.27826	3.59370	.29716	3.36516	.31626	3.16197	.33557	2.98004	.35510	2.81610	27
34	.27858	3.58966	.29748	3.36158	.31658	3.15877	.33589	2.97717	.35543	2.81350	26
35	.27889	3.58562	.29780	3.35800	.31690	3.15558	.33621	2.97430	.35576	2.81091	25
36	.27921	3.58160	.29811	3.35443	.31722	3.15240	.33654	2.97144	.35608	2.80833	24
37	.27952	3.57758	.29843	3.35087	.31754	3.14922	.33686	2.96858	.35641	2.80574	23
38	.27983	3.57357	.29875	3.34732	.31786	3.14605	.33718	2.96573	.35674	2.80316	22
39	.28015	3.56957	.29906	3.34377	.31818	3.14288	.33751	2.96288	.35707	2.80059	21
40	.28046	3.56557	.29938	3.34023	.31850	3.13972	.33783	2.96004	.35740	2.79802	20
41	.28077	3.56159	.29970	3.33670	.31882	3.13656	.33816	2.95721	.35772	2.79545	19
42	.28109	3.55761	.30001	3.33317	.31914	3.13341	.33848	2.95437	.35805	2.79289	18
43	.28140	3.55364	.30033	3.32965	.31946	3.13027	.33881	2.95155	.35838	2.79033	17
44	.28172	3.54968	.30065	3.32614	.31978	3.12713	.33913	2.94872	.35871	2.78778	16
45	.28203	3.54573	.30097	3.32264	.32010	3.12400	.33945	2.94591	.35904	2.78523	15
46	.28234	3.54179	.30128	3.31914	.32042	3.12087	.33978	2.94309	.35937	2.78269	14
47	.28266	3.53785	.30160	3.31565	.32074	3.11775	.34010	2.94028	.35969	2.78014	13
48	.28297	3.53393	.30192	3.31216	.32106	3.11464	.34043	2.93748	.36002	2.77761	12
49	.28329	3.53001	.30224	3.30868	.32139	3.11153	.34075	2.93468	.36035	2.77507	11
50	.28360	3.52609	.30255	3.30521	.32171	3.10842	.34108	2.93189	.36068	2.77254	10
51	.28391	3.52219	.30287	3.30174	.32203	3.10532	.34140	2.92910	.36101	2.77002	9
52	.28423	3.51829	.30319	3.29829	.32235	3.10223	.34173	2.92632	.36134	2.76750	8
53	.28454	3.51441	.30351	3.29483	.32267	3.09914	.34205	2.92354	.36167	2.76498	7
54	.28486	3.51053	.30382	3.29139	.32299	3.09606	.34238	2.92076	.36199	2.76247	6
55	.28517	3.50666	.30414	3.28795	.32331	3.09298	.34270	2.91799	.36232	2.75996	5
56	.28549	3.50279	.30446	3.28452	.32363	3.08991	.34303	2.91523	.36265	2.75746	4
57	.28580	3.49894	.30478	3.28109	.32396	3.08685	.34335	2.91246	.36298	2.75496	3
58	.28612	3.49509	.30509	3.27767	.32428	3.08379	.34368	2.90971	.36331	2.75246	2
59	.28643	3.49125	.30541	3.27426	.32460	3.08073	.34400	2.90696	.36364	2.74997	1
60	.28675	3.48741	.30573	3.27085	.32492	3.07768	.34433	2.90421	.36397	2.74748	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	74°		73°		72°		71°		70°		

′	20°		21°		22°		23°		24°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.36397	2.74748	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	60
1	.36430	2.74499	.38420	2.60283	.40436	2.47302	.42482	2.35395	.44558	2.24428	59
2	.36463	2.74251	.38453	2.60057	.40470	2.47095	.42516	2.35205	.44593	2.24252	58
3	.36496	2.74004	.38487	2.59831	.40504	2.46888	.42551	2.35015	.44627	2.24077	57
4	.36529	2.73756	.38520	2.59606	.40538	2.46682	.42585	2.34825	.44662	2.23902	56
5	.36562	2.73509	.38553	2.59381	.40572	2.46476	.42619	2.34636	.44697	2.23727	55
6	.36595	2.73263	.38587	2.59156	.40606	2.46270	.42654	2.34447	.44732	2.23553	54
7	.36628	2.73017	.38620	2.58932	.40640	2.46065	.42688	2.34258	.44767	2.23378	53
8	.36661	2.72771	.38654	2.58708	.40674	2.45860	.42722	2.34069	.44802	2.23204	52
9	.36694	2.72526	.38687	2.58484	.40707	2.45655	.42757	2.33881	.44837	2.23030	51
10	.36727	2.72281	.38721	2.58261	.40741	2.45451	.42791	2.33693	.44872	2.22857	50
11	.36760	2.72036	.38754	2.58038	.40775	2.45246	.42826	2.33505	.44907	2.22683	49
12	.36793	2.71792	.38787	2.57815	.40809	2.45043	.42860	2.33317	.44942	2.22510	48
13	.36826	2.71548	.38821	2.57593	.40843	2.44839	.42894	2.33130	.44977	2.22337	47
14	.36859	2.71305	.38854	2.57371	.40877	2.44636	.42929	2.32943	.45012	2.22164	46
15	.36892	2.71062	.38888	2.57150	.40911	2.44433	.42963	2.32756	.45047	2.21992	45
16	.36925	2.70819	.38921	2.56928	.40945	2.44230	.42998	2.32570	.45082	2.21819	44
17	.36958	2.70577	.38955	2.56707	.40979	2.44027	.43032	2.32383	.45117	2.21647	43
18	.36991	2.70335	.38988	2.56487	.41013	2.43825	.43067	2.32197	.45152	2.21475	42
19	.37024	2.70094	.39022	2.56266	.41047	2.43623	.43101	2.32012	.45187	2.21304	41
20	.37057	2.69853	.39055	2.56046	.41081	2.43422	.43136	2.31826	.45222	2.21132	40
21	.37090	2.69612	.39089	2.55827	.41115	2.43220	.43170	2.31641	.45257	2.20961	39
22	.37123	2.69371	.39122	2.55608	.41149	2.43019	.43205	2.31456	.45292	2.20790	38
23	.37157	2.69131	.39156	2.55389	.41183	2.42819	.43230	2.31271	.45327	2.20619	37
24	.37190	2.68892	.39190	2.55170	.41217	2.42618	.43274	2.31086	.45362	2.20449	36
25	.37223	2.68653	.39223	2.54952	.41251	2.42418	.43308	2.30902	.45397	2.20278	35
26	.37256	2.68414	.39257	2.54734	.41285	2.42218	.43343	2.30718	.45432	2.20108	34
27	.37289	2.68175	.39290	2.54516	.41319	2.42019	.43378	2.30534	.45467	2.19938	33
28	.37322	2.67937	.39324	2.54299	.41353	2.41819	.43412	2.30351	.45502	2.19769	32
29	.37355	2.67700	.39357	2.54082	.41387	2.41620	.43447	2.30167	.45538	2.19599	31
30	.37388	2.67462	.39391	2.53865	.41421	2.41421	.43481	2.29984	.45573	2.19430	30
31	.37422	2.67225	.39425	2.53648	.41455	2.41223	.43516	2.29801	.45608	2.19261	29
32	.37455	2.66989	.39458	2.53432	.41490	2.41025	.43550	2.29619	.45643	2.19092	28
33	.37488	2.66752	.39492	2.53217	.41524	2.40827	.43585	2.29437	.45678	2.18923	27
34	.37521	2.66516	.39526	2.53001	.41558	2.40629	.43620	2.29254	.45713	2.18755	26
35	.37554	2.66281	.39559	2.52786	.41592	2.40432	.43654	2.29073	.45748	2.18587	25
36	.37588	2.66046	.39593	2.52571	.41626	2.40235	.43689	2.28891	.45784	2.18419	24
37	.37621	2.65811	.39626	2.52357	.41660	2.40038	.43724	2.28710	.45819	2.18251	23
38	.37654	2.65576	.39660	2.52142	.41694	2.39841	.43758	2.28528	.45854	2.18084	22
39	.37687	2.65342	.39694	2.51929	.41728	2.39645	.43793	2.28348	.45889	2.17916	21
40	.37720	2.65109	.39727	2.51715	.41763	2.39449	.43828	2.28167	.45924	2.17749	20
41	.37754	2.64875	.39761	2.51502	.41797	2.39253	.43862	2.27987	.45960	2.17582	19
42	.37787	2.64642	.39795	2.51289	.41831	2.39058	.43897	2.27806	.45995	2.17416	18
43	.37820	2.64410	.39829	2.51076	.41865	2.38863	.43932	2.27626	.46030	2.17249	17
44	.37853	2.64177	.39862	2.50864	.41899	2.38668	.43966	2.27447	.46065	2.17083	16
45	.37887	2.63945	.39896	2.50652	.41933	2.38473	.44001	2.27267	.46101	2.16917	15
46	.37920	2.63714	.39930	2.50440	.41968	2.38279	.44036	2.27088	.46136	2.16751	14
47	.37953	2.63483	.39963	2.50229	.42002	2.38084	.44071	2.26909	.46171	2.16585	13
48	.37986	2.63252	.39997	2.50018	.42036	2.37891	.44105	2.26730	.46206	2.16420	12
49	.38020	2.63021	.40031	2.49807	.42070	2.37697	.44140	2.26552	.46242	2.16255	11
50	.38053	2.62791	.40065	2.49597	.42105	2.37504	.44175	2.26374	.46277	2.16090	10
51	.38086	2.62561	.40098	2.49386	.42139	2.37311	.44210	2.26196	.46312	2.15925	9
52	.38120	2.62332	.40132	2.49177	.42173	2.37118	.44244	2.26018	.46348	2.15760	8
53	.38153	2.62103	.40166	2.48967	.42207	2.36925	.44279	2.25840	.46383	2.15596	7
54	.38186	2.61874	.40200	2.48758	.42242	2.36733	.44314	2.25663	.46418	2.15432	6
55	.38220	2.61646	.40234	2.48549	.42276	2.36541	.44349	2.25486	.46454	2.15268	5
56	.38253	2.61418	.40267	2.48340	.42310	2.36349	.44384	2.25309	.46489	2.15104	4
57	.38286	2.61190	.40301	2.48132	.42345	2.36158	.44418	2.25132	.46525	2.14940	3
58	.38320	2.60963	.40335	2.47924	.42379	2.35967	.44453	2.24956	.46560	2.14777	2
59	.38353	2.60736	.40369	2.47716	.42413	2.35776	.44488	2.24780	.46595	2.14614	1
60	.38386	2.60509	.40403	2.47509	.42447	2.35585	.44523	2.24604	.46631	2.14451	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	69°		68°		67°		66°		65°		

′	25°		26°		27°		28°		29°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.46631	2.14451	.48773	2.05030	.50953	1.96261	.53171	1.88073	.55431	1.80405	60
1	.46666	2.14288	.48809	2.04879	.50989	1.96120	.53208	1.87941	.55469	1.80281	59
2	.46702	2.14125	.48845	2.04728	.51026	1.95979	.53246	1.87809	.55507	1.80158	58
3	.46737	2.13963	.48881	2.04577	.51063	1.95838	.53283	1.87677	.55545	1.80034	57
4	.46772	2.13801	.48917	2.04426	.51099	1.95698	.53320	1.87546	.55583	1.79911	56
5	.46808	2.13639	.48953	2.04276	.51136	1.95557	.53358	1.87415	.55621	1.79788	55
6	.46843	2.13477	.48989	2.04125	.51173	1.95417	.53395	1.87283	.55659	1.79665	54
7	.46879	2.13316	.49026	2.03975	.51209	1.95277	.53432	1.87152	.55697	1.79542	53
8	.46914	2.13154	.49062	2.03825	.51246	1.95137	.53470	1.87021	.55736	1.79419	52
9	.46950	2.12993	.49098	2.03675	.51283	1.94997	.53507	1.86891	.55774	1.79296	51
10	.46985	2.12832	.49134	2.03526	.51319	1.94858	.53545	1.86760	.55812	1.79174	50
11	.47021	2.12671	.49170	2.03376	.51356	1.94718	.53582	1.86630	.55850	1.79051	49
12	.47056	2.12511	.49206	2.03227	.51393	1.94579	.53620	1.86499	.55888	1.78929	48
13	.47092	2.12350	.49242	2.03078	.51430	1.94440	.53657	1.86369	.55926	1.78807	47
14	.47128	2.12190	.49278	2.02929	.51467	1.94301	.53694	1.86239	.55964	1.78685	46
15	.47163	2.12030	.49315	2.02780	.51503	1.94162	.53732	1.86109	.56003	1.78563	45
16	.47199	2.11871	.49351	2.02631	.51540	1.94023	.53769	1.85979	.56041	1.78441	44
17	.47234	2.11711	.49387	2.02483	.51577	1.93885	.53807	1.85850	.56079	1.78319	43
18	.47270	2.11552	.49423	2.02335	.51614	1.93746	.53844	1.85720	.56117	1.78198	42
19	.47305	2.11392	.49459	2.02187	.51651	1.93608	.53882	1.85591	.56156	1.78077	41
20	.47341	2.11233	.49495	2.02039	.51688	1.93470	.53920	1.85462	.56194	1.77955	40
21	.47377	2.11075	.49532	2.01891	.51724	1.93332	.53957	1.85333	.56232	1.77834	39
22	.47412	2.10916	.49568	2.01743	.51761	1.93195	.53995	1.85204	.56270	1.77713	38
23	.47448	2.10758	.49604	2.01596	.51798	1.93057	.54032	1.85075	.56309	1.77592	37
24	.47483	2.10600	.49640	2.01449	.51835	1.92920	.54070	1.84946	.56347	1.77471	36
25	.47519	2.10442	.49677	2.01302	.51872	1.92782	.54107	1.84818	.56385	1.77351	35
26	.47555	2.10284	.49713	2.01155	.51909	1.92645	.54145	1.84689	.56424	1.77230	34
27	.47590	2.10126	.49749	2.01008	.51946	1.92508	.54183	1.84561	.56462	1.77110	33
28	.47626	2.09969	.49786	2.00862	.51983	1.92371	.54220	1.84433	.56501	1.76990	32
29	.47662	2.09811	.49822	2.00715	.52020	1.92235	.54258	1.84305	.56539	1.76869	31
30	.47698	2.09654	.49858	2.00569	.52057	1.92098	.54296	1.84177	.56577	1.76749	30
31	.47733	2.09498	.49894	2.00423	.52094	1.91962	.54333	1.84049	.56616	1.76629	29
32	.47769	2.09341	.49931	2.00277	.52131	1.91826	.54371	1.83922	.56654	1.76510	28
33	.47805	2.09184	.49967	2.00131	.52168	1.91690	.54409	1.83794	.56693	1.76390	27
34	.47840	2.09028	.50004	1.99986	.52205	1.91554	.54446	1.83667	.56731	1.76271	26
35	.47876	2.08872	.50040	1.99841	.52242	1.91418	.54484	1.83540	.56769	1.76151	25
36	.47912	2.08716	.50076	1.99695	.52279	1.91282	.54522	1.83413	.56808	1.76032	24
37	.47948	2.08560	.50113	1.99550	.52316	1.91147	.54560	1.83286	.56846	1.75913	23
38	.47984	2.08405	.50149	1.99406	.52353	1.91012	.54597	1.83159	.56885	1.75794	22
39	.48019	2.08250	.50185	1.99261	.52390	1.90876	.54635	1.83033	.56923	1.75675	21
40	.48055	2.08094	.50222	1.99116	.52427	1.90741	.54673	1.82906	.56962	1.75556	20
41	.48091	2.07939	.50258	1.98972	.52464	1.90607	.54711	1.82780	.57000	1.75437	19
42	.48127	2.07785	.50295	1.98828	.52501	1.90472	.54748	1.82654	.57039	1.75319	18
43	.48163	2.07630	.50331	1.98684	.52538	1.90337	.54786	1.82528	.57078	1.75200	17
44	.48198	2.07476	.50368	1.98540	.52575	1.90203	.54824	1.82402	.57116	1.75082	16
45	.48234	2.07321	.50404	1.98396	.52613	1.90069	.54862	1.82276	.57155	1.74964	15
46	.48270	2.07167	.50441	1.98253	.52650	1.89935	.54900	1.82150	.57193	1.74846	14
47	.48306	2.07014	.50477	1.98110	.52687	1.89801	.54938	1.82025	.57232	1.74728	13
48	.48342	2.06860	.50514	1.97966	.52724	1.89667	.54975	1.81899	.57271	1.74610	12
49	.48378	2.06706	.50550	1.97823	.52761	1.89533	.55013	1.81774	.57309	1.74492	11
50	.48414	2.06553	.50587	1.97681	.52798	1.89400	.55051	1.81649	.57348	1.74375	10
51	.48450	2.06400	.50623	1.97538	.52836	1.89266	.55089	1.81524	.57386	1.74257	9
52	.48486	2.06247	.50660	1.97395	.52873	1.89133	.55127	1.81399	.57425	1.74140	8
53	.48521	2.06094	.50696	1.97253	.52910	1.89000	.55165	1.81274	.57464	1.74022	7
54	.48557	2.05942	.50733	1.97111	.52947	1.88867	.55203	1.81150	.57503	1.73905	6
55	.48593	2.05790	.50769	1.96969	.52985	1.88734	.55241	1.81025	.57541	1.73788	5
56	.48629	2.05637	.50806	1.96827	.53022	1.88602	.55279	1.80901	.57580	1.73671	4
57	.48665	2.05485	.50843	1.96685	.53059	1.88469	.55317	1.80777	.57619	1.73555	3
58	.48701	2.05333	.50879	1.96544	.53096	1.88337	.55355	1.80653	.57657	1.73438	2
59	.48737	2.05182	.50916	1.96402	.53134	1.88205	.55393	1.80529	.57696	1.73321	1
60	.48773	2.05030	.50953	1.96261	.53171	1.88073	.55431	1.80405	.57735	1.73205	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	64°		63°		62°		61°		60°		

′	30°		31°		32°		33°		34°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.57735	1.73205	.60086	1.66428	.62487	1.60033	.64941	1.53986	.67451	1.48256	60
1	.57774	1.73089	.60126	1.66318	.62527	1.59930	.64982	1.53888	.67493	1.48163	59
2	.57813	1.72973	.60165	1.66209	.62568	1.59826	.65024	1.53791	.67536	1.48070	58
3	.57851	1.72857	.60205	1.66099	.62608	1.59723	.65065	1.53693	.67578	1.47977	57
4	.57890	1.72741	.60245	1.65990	.62649	1.59620	.65106	1.53595	.67620	1.47885	56
5	.57929	1.72625	.60284	1.65881	.62689	1.59517	.65148	1.53497	.67663	1.47792	55
6	.57968	1.72509	.60324	1.65772	.62730	1.59414	.65189	1.53400	.67705	1.47699	54
7	.58007	1.72393	.60364	1.65663	.62770	1.59311	.65231	1.53302	.67748	1.47607	53
8	.58046	1.72278	.60403	1.65554	.62811	1.59208	.65272	1.53205	.67790	1.47514	52
9	.58085	1.72163	.60443	1.65445	.62852	1.59105	.65314	1.53107	.67832	1.47422	51
10	.58124	1.72047	.60483	1.65337	.62892	1.59002	.65355	1.53010	.67875	1.47330	50
11	.58162	1.71932	.60522	1.65228	.62933	1.58900	.65397	1.52913	.67917	1.47238	49
12	.58201	1.71817	.60562	1.65120	.62973	1.58797	.65438	1.52816	.67960	1.47146	48
13	.58240	1.71702	.60602	1.65011	.63014	1.58695	.65480	1.52719	.68002	1.47053	47
14	.58279	1.71588	.60642	1.64903	.63055	1.58593	.65521	1.52622	.68045	1.46962	46
15	.58318	1.71473	.60681	1.64795	.63095	1.58490	.65563	1.52525	.68088	1.46870	45
16	.58357	1.71358	.60721	1.64687	.63136	1.58388	.65604	1.52429	.68130	1.46778	44
17	.58396	1.71244	.60761	1.64579	.63177	1.58286	.65646	1.52332	.68173	1.46686	43
18	.58435	1.71129	.60801	1.64471	.63217	1.58184	.65688	1.52235	.68215	1.46595	42
19	.58474	1.71015	.60841	1.64363	.63258	1.58083	.65729	1.52139	.68258	1.46503	41
20	.58513	1.70901	.60881	1.64256	.63299	1.57981	.65771	1.52043	.68301	1.46411	40
21	.58552	1.70787	.60921	1.64148	.63340	1.57879	.65813	1.51946	.68343	1.46320	39
22	.58591	1.70673	.60960	1.64041	.63380	1.57778	.65854	1.51850	.68386	1.46229	38
23	.58631	1.70560	.61000	1.63934	.63421	1.57676	.65896	1.51754	.68429	1.46137	37
24	.58670	1.70446	.61040	1.63826	.63462	1.57575	.65938	1.51658	.68471	1.46046	36
25	.58709	1.70332	.61080	1.63719	.63503	1.57474	.65980	1.51562	.68514	1.45955	35
26	.58748	1.70219	.61120	1.63612	.63544	1.57372	.66021	1.51466	.68557	1.45864	34
27	.58787	1.70106	.61160	1.63505	.63584	1.57271	.66063	1.51370	.68600	1.45773	33
28	.58826	1.69992	.61200	1.63398	.63625	1.57170	.66105	1.51275	.68642	1.45682	32
29	.58865	1.69879	.61240	1.63292	.63666	1.57069	.66147	1.51179	.68685	1.45592	31
30	.58905	1.69766	.61280	1.63185	.63707	1.56969	.66189	1.51084	.68728	1.45501	30
31	.58944	1.69653	.61320	1.63079	.63748	1.56868	.66230	1.50988	.68771	1.45410	29
32	.58983	1.69541	.61360	1.62972	.63789	1.56767	.66272	1.50893	.68814	1.45320	28
33	.59022	1.69428	.61400	1.62866	.63830	1.56667	.66314	1.50797	.68857	1.45229	27
34	.59061	1.69316	.61440	1.62760	.63871	1.56566	.66356	1.50702	.68900	1.45139	26
35	.59101	1.69203	.61480	1.62654	.63912	1.56466	.66398	1.50607	.68942	1.45049	25
36	.59140	1.69091	.61520	1.62548	.63953	1.56366	.66440	1.50512	.68985	1.44958	24
37	.59179	1.68979	.61561	1.62442	.63994	1.56265	.66482	1.50417	.69028	1.44868	23
38	.59218	1.68866	.61601	1.62336	.64035	1.56165	.66524	1.50322	.69071	1.44778	22
39	.59258	1.68754	.61641	1.62230	.64076	1.56065	.66566	1.50228	.69114	1.44688	21
40	.59297	1.68643	.61681	1.62125	.64117	1.55966	.66608	1.50133	.69157	1.44598	20
41	.59336	1.68531	.61721	1.62019	.64158	1.55866	.66650	1.50038	.69200	1.44508	19
42	.59376	1.68419	.61761	1.61914	.64199	1.55766	.66692	1.49944	.69243	1.44418	18
43	.59415	1.68308	.61801	1.61808	.64240	1.55666	.66734	1.49849	.69286	1.44329	17
44	.59454	1.68196	.61842	1.61703	.64281	1.55567	.66776	1.49755	.69329	1.44239	16
45	.59494	1.68085	.61882	1.61598	.64322	1.55467	.66818	1.49661	.69372	1.44149	15
46	.59533	1.67974	.61922	1.61493	.64363	1.55368	.66860	1.49566	.69416	1.44060	14
47	.59573	1.67863	.61962	1.61388	.64404	1.55269	.66902	1.49472	.69459	1.43970	13
48	.59612	1.67752	.62003	1.61283	.64446	1.55170	.66944	1.49378	.69502	1.43881	12
49	.59651	1.67641	.62043	1.61179	.64487	1.55071	.66986	1.49284	.69545	1.43792	11
50	.59691	1.67530	.62083	1.61074	.64528	1.54972	.67028	1.49190	.69588	1.43703	10
51	.59730	1.67419	.62124	1.60970	.64569	1.54873	.67071	1.49097	.69631	1.43614	9
52	.59770	1.67309	.62164	1.60865	.64610	1.54774	.67113	1.49003	.69675	1.43525	8
53	.59809	1.67198	.62204	1.60761	.64652	1.54675	.67155	1.48909	.69718	1.43436	7
54	.59849	1.67088	.62245	1.60657	.64693	1.54576	.67197	1.48816	.69761	1.43347	6
55	.59888	1.66978	.62285	1.60553	.64734	1.54478	.67239	1.48722	.69804	1.43258	5
56	.59928	1.66867	.62325	1.60449	.64775	1.54379	.67282	1.48629	.69847	1.43169	4
57	.59967	1.66757	.62366	1.60345	.64817	1.54281	.67324	1.48536	.69891	1.43080	3
58	.60007	1.66647	.62406	1.60241	.64858	1.54183	.67366	1.48442	.69934	1.42992	2
59	.60046	1.66538	.62446	1.60137	.64899	1.54085	.67409	1.48349	.69977	1.42903	1
60	.60086	1.66428	.62487	1.60033	.64941	1.53986	.67451	1.48256	.70021	1.42815	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	59°		58°		57°		56°		55°		

′	35°		36°		37°		38°		39°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.70021	1.42815	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	60
1	.70064	1.42726	.72699	1.37554	.75401	1.32624	.78175	1.27917	.81027	1.23416	59
2	.70107	1.42638	.72743	1.37470	.75447	1.32544	.78222	1.27841	.81075	1.23343	58
3	.70151	1.42550	.72788	1.37386	.75492	1.32464	.78269	1.27764	.81123	1.23270	57
4	.70194	1.42462	.72832	1.37302	.75538	1.32384	.78316	1.27688	.81171	1.23196	56
5	.70238	1.42374	.72877	1.37218	.75584	1.32304	.78363	1.27611	.81220	1.23123	55
6	.70281	1.42286	.72921	1.37134	.75629	1.32224	.78410	1.27535	.81268	1.23050	54
7	.70325	1.42198	.72966	1.37050	.75675	1.32144	.78457	1.27458	.81316	1.22977	53
8	.70368	1.42110	.73010	1.36967	.75721	1.32064	.78504	1.27382	.81364	1.22904	52
9	.70412	1.42022	.73055	1.36883	.75767	1.31984	.78551	1.27306	.81413	1.22831	51
10	.70455	1.41934	.73100	1.36800	.75812	1.31904	.78598	1.27230	.81461	1.22758	50
11	.70499	1.41847	.73144	1.36716	.75858	1.31825	.78645	1.27153	.81510	1.22685	49
12	.70542	1.41759	.73189	1.36633	.75904	1.31745	.78692	1.27077	.81558	1.22612	48
13	.70586	1.41672	.73234	1.36549	.75950	1.31666	.78739	1.27001	.81606	1.22539	47
14	.70629	1.41584	.73278	1.36466	.75996	1.31586	.78786	1.26925	.81655	1.22467	46
15	.70673	1.41497	.73323	1.36383	.76042	1.31507	.78834	1.26849	.81703	1.22394	45
16	.70717	1.41409	.73368	1.36300	.76088	1.31427	.78881	1.26774	.81752	1.22321	44
17	.70760	1.41322	.73413	1.36217	.76134	1.31348	.78928	1.26698	.81800	1.22249	43
18	.70804	1.41235	.73457	1.36134	.76180	1.31269	.78975	1.26622	.81849	1.22176	42
19	.70848	1.41148	.73502	1.36051	.76226	1.31190	.79022	1.26546	.81898	1.22104	41
20	.70891	1.41061	.73547	1.35968	.76272	1.31110	.79070	1.26471	.81946	1.22031	40
21	.70935	1.40974	.73592	1.35885	.76318	1.31031	.79117	1.26395	.81995	1.21959	39
22	.70979	1.40887	.73637	1.35802	.76364	1.30952	.79164	1.26319	.82044	1.21886	38
23	.71023	1.40800	.73681	1.35719	.76410	1.30873	.79212	1.26244	.82092	1.21814	37
24	.71066	1.40714	.73726	1.35637	.76456	1.30795	.79259	1.26169	.82141	1.21742	36
25	.71110	1.40627	.73771	1.35554	.76502	1.30716	.79306	1.26093	.82190	1.21670	35
26	.71154	1.40540	.73816	1.35472	.76548	1.30637	.79354	1.26018	.82238	1.21598	34
27	.71198	1.40454	.73861	1.35389	.76594	1.30558	.79401	1.25943	.82287	1.21526	33
28	.71242	1.40367	.73906	1.35307	.76640	1.30480	.79449	1.25867	.82336	1.21454	32
29	.71285	1.40281	.73951	1.35224	.76686	1.30401	.79496	1.25792	.82385	1.21382	31
30	.71329	1.40195	.73996	1.35142	.76733	1.30323	.79544	1.25717	.82434	1.21310	30
31	.71373	1.40109	.74041	1.35060	.76779	1.30244	.79591	1.25642	.82483	1.21238	29
32	.71417	1.40022	.74086	1.34978	.76825	1.30166	.79639	1.25567	.82531	1.21166	28
33	.71461	1.39936	.74131	1.34896	.76871	1.30087	.79686	1.25492	.82580	1.21094	27
34	.71505	1.39850	.74176	1.34814	.76918	1.30009	.79734	1.25417	.82629	1.21023	26
35	.71549	1.39764	.74221	1.34732	.76964	1.29931	.79781	1.25343	.82678	1.20951	25
36	.71593	1.39679	.74267	1.34650	.77010	1.29853	.79829	1.25268	.82727	1.20879	24
37	.71637	1.39593	.74312	1.34568	.77057	1.29775	.79877	1.25193	.82776	1.20808	23
38	.71681	1.39507	.74357	1.34487	.77103	1.29696	.79924	1.25118	.82825	1.20736	22
39	.71725	1.39421	.74402	1.34405	.77149	1.29618	.79972	1.25044	.82874	1.20665	21
40	.71769	1.39336	.74447	1.34323	.77196	1.29541	.80020	1.24969	.82923	1.20593	20
41	.71813	1.39250	.74492	1.34242	.77242	1.29463	.80067	1.24895	.82972	1.20522	19
42	.71857	1.39165	.74538	1.34160	.77289	1.29385	.80115	1.24820	.83022	1.20451	18
43	.71901	1.39079	.74583	1.34079	.77335	1.29307	.80163	1.24746	.83071	1.20379	17
44	.71946	1.38994	.74628	1.33998	.77382	1.29229	.80211	1.24672	.83120	1.20308	16
45	.71990	1.38909	.74674	1.33916	.77428	1.29152	.80258	1.24597	.83169	1.20237	15
46	.72034	1.38824	.74719	1.33835	.77475	1.29074	.80306	1.24523	.83218	1.20166	14
47	.72078	1.38738	.74764	1.33754	.77521	1.28997	.80354	1.24449	.83268	1.20095	13
48	.72122	1.38653	.74810	1.33673	.77568	1.28919	.80402	1.24375	.83317	1.20024	12
49	.72167	1.38568	.74855	1.33592	.77615	1.28842	.80450	1.24301	.83366	1.19953	11
50	.72211	1.38484	.74900	1.33511	.77661	1.28764	.80498	1.24227	.83415	1.19882	10
51	.72255	1.38399	.74946	1.33430	.77708	1.28687	.80546	1.24153	.83465	1.19811	9
52	.72299	1.38314	.74991	1.33349	.77754	1.28610	.80594	1.24079	.83514	1.19740	8
53	.72344	1.38229	.75037	1.33268	.77801	1.28533	.80642	1.24005	.83564	1.19669	7
54	.72388	1.38145	.75082	1.33187	.77848	1.28456	.80690	1.23931	.83613	1.19599	6
55	.72432	1.38060	.75128	1.33107	.77895	1.28379	.80738	1.23858	.83662	1.19528	5
56	.72477	1.37976	.75173	1.33026	.77941	1.28302	.80786	1.23784	.83712	1.19457	4
57	.72521	1.37891	.75219	1.32946	.77988	1.28225	.80834	1.23710	.83761	1.19387	3
58	.72565	1.37807	.75264	1.32865	.78035	1.28148	.80882	1.23637	.83811	1.19316	2
59	.72610	1.37722	.75310	1.32785	.78082	1.28071	.80930	1.23563	.83860	1.19246	1
60	.72654	1.37638	.75355	1.32704	.78129	1.27994	.80978	1.23490	.83910	1.19175	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	54°		53°		52°		51°		50°		

′	40°		41°		42°		43°		44°		′
	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	
0	.83910	1.19175	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	60
1	.83960	1.19105	.86980	1.14969	.90093	1.10996	.93306	1.07174	.96625	1.03493	59
2	.84009	1.19035	.87031	1.14902	.90146	1.10931	.93360	1.07112	.96681	1.03433	58
3	.84059	1.18964	.87082	1.14834	.90199	1.10867	.93415	1.07049	.96738	1.03372	57
4	.84108	1.18894	.87133	1.14767	.90251	1.10802	.93469	1.06987	.96794	1.03312	56
5	.84158	1.18824	.87184	1.14699	.90304	1.10737	.93524	1.06925	.96850	1.03252	55
6	.84208	1.18754	.87236	1.14632	.90357	1.10672	.93578	1.06862	.96907	1.03192	54
7	.84258	1.18684	.87287	1.14565	.90410	1.10607	.93633	1.06800	.96963	1.03132	53
8	.84307	1.18614	.87338	1.14498	.90463	1.10543	.93688	1.06738	.97020	1.03072	52
9	.84357	1.18544	.87389	1.14430	.90516	1.10478	.93742	1.06676	.97076	1.03012	51
10	.84407	1.18474	.87441	1.14363	.90569	1.10414	.93797	1.06613	.97133	1.02952	50
11	.84457	1.18404	.87492	1.14296	.90621	1.10349	.93852	1.06551	.97189	1.02892	49
12	.84507	1.18334	.87543	1.14229	.90674	1.10285	.93906	1.06489	.97246	1.02832	48
13	.84556	1.18264	.87595	1.14162	.90727	1.10220	.93961	1.06427	.97302	1.02772	47
14	.84606	1.18194	.87646	1.14095	.90781	1.10156	.94016	1.06365	.97359	1.02713	46
15	.84656	1.18125	.87698	1.14028	.90834	1.10091	.94071	1.06303	.97416	1.02653	45
16	.84706	1.18055	.87749	1.13961	.90887	1.10027	.94125	1.06241	.97472	1.02593	44
17	.84756	1.17986	.87801	1.13894	.90940	1.09963	.94180	1.06179	.97529	1.02533	43
18	.84806	1.17916	.87852	1.13828	.90993	1.09899	.94235	1.06117	.97586	1.02474	42
19	.84856	1.17846	.87904	1.13761	.91046	1.09834	.94290	1.06056	.97643	1.02414	41
20	.84906	1.17777	.87955	1.13694	.91099	1.09770	.94345	1.05994	.97700	1.02355	40
21	.84956	1.17708	.88007	1.13627	.91153	1.09706	.94400	1.05932	.97756	1.02295	39
22	.85006	1.17638	.88059	1.13561	.91206	1.09642	.94455	1.05870	.97813	1.02236	38
23	.85057	1.17569	.88110	1.13494	.91259	1.09578	.94510	1.05809	.97870	1.02176	37
24	.85107	1.17500	.88162	1.13428	.91313	1.09514	.94565	1.05747	.97927	1.02117	36
25	.85157	1.17430	.88214	1.13361	.91366	1.09450	.94620	1.05685	.97984	1.02057	35
26	.85207	1.17361	.88265	1.13295	.91419	1.09386	.94676	1.05624	.98041	1.01998	34
27	.85257	1.17292	.88317	1.13228	.91473	1.09322	.94731	1.05562	.98098	1.01939	33
28	.85308	1.17223	.88369	1.13162	.91526	1.09258	.94786	1.05501	.98155	1.01879	32
29	.85358	1.17154	.88421	1.13096	.91580	1.09195	.94841	1.05439	.98213	1.01820	31
30	.85408	1.17085	.88473	1.13029	.91633	1.09131	.94896	1.05378	.98270	1.01761	30
31	.85458	1.17016	.88524	1.12963	.91687	1.09067	.94952	1.05317	.98327	1.01702	29
32	.85509	1.16947	.88576	1.12897	.91740	1.09003	.95007	1.05255	.98384	1.01642	28
33	.85559	1.16878	.88628	1.12831	.91794	1.08940	.95062	1.05194	.98441	1.01583	27
34	.85609	1.16809	.88680	1.12765	.91847	1.08876	.95118	1.05133	.98499	1.01524	26
35	.85660	1.16741	.88732	1.12699	.91901	1.08813	.95173	1.05072	.98556	1.01465	25
36	.85710	1.16672	.88784	1.12633	.91955	1.08749	.95229	1.05010	.98613	1.01406	24
37	.85761	1.16603	.88836	1.12567	.92008	1.08686	.95284	1.04949	.98671	1.01347	23
38	.85811	1.16535	.88888	1.12501	.92062	1.08622	.95340	1.04888	.98728	1.01288	22
39	.85862	1.16466	.88940	1.12435	.92116	1.08559	.95395	1.04827	.98786	1.01229	21
40	.85912	1.16398	.88992	1.12369	.92170	1.08496	.95451	1.04766	.98843	1.01170	20
41	.85963	1.16329	.89045	1.12303	.92224	1.08432	.95506	1.04705	.98901	1.01112	19
42	.86014	1.16261	.89097	1.12238	.92277	1.08369	.95562	1.04644	.98958	1.01053	18
43	.86064	1.16192	.89149	1.12172	.92331	1.08306	.95618	1.04583	.99016	1.00994	17
44	.86115	1.16124	.89201	1.12106	.92385	1.08243	.95673	1.04522	.99073	1.00935	16
45	.86166	1.16056	.89253	1.12041	.92439	1.08179	.95729	1.04461	.99131	1.00876	15
46	.86216	1.15987	.89306	1.11975	.92493	1.08116	.95785	1.04401	.99189	1.00818	14
47	.86267	1.15919	.89358	1.11909	.92547	1.08053	.95841	1.04340	.99247	1.00759	13
48	.86318	1.15851	.89410	1.11844	.92601	1.07990	.95897	1.04279	.99304	1.00701	12
49	.86368	1.15783	.89463	1.11778	.92655	1.07927	.95952	1.04218	.99362	1.00642	11
50	.86419	1.15715	.89515	1.11713	.92709	1.07864	.96008	1.04158	.99420	1.00583	10
51	.86470	1.15647	.89567	1.11648	.92763	1.07801	.96064	1.04097	.99478	1.00525	9
52	.86521	1.15579	.89620	1.11582	.92817	1.07738	.96120	1.04036	.99536	1.00467	8
53	.86572	1.15511	.89672	1.11517	.92872	1.07676	.96176	1.03976	.99594	1.00408	7
54	.86623	1.15443	.89725	1.11452	.92926	1.07613	.96232	1.03915	.99652	1.00350	6
55	.86674	1.15375	.89777	1.11387	.92980	1.07550	.96288	1.03855	.99710	1.00291	5
56	.86725	1.15308	.89830	1.11321	.93034	1.07487	.96344	1.03794	.99768	1.00233	4
57	.86776	1.15240	.89883	1.11256	.93088	1.07425	.96400	1.03734	.99826	1.00175	3
58	.86827	1.15172	.89935	1.11191	.93143	1.07362	.96457	1.03674	.99884	1.00116	2
59	.86878	1.15104	.89988	1.11126	.93197	1.07299	.96513	1.03613	.99942	1.00058	1
60	.86929	1.15037	.90040	1.11061	.93252	1.07237	.96569	1.03553	1.00000	1.00000	0
′	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	Cotang	Tang	′
	49°		48°		47°		46°		45°		

LOGARITHMIC TRIGONOMETRIC FUNCTIONS

″	′	log sin	d	S	T	log tan	c. d.	log cot	C	log cos	
				6.685					5.314		
0	0									0.00000	60
60	1	4.46373	30103	57	57	4.46373	30103	3.53627	43	0.00000	59
120	2	4.76476	17609	57	57	4.76476	17609	3.23524	43	0.00000	58
180	3	4.94085	12494	57	58	4.94085	12494	3.05915	43	0.00000	57
240	4	5.06579	9691	57	58	5.06579	9691	2.93421	42	0.00000	56
300	5	5.16270	7918	57	58	5.16270	7918	2.83730	42	0.00000	55
360	6	5.24188	6694	57	58	5.24188	6694	2.75812	42	0.00000	54
420	7	5.30882	5800	57	58	5.30882	5800	2.69118	42	0.00000	53
480	8	5.36682	5115	57	58	5.36682	5115	2.63318	42	0.00000	52
540	9	5.41797	4576	57	58	5.41797	4576	2.58203	42	0.00000	51
600	10	5.46373	4139	57	58	5.46373	4139	2.53627	42	0.00000	50
660	11	5.50512	3779	57	58	5.50512	3779	2.49488	42	0.00000	49
720	12	5.54291	3476	57	58	5.54291	3476	2.45709	42	0.00000	48
780	13	5.57767	3218	57	58	5.57767	3219	2.42233	42	0.00000	47
840	14	5.60985	2997	57	58	5.60986	2996	2.39014	42	0.00000	46
900	15	5.63982	2802	57	58	5.63982	2803	2.36018	42	0.00000	45
960	16	5.66784	2633	57	58	5.66785	2633	2.33215	42	0.00000	44
1020	17	5.69417	2483	57	58	5.69418	2482	2.30582	42	1.99999	43
1080	18	5.71900	2348	57	58	5.71900	2348	2.28100	42	1.99999	42
1140	19	5.74248	2227	57	58	5.74248	2228	2.25752	42	1.99999	41
1200	20	5.76475	2119	57	58	5.76476	2119	2.23524	42	1.99999	40
1260	21	5.78594	2021	57	58	5.78595	2020	2.21405	42	1.99999	39
1320	22	5.80615	1930	57	58	5.80615	1931	2.19385	42	1.99999	38
1380	23	5.82545	1848	57	58	5.82546	1848	2.17454	42	1.99999	37
1440	24	5.84393	1773	57	58	5.84394	1773	2.15606	42	1.99999	36
1500	25	5.86166	1704	57	58	5.86167	1704	2.13833	42	1.99999	35
1560	26	5.87870	1639	57	58	5.87871	1639	2.12129	42	1.99999	34
1620	27	5.89509	1579	57	58	5.89510	1579	2.10490	42	1.99999	33
1680	28	5.91088	1524	57	58	5.91089	1524	2.08911	42	1.99999	32
1740	29	5.92612	1472	57	59	5.92613	1473	2.07387	41	1.99998	31
1800	30	5.94084	1424	57	59	5.94086	1424	2.05914	41	1.99998	30
1860	31	5.95508	1379	57	59	5.95510	1379	2.04490	41	1.99998	29
1920	32	5.96887	1336	57	59	5.96889	1336	2.03111	41	1.99998	28
1980	33	5.98223	1297	57	59	5.98225	1297	2.01775	41	1.99998	27
2040	34	5.99520	1259	57	59	5.99522	1259	2.00478	41	1.99998	26
2100	35	6.00779	1223	57	59	6.00781	1223	1.99219	41	1.99998	25
2160	36	6.02002	1190	57	59	6.02004	1190	1.97996	41	1.99998	24
2220	37	6.03192	1158	57	59	6.03194	1159	1.96806	41	1.99997	23
2280	38	6.04350	1128	57	59	6.04353	1128	1.95647	41	1.99997	22
2340	39	6.05478	1100	57	59	6.05481	1100	1.94519	41	1.99997	21
2400	40	6.06578	1072	57	59	6.06581	1072	1.93419	41	1.99997	20
2460	41	6.07650	1046	56	60	6.07653	1047	1.92347	40	1.99997	19
2520	42	6.08696	1022	56	60	6.08700	1022	1.91300	40	1.99997	18
2580	43	6.09718	999	56	60	6.09722	998	1.90278	40	1.99997	17
2640	44	6.10717	976	56	60	6.10720	976	1.89280	40	1.99996	16
2700	45	6.11693	954	56	60	6.11696	955	1.88304	40	1.99996	15
2760	46	6.12647	934	56	60	6.12651	934	1.87349	40	1.99996	14
2820	47	6.13581	914	56	60	6.13585	915	1.86415	40	1.99996	13
2880	48	6.14495	896	56	60	6.14500	895	1.85500	40	1.99996	12
2940	49	6.15391	877	56	60	6.15395	878	1.84605	40	1.99996	11
3000	50	6.16268	860	56	61	6.16273	860	1.83727	39	1.99995	10
3060	51	6.17128	843	56	61	6.17133	843	1.82867	39	1.99995	9
3120	52	6.17971	827	56	61	6.17976	828	1.82024	39	1.99995	8
3180	53	6.18798	812	56	61	6.18804	812	1.81196	39	1.99995	7
3240	54	6.19610	797	56	61	6.19616	797	1.80384	39	1.99995	6
3300	55	6.20407	782	56	61	6.20413	782	1.79587	39	1.99994	5
3360	56	6.21189	769	56	61	6.21195	769	1.78805	39	1.99994	4
3420	57	6.21958	755	55	61	6.21964	756	1.78036	39	1.99994	3
3480	58	6.22713	743	55	62	6.22720	742	1.77280	38	1.99994	2
3540	59	6.23456	730	55	62	6.23462	730	1.76538	38	1.99994	1
3600	60	6.24186		55	62	6.24192		1.75808	38	1.99993	0
				6.685					5.314		
		log cos	d	S	T	log cot	c. d.	log tan	C	log sin	′

″	′	log sin	d	S	T	log tan	c. d.	log cot	C	log cos	
				6.685					5.314		
3600	0	2.24186	717	55	62	2.24192	718	1.75808	38	1.99993	60
3660	1	2.24903	706	55	62	2.24910	706	1.75090	38	1.99993	59
3720	2	2.25609	695	55	62	2.25616	696	1.74384	38	1.99993	58
3780	3	2.26304	684	55	62	2.26312	684	1.73688	38	1.99993	57
3840	4	2.26988	673	55	63	2.26996	673	1.73004	37	1.99992	56
3900	5	2.27661	663	55	63	2.27669	663	1.72331	37	1.99992	55
3960	6	2.28324	653	55	63	2.28332	654	1.71668	37	1.99992	54
4020	7	2.28977	644	55	63	2.28986	643	1.71014	37	1.99992	53
4080	8	2.29621	634	55	63	2.29629	634	1.70371	37	1.99992	52
4140	9	2.30255	624	55	63	2.30263	625	1.69737	37	1.99991	51
4200	10	2.30879	616	54	63	2.30888	617	1.69112	37	1.99991	50
4260	11	2.31495	608	54	64	2.31505	607	1.68495	36	1.99991	49
4320	12	2.32103	599	54	64	2.32112	599	1.67888	36	1.99990	48
4380	13	2.32702	590	54	64	2.32711	591	1.67289	36	1.99990	47
4440	14	2.33292	583	54	64	2.33302	584	1.66698	36	1.99990	46
4500	15	2.33875	575	54	64	2.33886	575	1.66114	36	1.99990	45
4560	16	2.34450	568	54	65	2.34461	568	1.65539	35	1.99989	44
4620	17	2.35018	560	54	65	2.35029	561	1.64971	35	1.99989	43
4680	18	2.35578	553	54	65	2.35590	553	1.64410	35	1.99989	42
4740	19	2.36131	547	54	65	2.36143	546	1.63857	35	1.99989	41
4800	20	2.36678	539	54	65	2.36689	540	1.63311	35	1.99988	40
4860	21	2.37217	533	53	66	2.37229	533	1.62771	34	1.99988	39
4920	22	2.37750	526	53	66	2.37762	527	1.62238	34	1.99988	38
4980	23	2.38276	520	53	66	2.38289	520	1.61711	34	1.99987	37
5040	24	2.38796	514	53	66	2.38809	514	1.61191	34	1.99987	36
5100	25	2.39310	508	53	66	2.39323	509	1.60677	34	1.99987	35
5160	26	2.39818	502	53	67	2.39832	502	1.60168	33	1.99986	34
5220	27	2.40320	496	53	67	2.40334	496	1.59666	33	1.99986	33
5280	28	2.40816	491	53	67	2.40830	491	1.59170	33	1.99986	32
5340	29	2.41307	485	53	67	2.41321	486	1.58679	33	1.99985	31
5400	30	2.41792	480	53	67	2.41807	480	1.58193	33	1.99985	30
5460	31	2.42272	474	52	68	2.42287	475	1.57713	32	1.99985	29
5520	32	2.42746	470	52	68	2.42762	470	1.57238	32	1.99984	28
5580	33	2.43216	464	52	68	2.43232	464	1.56768	32	1.99984	27
5640	34	2.43680	459	52	68	2.43696	460	1.56304	32	1.99984	26
5700	35	2.44139	455	52	69	2.44156	455	1.55844	31	1.99983	25
5760	36	2.44594	450	52	69	2.44611	450	1.55389	31	1.99983	24
5820	37	2.45044	445	52	69	2.45061	446	1.54939	31	1.99983	23
5880	38	2.45489	441	52	69	2.45507	441	1.54493	31	1.99982	22
5940	39	2.45930	436	51	69	2.45948	437	1.54052	31	1.99982	21
6000	40	2.46366	433	51	70	2.46385	432	1.53615	30	1.99982	20
6060	41	2.46799	427	51	70	2.46817	428	1.53183	30	1.99981	19
6120	42	2.47226	424	51	70	2.47245	424	1.52755	30	1.99981	18
6180	43	2.47650	419	51	70	2.47669	420	1.52331	30	1.99981	17
6240	44	2.48069	416	51	71	2.48089	416	1.51911	29	1.99980	16
6300	45	2.48485	411	51	71	2.48505	412	1.51495	29	1.99980	15
6360	46	2.48896	408	51	71	2.48917	408	1.51083	29	1.99979	14
6420	47	2.49304	404	50	72	2.49325	404	1.50675	28	1.99979	13
6480	48	2.49708	400	50	72	2.49729	401	1.50271	28	1.99979	12
6540	49	2.50108	396	50	72	2.50130	397	1.49870	28	1.99978	11
6600	50	2.50504	393	50	72	2.50527	393	1.49473	28	1.99978	10
6660	51	2.50897	390	50	73	2.50920	390	1.49080	27	1.99977	9
6720	52	2.51287	386	50	73	2.51310	386	1.48690	27	1.99977	8
6780	53	2.51673	382	50	73	2.51696	383	1.48304	27	1.99977	7
6840	54	2.52055	379	50	73	2.52079	380	1.47921	27	1.99976	6
6900	55	2.52434	376	49	74	2.52459	376	1.47541	26	1.99976	5
6960	56	2.52810	373	49	74	2.52835	373	1.47165	26	1.99975	4
7020	57	2.53183	369	49	74	2.53208	370	1.46792	26	1.99975	3
7080	58	2.53552	367	49	75	2.53578	367	1.46422	25	1.99974	2
7140	59	2.53919	363	49	75	2.53945	363	1.46055	25	1.99974	1
7200	60	2.54282		49	75	2.54308		1.45692	25	1.99974	0
				6.685					5.314		
		log cos	d	S	T	log cot	c. d.	log tan	C	log sin	′

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″	′	log sin	d	S	T	log tan	c. d.	log cot	C	log cos	
				$\bar{6}.685$					5.314		
7200	0	$\bar{3}.54282$	360	49	75	$\bar{3}.54308$	361	1.45692	25	$\bar{1}.99974$	60
7260	1	3.54642	357	49	75	3.54669	358	1.45331	25	$\bar{1}.99973$	59
7320	2	3.54999	355	48	76	3.55027	355	1.44973	24	$\bar{1}.99973$	58
7380	3	$\bar{3}.55354$	351	48	76	3.55382	352	1.44618	24	$\bar{1}.99972$	57
7440	4	$\bar{3}.55705$	349	48	76	$\bar{3}.55734$	349	1.44266	24	$\bar{1}.99972$	56
7500	5	3.56054	346	48	77	$\bar{3}.56083$	346	1.43917	23	$\bar{1}.99971$	55
7560	6	3.56400	343	48	77	$\bar{3}.56429$	344	1.43571	23	$\bar{1}.99971$	54
7620	7	3.56743	341	48	77	$\bar{3}.56773$	341	1.43227	23	$\bar{1}.99970$	53
7680	8	3.57084	337	47	78	$\bar{3}.57114$	338	1.42886	22	$\bar{1}.99970$	52
7740	9	$\bar{3}.57421$	336	47	78	$\bar{3}.57452$	336	1.42548	22	$\bar{1}.99969$	51
7800	10	$\bar{3}.57757$	332	47	78	$\bar{3}.57788$	333	1.42212	22	$\bar{1}.99969$	50
7860	11	$\bar{3}.58089$	330	47	79	$\bar{3}.58121$	330	1.41879	21	$\bar{1}.99968$	49
7920	12	$\bar{3}.58419$	328	47	79	$\bar{3}.58451$	328	1.41549	21	$\bar{1}.99968$	48
7980	13	$\bar{3}.58747$	325	47	79	$\bar{3}.58779$	326	1.41221	21	$\bar{1}.99967$	47
8040	14	3.59072	323	46	79	3.59105	323	1.40895	21	$\bar{1}.99967$	46
8100	15	3.59395	320	46	80	3.59428	321	1.40572	20	$\bar{1}.99967$	45
8160	16	3.59715	318	46	80	3.59749	319	1.40251	20	$\bar{1}.99966$	44
8220	17	3.60033	316	46	80	3.60068	316	1.39932	20	$\bar{1}.99966$	43
8280	18	3.60349	313	46	81	3.60384	314	1.39616	19	$\bar{1}.99965$	42
8340	19	3.60662	311	46	81	3.60698	311	1.39302	19	$\bar{1}.99964$	41
8400	20	3.60973	309	45	82	3.61009	310	1.38991	18	$\bar{1}.99964$	40
8460	21	3.61282	307	45	82	3.61319	307	1.38681	18	$\bar{1}.99963$	39
8520	22	3.61589	305	45	82	3.61626	305	1.38374	18	$\bar{1}.99963$	38
8580	23	3.61894	302	45	83	3.61931	303	1.38069	17	$\bar{1}.99962$	37
8640	24	3.62196	301	45	83	3.62234	301	1.37766	17	$\bar{1}.99962$	36
8700	25	3.62497	298	45	83	3.62535	299	1.37465	17	$\bar{1}.99961$	35
8760	26	3.62795	296	44	84	3.62834	297	1.37166	16	$\bar{1}.99961$	34
8820	27	3.63091	294	44	84	3.63131	295	1.36869	16	$\bar{1}.99960$	33
8880	28	3.63385	293	44	84	3.63426	292	1.36574	16	$\bar{1}.99960$	32
8940	29	3.63678	290	44	85	3.63718	291	1.36282	15	$\bar{1}.99959$	31
9000	30	3.63968	288	44	85	3.64009	289	1.35991	15	$\bar{1}.99959$	30
9060	31	$\bar{3}.64256$	287	44	85	$\bar{3}.64298$	287	1.35702	15	$\bar{1}.99958$	29
9120	32	$\bar{3}.64543$	284	43	86	$\bar{3}.64585$	285	1.35415	14	$\bar{1}.99958$	28
9180	33	$\bar{3}.64827$	283	43	86	$\bar{3}.64870$	284	1.35130	14	$\bar{1}.99957$	27
9240	34	3.65110	281	43	87	3.65154	281	1.34846	13	$\bar{1}.99956$	26
9300	35	3.65391	279	43	87	3.65435	280	1.34565	13	$\bar{1}.99956$	25
9360	36	3.65670	277	43	87	3.65715	278	1.34285	13	$\bar{1}.99955$	24
9420	37	3.65947	276	42	88	3.65993	276	1.34007	12	$\bar{1}.99955$	23
9480	38	3.66223	274	42	88	3.66269	274	1.33731	12	$\bar{1}.99954$	22
9540	39	3.66497	272	42	88	3.66543	273	1.33457	12	$\bar{1}.99954$	21
9600	40	3.66769	270	42	89	3.66816	271	1.33184	11	$\bar{1}.99953$	20
9660	41	$\bar{3}.67039$	269	42	89	$\bar{3}.67087$	269	1.32913	11	$\bar{1}.99952$	19
9720	42	3.67308	267	41	90	3.67356	268	1.32644	10	$\bar{1}.99952$	18
9780	43	3.67575	266	41	90	3.67624	266	1.32376	10	$\bar{1}.99951$	17
9840	44	3.67841	263	41	90	3.67890	264	1.32110	10	$\bar{1}.99951$	16
9900	45	3.68104	263	41	91	3.68154	263	1.31846	09	$\bar{1}.99950$	15
9960	46	$\bar{3}.68367$	260	41	91	$\bar{3}.68417$	261	1.31583	09	$\bar{1}.99949$	14
10020	47	$\bar{3}.68627$	259	40	92	$\bar{3}.68678$	260	1.31322	08	$\bar{1}.99949$	13
10080	48	3.68886	258	40	92	3.68938	258	1.31062	08	$\bar{1}.99948$	12
10141	49	3.69144	256	40	92	3.69196	257	1.30804	08	$\bar{1}.99948$	11
10200	50	3.69400	254	40	93	3.69453	255	1.30547	07	$\bar{1}.99947$	10
10260	51	$\bar{3}.69654$	253	40	93	$\bar{3}.69708$	254	1.30292	07	$\bar{1}.99946$	9
10320	52	3.69907	252	39	94	3.69962	252	1.30038	06	$\bar{1}.99946$	8
10380	53	3.70159	250	39	94	3.70214	251	1.29786	06	$\bar{1}.99945$	7
10440	54	3.70409	249	39	95	3.70465	249	1.29535	05	$\bar{1}.99944$	6
10500	55	3.70658	247	39	95	3.70714	248	1.29286	05	$\bar{1}.99944$	5
10560	56	3.70905	246	39	95	3.70962	246	1.29038	05	$\bar{1}.99943$	4
10620	57	$\bar{3}.71151$	244	38	96	$\bar{3}.71208$	245	1.28792	04	$\bar{1}.99942$	3
10680	58	$\bar{3}.71395$	243	38	96	$\bar{3}.71453$	244	1.28547	04	$\bar{1}.99942$	2
10740	59	$\bar{3}.71638$	242	38	97	$\bar{3}.71697$	243	1.28303	03	$\bar{1}.99941$	1
10800	60	$\bar{3}.71880$		38	97	$\bar{3}.71940$		1.28060	03	$\bar{1}.99940$	0
				$\bar{6}.685$					5.314		
		log cos	d	S	T	log cot	c. d.	log tan	C	log sin	′

′	log sin	d	log tan	c. d.	log cot	log cos		p. p.			
									238	234	229
0	$\bar{2}.71880$	240	$\bar{2}.71940$	241	1.28060	$\bar{1}.99940$	60				
1	$\bar{2}.72120$	239	$\bar{2}.72181$	239	1.27819	$\bar{1}.99940$	59	6	23.8	23.4	22.9
2	$\bar{2}.72359$	238	$\bar{2}.72420$	239	1.27580	$\bar{1}.99939$	58	7	27.8	27.3	26.7
3	$\bar{2}.72597$	237	$\bar{2}.72659$	237	1.27341	$\bar{1}.99938$	57	8	31.7	31.2	30.5
4	$\bar{2}.72834$	235	$\bar{2}.72896$	236	1.27104	$\bar{1}.99938$	56	9	35.7	35.1	34.4
5	$\bar{2}.73069$	234	$\bar{2}.73132$	234	1.26868	$\bar{1}.99937$	55	10	39.7	39.0	38.2
6	$\bar{2}.73303$	232	$\bar{2}.73366$	234	1.26634	$\bar{1}.99936$	54	20	79.3	78.0	76.3
7	$\bar{2}.73535$	232	$\bar{2}.73600$	232	1.26400	$\bar{1}.99936$	53	30	119.0	117.0	114.5
8	$\bar{2}.73767$	230	$\bar{2}.73832$	231	1.26168	$\bar{1}.99935$	52	40	158.7	156.0	152.7
9	$\bar{2}.73997$	229	$\bar{2}.74063$	229	1.25937	$\bar{1}.99934$	51	50	198.3	195.0	190.8
10	$\bar{2}.74226$	228	$\bar{2}.74292$	229	1.25708	$\bar{1}.99934$	50				
									225	220	216
11	$\bar{2}.74454$	226	$\bar{2}.74521$	227	1.25479	$\bar{1}.99933$	49	6	22.5	22.0	21.6
12	$\bar{2}.74680$	226	$\bar{2}.74748$	226	1.25252	$\bar{1}.99932$	48	7	26.3	25.7	25.2
13	$\bar{2}.74906$	224	$\bar{2}.74974$	225	1.25026	$\bar{1}.99932$	47	8	30.0	29.3	28.8
14	$\bar{2}.75130$	223	$\bar{2}.75199$	224	1.24801	$\bar{1}.99931$	46	9	33.8	33.0	32.4
15	$\bar{2}.75353$	222	$\bar{2}.75423$	222	1.24577	$\bar{1}.99930$	45	10	37.5	36.7	36.0
16	$\bar{2}.75575$	220	$\bar{2}.75645$	222	1.24355	$\bar{1}.99929$	44	20	75.0	73.3	72.0
17	$\bar{2}.75795$	220	$\bar{2}.75867$	220	1.24133	$\bar{1}.99929$	43	30	112.5	110.0	108.0
18	$\bar{2}.76015$	219	$\bar{2}.76087$	219	1.23913	$\bar{1}.99928$	42	40	150.0	146.7	144.0
19	$\bar{2}.76234$	217	$\bar{2}.76306$	219	1.23694	$\bar{1}.99927$	41	50	187.5	183.3	180.0
20	$\bar{2}.76451$	216	$\bar{2}.76525$	217	1.23475	$\bar{1}.99926$	40				
									212	208	204
21	$\bar{2}.76667$	216	$\bar{2}.76742$	216	1.23258	$\bar{1}.99926$	39	6	21.2	20.8	20.4
22	$\bar{2}.76883$	214	$\bar{2}.76958$	215	1.23042	$\bar{1}.99925$	38	7	24.7	24.3	23.8
23	$\bar{2}.77097$	213	$\bar{2}.77173$	214	1.22827	$\bar{1}.99924$	37	8	28.3	27.7	27.2
24	$\bar{2}.77310$	212	$\bar{2}.77387$	213	1.22613	$\bar{1}.99923$	36	9	31 8	31.2	30.6
25	$\bar{2}.77522$	211	$\bar{2}.77600$	211	1.22400	$\bar{1}.99923$	35	10	35.3	34.7	34.0
26	$\bar{2}.77733$	210	$\bar{2}.77811$	211	1.22189	$\bar{1}.99922$	34	20	70.7	69.3	68.0
27	$\bar{2}.77943$	209	$\bar{2}.78022$	210	1.21978	$\bar{1}.99921$	33	30	106.0	104.0	102.0
28	$\bar{2}.78152$	208	$\bar{2}.78232$	209	1.21768	$\bar{1}.99920$	32	40	141.3	138.7	136.0
29	$\bar{2}.78360$	208	$\bar{2}.78441$	208	1.21559	$\bar{1}.99920$	31	50	176.7	173.3	170.0
30	$\bar{2}.78568$	206	$\bar{2}.78649$	206	1.21351	$\bar{1}.99919$	30				
									201	197	193
31	$\bar{2}.78774$	205	$\bar{2}.78855$	206	1.21145	$\bar{1}.99918$	29	6	20.1	19.7	19.3
32	$\bar{2}.78979$	204	$\bar{2}.79061$	205	1.20939	$\bar{1}.99917$	28	7	23.5	23.0	22.5
33	$\bar{2}.79183$	203	$\bar{2}.79266$	204	1.20734	$\bar{1}.99917$	27	8	26.8	26.3	25.7
34	$\bar{2}.79386$	202	$\bar{2}.79470$	203	1.20530	$\bar{1}.99916$	26	9	30.2	29.6	29.0
35	$\bar{2}.79588$	201	$\bar{2}.79673$	202	1.20327	$\bar{1}.99915$	25	10	33.5	32.8	32.2
36	$\bar{2}.79789$	201	$\bar{2}.79875$	201	1.20125	$\bar{1}.99914$	24	20	67.0	65.7	64.3
37	$\bar{2}.79990$	199	$\bar{2}.80076$	201	1.19924	$\bar{1}.99913$	23	30	100.5	98.5	96.5
38	$\bar{2}.80189$	199	$\bar{2}.80277$	199	1.19723	$\bar{1}.99913$	22	40	134.0	131.3	128.7
39	$\bar{2}.80388$	197	$\bar{2}.80476$	199	1.19524	$\bar{1}.99912$	21	50	167.5	164.2	160.8
40	$\bar{2}.80585$	197	$\bar{2}.80674$	198	1.19326	$\bar{1}.99911$	20				
									189	185	181
41	$\bar{2}.80782$	196	$\bar{2}.80872$	196	1.19128	$\bar{1}.99910$	19	6	18.9	18.5	18.1
42	$\bar{2}.80978$	195	$\bar{2}.81068$	196	1.18932	$\bar{1}.99909$	18	7	22.1	21.6	21.1
43	$\bar{2}.81173$	194	$\bar{2}.81264$	195	1.18736	$\bar{1}.99909$	17	8	25.2	24.7	24.1
44	$\bar{2}.81367$	193	$\bar{2}.81459$	194	1.18541	$\bar{1}.99908$	16	9	28.4	27.8	27.2
45	$\bar{2}.81560$	192	$\bar{2}.81653$	193	1.18347	$\bar{1}.99907$	15	10	31.5	30.8	30.2
46	$\bar{2}.81752$	192	$\bar{2}.81846$	192	1.18154	$\bar{1}.99906$	14	20	63.0	61.7	60.3
47	$\bar{2}.81944$	190	$\bar{2}.82038$	192	1.17962	$\bar{1}.99905$	13	30	94.5	92.5	90.5
48	$\bar{2}.82134$	190	$\bar{2}.82230$	190	1.17770	$\bar{1}.99904$	12	40	126.0	123.3	120.7
49	$\bar{2}.82324$	189	$\bar{2}.82420$	190	1.17580	$\bar{1}.99904$	11	50	157.5	154.2	150.8
50	$\bar{2}.82513$	188	$\bar{2}.82610$	190	1.17390	$\bar{1}.99903$	10				
									4	3	2	1
51	$\bar{2}.82701$	187	$\bar{2}.82799$	189	1.17201	$\bar{1}.99902$	9					
52	$\bar{2}.82888$	187	$\bar{2}.82987$	188	1.17013	$\bar{1}.99901$	8	6	0.4	0.3	0.2	0.1
53	$\bar{2}.83075$	186	$\bar{2}.83175$	188	1.16825	$\bar{1}.99900$	7	7	0.5	0.4	0.2	0.1
54	$\bar{2}.83261$	185	$\bar{2}.83361$	186	1.16639	$\bar{1}.99899$	6	8	0.5	0 4	0.3	0.1
55	$\bar{2}.83446$	184	$\bar{2}.83547$	185	1.16453	$\bar{1}.99898$	5	9	0.6	0.5	0.3	0.2
56	$\bar{2}.83630$	183	$\bar{2}.83732$	184	1.16268	$\bar{1}.99898$	4	10	0.7	0.5	0.3	0.2
57	$\bar{2}.83813$	183	$\bar{2}.83916$	184	1.16084	$\bar{1}.99897$	3	20	1.3	1.0	0.7	0.3
58	$\bar{2}.83996$	181	$\bar{2}.84100$	182	1.15900	$\bar{1}.99896$	2	30	2 0	1.5	1.0	0.5
59	$\bar{2}.84177$	181	$\bar{2}.84282$	182	1.15718	$\bar{1}.99895$	1	40	2.7	2.0	1.3	0.7
60	$\bar{2}.84358$		$\bar{2}.84464$		1.15536	$\bar{1}.99894$	0	50	3.3	2.5	1.7	0.8
	log cos	d	log cot	c. d.	log tan	log sin	′	p. p.			

′	log sin	d	log tan	c. d.	log cot	log cos		p. p.		
									181	179	177
0	2.84358	181	2.84464	182	1.15536	1.99894	60	6	18.1	17.9	17.7
1	2.84539	179	2.84646	180	1.15354	1.99893	59	7	21.1	20.9	20.7
2	2.84718	179	2.84826	180	1.15174	1.99892	58	8	24.1	23.9	23.6
3	2.84897	178	2.85006	179	1.14994	1.99891	57	9	27.2	26.9	26.6
4	2.85075	177	2.85185	178	1.14815	1.99891	56	10	30.2	29.8	29.5
5	2.85252	177	2.85363	177	1.14637	1.99890	55	20	60.3	59.7	59.0
6	2.85429	176	2.85540	177	1.14460	1.99889	54	30	90.5	89.5	88.5
7	2.85605	175	2.85717	176	1.14283	1.99888	53	40	120.7	119.3	118.0
8	2.85780	175	2.85893	176	1.14107	1.99887	52	50	150.8	149.2	147.5
9	2.85955	173	2.86069	174	1.13931	1.99886	51				
10	2.86128		2.86243		1.13757	1.99885	50				
		173		174					175	173	171
11	2.86301		2.86417		1.13583	1.99884	49	6	17.5	17.3	17.1
12	2.86474	173	2.86591	174	1.13409	1.99883	48	7	20.4	20.2	20.0
13	2.86645	171	2.86763	172	1.13237	1.99882	47	8	23.3	23.1	22.8
14	2.86816	171	2.86935	172	1.13065	1.99881	46	9	26.3	26.0	25.7
15	2.86987	171	2.87106	171	1.12894	1.99880	45	10	29.2	28.8	28.5
16	2.87156	169	2.87277	171	1.12723	1.99879	44	20	58.3	57.7	57.0
17	2.87325	169	2.87447	170	1.12553	1.99879	43	30	87.5	86.5	85.5
18	2.87494	169	2.87616	169	1.12384	1.99878	42	40	116.7	115.3	114.0
19	2.87661	167	2.87785	169	1.12215	1.99877	41	50	145.8	144.2	142.5
20	2.87829	168	2.87953	168	1.12047	1.99876	40				
		166		167					168	166	164
21	2.87995		2.88120		1.11880	1.99875	39	6	16.8	16.6	16.4
22	2.88161	166	2.88287	167	1.11713	1.99874	38	7	19.6	19.4	19.1
23	2.88326	165	2.88453	166	1.11547	1.99873	37	8	22.4	22.1	21.9
24	2.88490	164	2.88618	165	1.11382	1.99872	36	9	25.2	24.9	24.6
25	2.88654	164	2.88783	165	1.11217	1.99871	35	10	28.0	27.7	27.3
26	2.88817	163	2.88948	165	1.11052	1.99870	34	20	56.0	55.3	54.7
27	2.88980	163	2.89111	163	1.10889	1.99869	33	30	84.0	83.0	82.0
28	2.89142	162	2.89274	163	1.10726	1.99868	32	40	112.0	110.7	109.3
29	2.89304	162	2.89437	163	1.10563	1.99867	31	50	140.0	138.3	136.7
30	2.89464	160	2.89598	161	1.10402	1.99866	30				
		161		162					162	159	157
31	2.89625		2.89760		1.10240	1.99865	29	6	16.2	15.9	15.7
32	2.89784	159	2.89920	160	1.10080	1.99864	28	7	18.9	18.6	18.3
33	2.89943	159	2.90080	160	1.09920	1.99863	27	8	21.6	21.2	20.9
34	2.90102	159	2.90240	160	1.09760	1.99862	26	9	24.3	23.9	23.6
35	2.90260	158	2.90399	159	1.09601	1.99861	25	10	27.0	26.5	26.2
36	2.90417	157	2.90557	158	1.09443	1.99860	24	20	54.0	53.0	52.3
37	2.90574	157	2.90715	158	1.09285	1.99859	23	30	81.0	79.5	78.5
38	2.90730	156	2.90872	157	1.09128	1.99858	22	40	108.0	106.0	104.7
39	2.90885	155	2.91029	157	1.08971	1.99857	21	50	135.0	132.5	130.8
40	2.91040	155	2.91185	156	1.08815	1.99856	20				
		155		155					155	153	151
41	2.91195		2.91340		1.08660	1.99855	19	6	15.5	15.3	15.1
42	2.91349	154	2.91495	155	1.08505	1.99854	18	7	18.1	17.9	17.6
43	2.91502	153	2.91650	155	1.08350	1.99853	17	8	20.7	20.4	20.1
44	2.91655	153	2.91803	153	1.08197	1.99852	16	9	23.3	23.0	22.7
45	2.91807	152	2.91957	154	1.08043	1.99851	15	10	25.8	25.5	25.2
46	2.91959	152	2.92110	153	1.07890	1.99850	14	20	51.7	51.0	50.3
47	2.92110	151	2.92262	152	1.07738	1.99848	13	30	77.5	76.5	75.5
48	2.92261	151	2.92414	152	1.07586	1.99847	12	40	103.3	102.0	100.7
49	2.92411	150	2.92565	151	1.07435	1.99846	11	50	129.2	127.5	125.8
50	2.92561	150	2.92716	151	1.07284	1.99845	10				
		149		150					149	147	1
51	2.92710		2.92866		1.07134	1.99844	9	6	14.9	14.7	0.1
52	2.92859	149	2.93016	150	1.06984	1.99843	8	7	17.4	17.2	0.1
53	2.93007	148	2.93165	149	1.06835	1.99842	7	8	19.9	19.6	0.1
54	2.93154	147	2.93313	148	1.06687	1.99841	6	9	22.4	22.1	0.2
55	2.93301	147	2.93462	149	1.06538	1.99840	5	10	24.8	24.5	0.2
56	2.93448	147	2.93609	147	1.06391	1.99839	4	20	49.7	49.0	0.3
57	2.93594	146	2.93756	147	1.06244	1.99838	3	30	74.5	73.5	0.5
58	2.93740	146	2.93903	147	1.06097	1.99837	2	40	99.3	98.0	0.7
59	2.93885	145	2.94049	146	1.05951	1.99836	1	50	124.2	122.5	0.8
60	2.94030	145	2.94195	146	1.05805	1.99834	0				
	log cos	d	log cot	c. d.	log tan	log sin	′	p. p.			

′	log sin	d	log tan	c. d.	log cot	log cos		p. p.			
0	$\bar{2}.94030$		$\bar{2}.94195$		1.05805	$\bar{1}.99834$	60		145	143	141
1	$\bar{2}.94174$	144	$\bar{2}.94340$	145	1.05660	$\bar{1}.99833$	59	6	14.5	14.3	14.1
2	$\bar{2}.94317$	143	$\bar{2}.94485$	145	1.05515	$\bar{1}.99832$	58	7	16.9	16.7	16.5
3	$\bar{2}.94461$	144	$\bar{2}.94630$	145	1.05370	$\bar{1}.99831$	57	8	19.3	19.1	18.8
4	$\bar{2}.94603$	142	$\bar{2}.94773$	143	1.05227	$\bar{1}.99830$	56	9	21.8	21.5	21.2
5	$\bar{2}.94746$	143	$\bar{2}.94917$	144	1.05083	$\bar{1}.99829$	55	10	24.2	23.8	23.5
6	$\bar{2}.94887$	141	$\bar{2}.95060$	143	1.04940	$\bar{1}.99828$	54	20	48.3	47.7	47.0
7	$\bar{2}.95029$	142	$\bar{2}.95202$	142	1.04798	$\bar{1}.99827$	53	30	72.5	71.5	70.5
8	$\bar{2}.95170$	141	$\bar{2}.95344$	142	1.04656	$\bar{1}.99825$	52	40	96.7	95.3	94.0
9	$\bar{2}.95310$	140	$\bar{2}.95486$	142	1.04514	$\bar{1}.99824$	51	50	120.8	119.2	117.5
10	$\bar{2}.95450$	140	$\bar{2}.95627$	141	1.04373	$\bar{1}.99823$	50				
		139		140					139	138	136
11	$\bar{2}.95589$		$\bar{2}.95767$		1.04233	$\bar{1}.99822$	49	6	13.9	13.8	13.6
12	$\bar{2}.95728$	139	$\bar{2}.95908$	141	1.04092	$\bar{1}.99821$	48	7	16.2	16.1	15.9
13	$\bar{2}.95867$	139	$\bar{2}.96047$	139	1.03953	$\bar{1}.99820$	47	8	18.5	18.4	18.1
14	$\bar{2}.96005$	138	$\bar{2}.96187$	140	1.03813	$\bar{1}.99819$	46	9	20.9	20.7	20.4
15	$\bar{2}.96143$	138	$\bar{2}.96325$	138	1.03675	$\bar{1}.99817$	45	10	23.2	23.0	22.7
16	$\bar{2}.96280$	137	$\bar{2}.96464$	139	1.03536	$\bar{1}.99816$	44	20	46.3	46.0	45.3
17	$\bar{2}.96417$	137	$\bar{2}.96602$	138	1.03398	$\bar{1}.99815$	43	30	69.5	69.0	68.0
18	$\bar{2}.96553$	136	$\bar{2}.96739$	137	1.03261	$\bar{1}.99814$	42	40	92.7	92.0	90.7
19	$\bar{2}.96689$	136	$\bar{2}.96877$	138	1.03123	$\bar{1}.99813$	41	50	115.8	115.0	113.3
20	$\bar{2}.96825$	136	$\bar{2}.97013$	136	1.02987	$\bar{1}.99812$	40				
		135		137					135	133	131
21	$\bar{2}.96960$		$\bar{2}.97150$		1.02850	$\bar{1}.99810$	39	6	13.5	13.3	13.1
22	$\bar{2}.97095$	135	$\bar{2}.97285$	135	1.02715	$\bar{1}.99809$	38	7	15.8	15.5	15.3
23	$\bar{2}.97229$	134	$\bar{2}.97421$	136	1.02579	$\bar{1}.99808$	37	8	18.0	17.7	17.5
24	$\bar{2}.97363$	134	$\bar{2}.97556$	135	1.02444	$\bar{1}.99807$	36	9	20.3	20.0	19.7
25	$\bar{2}.97496$	133	$\bar{2}.97691$	135	1.02309	$\bar{1}.99806$	35	10	22.5	22.2	21.8
26	$\bar{2}.97629$	133	$\bar{2}.97825$	134	1.02175	$\bar{1}.99804$	34	20	45.0	44.3	43.7
27	$\bar{2}.97762$	133	$\bar{2}.97959$	134	1.02041	$\bar{1}.99803$	33	30	67.5	66.5	65.5
28	$\bar{2}.97894$	132	$\bar{2}.98092$	133	1.01908	$\bar{1}.99802$	32	40	90.0	88.7	87.3
29	$\bar{2}.98026$	132	$\bar{2}.98225$	133	1.01775	$\bar{1}.99801$	31	50	112.5	110.8	109.2
30	$\bar{2}.98157$	131	$\bar{2}.98358$	133	1.01642	$\bar{1}.99800$	30				
		131		132					129	128	126
31	$\bar{2}.98288$		$\bar{2}.98490$		1.01510	$\bar{1}.99798$	29	6	12.9	12.8	12.6
32	$\bar{2}.98419$	131	$\bar{2}.98622$	132	1.01378	$\bar{1}.99797$	28	7	15.1	14.9	14.7
33	$\bar{2}.98549$	130	$\bar{2}.98753$	131	1.01247	$\bar{1}.99796$	27	8	17.2	17.1	16.8
34	$\bar{2}.98679$	130	$\bar{2}.98884$	131	1.01116	$\bar{1}.99795$	26	9	19.4	19.2	18.9
35	$\bar{2}.98808$	129	$\bar{2}.99015$	131	1.00985	$\bar{1}.99793$	25	10	21.5	21.3	21.0
36	$\bar{2}.98937$	129	$\bar{2}.99145$	130	1.00855	$\bar{1}.99792$	24	20	43.0	42.7	42.0
37	$\bar{2}.99066$	129	$\bar{2}.99275$	130	1.00725	$\bar{1}.99791$	23	30	64.5	64.0	63.0
38	$\bar{2}.99194$	128	$\bar{2}.99405$	130	1.00595	$\bar{1}.99790$	22	40	86.0	85.3	84.0
39	$\bar{2}.99322$	128	$\bar{2}.99534$	129	1.00466	$\bar{1}.99788$	21	50	107.5	106.7	105.0
40	$\bar{2}.99450$	128	$\bar{2}.99662$	128	1.00338	$\bar{1}.99787$	20				
		127		129					125	123	122
41	$\bar{2}.99577$		$\bar{2}.99791$		1.00209	$\bar{1}.99786$	19	6	12.5	12.3	12.2
42	$\bar{2}.99704$	127	$\bar{2}.99919$	128	1.00081	$\bar{1}.99785$	18	7	14.6	14.4	14.2
43	$\bar{2}.99830$	126	$\bar{1}.00046$	127	0.99954	$\bar{1}.99783$	17	8	16.7	16.4	16.3
44	$\bar{2}.99956$	126	$\bar{1}.00174$	128	0.99826	$\bar{1}.99782$	16	9	18.8	18.5	18.3
45	$\bar{1}.00082$	126	$\bar{1}.00301$	127	0.99699	$\bar{1}.99781$	15	10	20.8	20.5	20.3
46	$\bar{1}.00207$	125	$\bar{1}.00427$	126	0.99573	$\bar{1}.99780$	14	20	41.7	41.0	40.7
47	$\bar{1}.00332$	125	$\bar{1}.00553$	126	0.99447	$\bar{1}.99778$	13	30	62.5	61.5	61.0
48	$\bar{1}.00456$	124	$\bar{1}.00679$	126	0.99321	$\bar{1}.99777$	12	40	83.3	82.0	81.3
49	$\bar{1}.00581$	125	$\bar{1}.00805$	126	0.99195	$\bar{1}.99776$	11	50	104.2	102.5	101.7
50	$\bar{1}.00704$	123	$\bar{1}.00930$	125	0.99070	$\bar{1}.99775$	10				
		124		125					121	120	1
51	$\bar{1}.00828$		$\bar{1}.01055$		0.98945	$\bar{1}.99773$	9	6	12.1	12.0	0.1
52	$\bar{1}.00951$	123	$\bar{1}.01179$	124	0.98821	$\bar{1}.99772$	8	7	14.1	14.0	0.1
53	$\bar{1}.01074$	123	$\bar{1}.01303$	124	0.98697	$\bar{1}.99771$	7	8	16.1	16.0	0.1
54	$\bar{1}.01196$	122	$\bar{1}.01427$	124	0.98573	$\bar{1}.99769$	6	9	18.2	18.0	0.2
55	$\bar{1}.01318$	122	$\bar{1}.01550$	123	0.98450	$\bar{1}.99768$	5	10	20.2	20.0	0.2
56	$\bar{1}.01440$	122	$\bar{1}.01673$	123	0.98327	$\bar{1}.99767$	4	20	40.3	40.0	0.3
57	$\bar{1}.01561$	121	$\bar{1}.01796$	123	0.98204	$\bar{1}.99765$	3	30	60.5	60.0	0.5
58	$\bar{1}.01682$	121	$\bar{1}.01918$	122	0.98082	$\bar{1}.99764$	2	40	80.7	80.0	0.7
59	$\bar{1}.01803$	121	$\bar{1}.02040$	122	0.97960	$\bar{1}.99763$	1	50	100.8	100.0	0.8
60	$\bar{1}.01923$	120	$\bar{1}.02162$	122	0.97838	$\bar{1}.99761$	0				
	log cos	d	log cot	c. d.	log tan	log sin	′	p. p.			

′	log sin	d	log tan	c. d.	log cot	log cos		p. p.		
									121	120	119
0	T.01923	120	T.02162	121	0.97838	T.99761	60				
1	T.02043	120	T.02283	121	0.97717	T.99760	59	6	12.1	12.0	11.9
2	T.02163	120	T.02404	121	0.97596	T.99759	58	7	14.1	14.0	13.9
3	T.02283	119	T.02525	120	0.97475	T.99757	57	8	16.1	16.0	15.9
4	T.02402	118	T.02645	121	0.97355	T.99756	56	9	18.2	18.0	17.9
5	T.02520	119	T.02766	119	0.97234	T.99755	55	10	20.2	20.0	19.8
6	T.02639	118	T.02885	120	0.97115	T.99753	54	20	40.3	40.0	39.7
7	T.02757	117	T.03005	119	0.96995	T.99752	53	30	60.5	60.0	59.5
8	T.02874	118	T.03124	118	0.96876	T.99751	52	40	80.7	80.0	79.3
9	T.02992	117	T.03242	119	0.96758	T.99749	51	50	100.8	100.0	99.2
10	T.03109	117	T.03361	118	0.96639	T.99748	50				
									118	117	116
11	T.03226	116	T.03479	118	0.96521	T.99747	49	6	11.8	11.7	11.6
12	T.03342	116	T.03597	117	0.96403	T.99745	48	7	13.8	13.7	13.5
13	T.03458	116	T.03714	118	0.96286	T.99744	47	8	15.7	15.6	15.5
14	T.03574	116	T.03832	116	0.96168	T.99742	46	9	17.7	17.6	17.4
15	T.03690	115	T.03948	117	0.96052	T.99741	45	10	19.7	19.5	19.3
16	T.03805	115	T.04065	116	0.95935	T.99740	44	20	39.3	39.0	38.7
17	T.03920	114	T.04181	116	0.95819	T.99738	43	30	59.0	58.5	58.0
18	T.04034	115	T.04297	116	0.95703	T.99737	42	40	78.7	78.0	77.3
19	T.04149	113	T.04413	115	0.95587	T.99736	41	50	98.3	97.5	96.7
20	T.04262	114	T.04528	115	0.95472	T.99734	40				
									115	114	113
21	T.04376	114	T.04643	115	0.95357	T.99733	39	6	11.5	11.4	11.3
22	T.04490	113	T.04758	115	0.95242	T.99731	38	7	13.4	13.3	13.2
23	T.04603	112	T.04873	114	0.95127	T.99730	37	8	15.3	15.2	15.1
24	T.04715	113	T.04987	114	0.95013	T.99728	36	9	17.3	17.1	17.0
25	T.04828	112	T.05101	113	0.94899	T.99727	35	10	19.2	19.0	18.8
26	T.04940	112	T.05214	114	0.94786	T.99726	34	20	38.3	38.0	37.7
27	T.05052	112	T.05328	113	0.94672	T.99724	33	30	57.5	57.0	56.5
28	T.05164	111	T.05441	112	0.94559	T.99723	32	40	76.7	76.0	75.3
29	T.05275	111	T.05553	113	0.94447	T.99721	31	50	95 8	95.0	94.2
30	T.05386	111	T.05666	112	0.94334	T.99720	30				
									112	111	110
31	T.05497	110	T.05778	112	0.94222	T.99718	29	6	11.2	11.1	11.0
32	T.05607	110	T.05890	112	0.94110	T.99717	28	7	13.1	13.0	12.8
33	T.05717	110	T.06002	111	0.93998	T.99716	27	8	14.9	14.8	14.7
34	T.05827	110	T.06113	111	0.93887	T.99714	26	9	16.8	16.7	16.5
35	T.05937	109	T.06224	111	0.93776	T.99713	25	10	18.7	18.5	18.3
36	T.06046	109	T.06335	111	0.93665	T.99711	24	20	37.3	37.0	36.7
37	T.06155	109	T.06445	110	0.93555	T.99710	23	30	56.0	55.5	55.0
38	T.06264	108	T.06556	111	0.93444	T.99708	22	40	74.7	74.0	73.3
39	T.06372	109	T.06666	110	0.93334	T.99707	21	50	93.3	92.5	91.7
40	T.06481	108	T.06775	109	0.93225	T.99705	20				
									109	108	107
41	T.06589	107	T.06885	110	0.93115	T.99704	19	6	10.9	10.8	10.7
42	T.06696	108	T.06994	109	0.93006	T.99702	18	7	12.7	12.6	12.5
43	T.06804	107	T.07103	109	0.92897	T.99701	17	8	14.5	14 4	14.3
44	T.06911	107	T.07211	108	0.92789	T.99699	16	9	16.4	16.2	16.1
45	T.07018	106	T.07320	109	0.92680	T.99698	15	10	18.2	18.0	17.8
46	T.07124	107	T.07428	108	0.92572	T.99696	14	20	36.3	36.0	35.7
47	T.07231	106	T.07536	108	0.92464	T.99695	13	30	54.5	54.0	53.5
48	T.07337	106	T.07643	107	0.92357	T.99693	12	40	72.7	72.0	71 3
49	T.07442	105	T.07751	108	0.92249	T.99692	11	50	90.8	90.0	89.2
50	T.07548	106	T.07858	107	0.92142	T.99690	10				
		105		106							
51	T.07653	105	T.07964	107	0.92036	T.99689	9		106	105	104
52	T.07758	105	T.08071	106	0.91929	T.99687	8	6	10.6	10.5	10.4
53	T.07863	105	T.08177	106	0.91823	T.99686	7	7	12.4	12.3	12.1
54	T.07968	104	T.08283	106	0.91717	T.99684	6	8	14.1	14.0	13.9
55	T.08072	104	T.08389	106	0.91611	T.99683	5	9	15.9	15.8	15.6
56	T.08176	104	T.08495	105	0.91505	T.99681	4	10	17.7	17.5	17.3
57	T.08280	103	T.08600	105	0.91400	T.99680	3	20	35.3	35.0	34.7
58	T.08383	103	T.08705	105	0.91295	T.99678	2	30	53.0	52.5	52.0
59	T.08486	103	T.08810	104	0.91190	T.99677	1	40	70.7	70.0	69.3
60	T.08589		T.08914		0.91086	T.99675	0	50	88.3	87.5	86.7
	log cos	d	log cot	c. d.	log tan	log sin	′		p. p.		

′	log sin	d	log tan	c. d.	log cot	log cos		p. p.
0	T.08589		T.08914		0.91086	T.99675	60	105 104 103
1	T.08692	103	T.09019	105	0.90981	T.99674	59	6 10.5 10.4 10.3
2	T.08795	103	T.09123	104	0.90877	T.99672	58	7 12.3 12.1 12.0
3	T.08897	102	T.09227	104	0.90773	T.99670	57	8 14.0 13.9 13.7
4	T.08999	102	T.09330	103	0.90670	T.99669	56	9 15.8 15.6 15.5
5	T.09101	102	T.09434	104	0.90566	T.99667	55	10 17.5 17.3 17.2
6	T.09202	101	T.09537	103	0.90463	T.99666	54	20 35.0 34.7 34.3
7	T.09304	102	T.09640	103	0.90360	T.99664	53	30 52.5 52.0 51.5
8	T.09405	101	T.09742	102	0.90258	T.99663	52	40 70.0 69.3 68.7
9	T.09506	101	T.09845	103	0.90155	T.99661	51	50 87.5 86.7 85.8
10	T.09606	100	T.09947	102	0.90053	T.99659	50	
		101		102				102 101 100
11	T.09707		T.10049		0.89951	T.99658	49	6 10.2 10.1 10.0
12	T.09807	100	T.10150	101	0.89850	T.99656	48	7 11.9 11.8 11.7
13	T.09907	100	T.10252	102	0.89748	T.99655	47	8 13.6 13.5 13.3
14	T.10006	99	T.10353	101	0.89647	T.99653	46	9 15.3 15.2 15.0
15	T.10106	100	T.10454	101	0.89546	T.99651	45	10 17.0 16.8 16.7
16	T.10205	99	T.10555	101	0.89445	T.99650	44	20 34.0 33.7 33.3
17	T.10304	99	T.10656	101	0.89344	T.99648	43	30 51.0 50.5 50.0
18	T.10402	98	T.10756	100	0.89244	T.99647	42	40 68.0 67.3 66.7
19	T.10501	99	T.10856	100	0.89144	T.99645	41	50 85.0 84.2 83.3
20	T.10599	98	T.10956	100	0.89044	T.99643	40	
		98		100				99 98
21	T.10697		T.11056		0.88944	T.99642	39	6 9.9 9.8
22	T.10795	98	T.11155	99	0.88845	T.99640	38	7 11.6 11.4
23	T.10893	98	T.11254	99	0.88746	T.99638	37	8 13.2 13.1
24	T.10990	97	T.11353	99	0.88647	T.99637	36	9 14.9 14.7
25	T.11087	97	T.11452	99	0.88548	T.99635	35	10 16.5 16.3
26	T.11184	97	T.11551	99	0.88449	T.99633	34	20 33.0 32.7
27	T.11281	97	T.11649	98	0.88351	T.99632	33	30 49.5 49.0
28	T.11377	96	T.11747	98	0.88253	T.99630	32	40 66.0 65.3
29	T.11474	97	T.11845	98	0.88155	T.99629	31	50 82.5 81.7
30	T.11570	96	T.11943	98	0.88057	T.99627	30	
		96		97				97 96 95
31	T.11666		T.12040		0.87960	T.99625	29	6 9.7 9.6 9.5
32	T.11761	95	T.12138	98	0.87862	T.99624	28	7 11.3 11.2 11.1
33	T.11857	96	T.12235	97	0.87765	T.99622	27	8 12.9 12.8 12.7
34	T.11952	95	T.12332	97	0.87668	T.99620	26	9 14.6 14.4 14.3
35	T.12047	95	T.12428	96	0.87572	T.99618	25	10 16.2 16.0 15.8
36	T.12142	95	T.12525	97	0.87475	T.99617	24	20 32.3 32.0 31.7
37	T.12236	94	T.12621	96	0.87379	T.99615	23	30 48.5 48.0 47.5
38	T.12331	95	T.12717	96	0.87283	T.99613	22	40 64.7 64.0 63.3
39	T.12425	94	T.12813	96	0.87187	T.99612	21	50 80.8 80.0 79.2
40	T.12519	94	T.12909	96	0.87091	T.99610	20	
		93		95				94 93 92
41	T.12612		T.13004		0.86996	T.99608	19	6 9.4 9.3 9.2
42	T.12706	94	T.13099	95	0.86901	T.99607	18	7 11.0 10.9 10.7
43	T.12799	93	T.13194	95	0.86806	T.99605	17	8 12.5 12.4 12.3
44	T.12892	93	T.13289	95	0.86711	T.99603	16	9 14.1 14.0 13.8
45	T.12985	93	T.13384	95	0.86616	T.99601	15	10 15.7 15.5 15.3
46	T.13078	93	T.13478	94	0.86522	T.99600	14	20 31.3 31.0 30.7
47	T.13171	93	T.13573	95	0.86427	T.99598	13	30 47.0 46.5 46.0
48	T.13263	92	T.13667	94	0.86333	T.99596	12	40 62.7 62.0 61.3
49	T.13355	92	T.13761	94	0.86239	T.99595	11	50 78.3 77.5 76.7
50	T.13447	92	T.13854	93	0.86146	T.99593	10	
		92		94				91 90 2
51	T.13539		T.13948		0.86052	T.99591	9	6 9.1 9.0 0.2
52	T.13630	91	T.14041	93	0.85959	T.99589	8	7 10.6 10.5 0.2
53	T.13722	92	T.14134	93	0.85866	T.99588	7	8 12.1 12.0 0.3
54	T.13813	91	T.14227	93	0.85773	T.99586	6	9 13.7 13.5 0.3
55	T.13904	91	T.14320	93	0.85680	T.99584	5	10 15.2 15.0 0.3
56	T.13994	90	T.14412	92	0.85588	T.99582	4	20 30.3 30.0 0.7
57	T.14085	91	T.14504	92	0.85496	T.99581	3	30 45.5 45.0 1.0
58	T.14175	90	T.14597	93	0.85403	T.99579	2	40 60.7 60.0 1.3
59	T.14266	91	T.14688	91	0.85312	T.99577	1	50 75.8 75.0 1.7
60	T.14356	90	T.14780	92	0.85220	T.99575	0	
	log cos	d	log cot	c. d.	log tan	log sin	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos		p. p.
0	T.14356		T.14780		0.85220	T.99575	60	
1	T.14445	89	T.14872	92	0.85128	T.99574	59	92 91 90
2	T.14535	90	T.14963	91	0.85037	T.99572	58	6 9.2 9.1 9.0
3	T.14624	89	T.15054	91	0.84946	T.99570	57	7 10.7 10.6 10.5
4	T.14714	90	T.15145	91	0.84855	T.99568	56	8 12.3 12.1 12.0
5	T.14803	89	T.15236	91	0.84764	T.99566	55	9 13.8 13.7 13.5
6	T.14891	88	T.15327	90	0.84673	T.99565	54	10 15.3 15.2 15.0
7	T.14980	89	T.15417	91	0.84583	T.99563	53	20 30.7 30.3 30.0
8	T.15069	89	T.15508	90	0.84492	T.99561	52	30 46.0 45.5 45.0
9	T.15157	88	T.15598	90	0.84402	T.99559	51	40 61.3 60.7 60.0
10	T.15245	88	T.15688	90	0.84312	T.99557	50	50 76.7 75.8 75.0
11	T.15333	88	T.15777	89	0.84223	T.99556	49	89 88
12	T.15421	87	T.15867	90	0.84133	T.99554	48	6 8.9 8.8
13	T.15508	88	T.15956	89	0.84044	T.99552	47	7 10.4 10.3
14	T.15596	87	T.16046	90	0.83954	T.99550	46	8 11.9 11.7
15	T.15683	87	T.16135	89	0.83865	T.99548	45	9 13.4 13.2
16	T.15770	87	T.16224	89	0.83776	T.99546	44	10 14.8 14.7
17	T.15857	87	T.16312	88	0.83688	T.99545	43	20 29.7 29.3
18	T.15944	86	T.16401	89	0.83599	T.99543	42	30 44.5 44.0
19	T.16030	86	T.16489	88	0.83511	T.99541	41	40 59.3 58.7
20	T.16116	87	T.16577	88	0.83423	T.99539	40	50 74.2 73.3
21	T.16203	86	T.16665	88	0.83335	T.99537	39	87 86
22	T.16289	85	T.16753	88	0.83247	T.99535	38	6 8.7 8.6
23	T.16374	86	T.16841	87	0.83159	T.99533	37	7 10.2 10.0
24	T.16460	85	T.16928	88	0.83072	T.99532	36	8 11.6 11.5
25	T.16545	86	T.17016	87	0.82984	T.99530	35	9 13.1 12.9
26	T.16631	85	T.17103	87	0.82897	T.99528	34	10 14.5 14.3
27	T.16716	85	T.17190	87	0.82810	T.99526	33	20 29.0 28.7
28	T.16801	85	T.17277	87	0.82723	T.99524	32	30 43.5 43.0
29	T.16886	84	T.17363	86	0.82637	T.99522	31	40 58.0 57.3
30	T.16970	85	T.17450	87	0.82550	T.99520	30	50 72.5 71.7
31	T.17055	84	T.17536	86	0.82464	T.99518	29	85 84
32	T.17139	84	T.17622	86	0.82378	T.99517	28	6 8.5 8.4
33	T.17223	84	T.17708	86	0.82292	T.99515	27	7 9.9 9.8
34	T.17307	84	T.17794	86	0.82206	T.99513	26	8 11.3 11.2
35	T.17391	83	T.17880	86	0.82120	T.99511	25	9 12.8 12.6
36	T.17474	84	T.17965	85	0.82035	T.99509	24	10 14.2 14.0
37	T.17558	83	T.18051	86	0.81949	T.99507	23	20 28.3 28.0
38	T.17641	83	T.18136	85	0.81864	T.99505	22	30 42.5 42.0
39	T.17724	83	T.18221	85	0.81779	T.99503	21	40 56.7 56.0
40	T.17807	83	T.18306	85	0.81694	T.99501	20	50 70.8 70.0
41	T.17890	83	T.18391	84	0.81609	T.99499	19	83 82
42	T.17973	82	T.18475	85	0.81525	T.99497	18	6 8.3 8.2
43	T.18055	82	T.18560	84	0.81440	T.99495	17	7 9.7 9.6
44	T.18137	83	T.18644	84	0.81356	T.99494	16	8 11.1 10.9
45	T.18220	82	T.18728	84	0.81272	T.99492	15	9 12.5 12.3
46	T.18302	81	T.18812	84	0.81188	T.99490	14	10 13.8 13.7
47	T.18383	82	T.18896	84	0.81104	T.99488	13	20 27.7 27.3
48	T.18465	82	T.18979	83	0.81021	T.99486	12	30 41.5 41.0
49	T.18547	81	T.19063	84	0.80937	T.99484	11	40 55.3 54.7
50	T.18628	81	T.19146	83	0.80854	T.99482	10	50 69.2 68.3
51	T.18709	81	T.19229	83	0.80771	T.99480	9	81 80 2
52	T.18790	81	T.19312	83	0.80688	T.99478	8	6 8.1 8.0 0.2
53	T.18871	81	T.19395	83	0.80605	T.99476	7	7 9.5 9.3 0.2
54	T.18952	81	T.19478	83	0.80522	T.99474	6	8 10.8 10.7 0.3
55	T.19033	80	T.19561	82	0.80439	T.99472	5	9 12.2 12.0 0.3
56	T.19113	80	T.19643	82	0.80357	T.99470	4	10 13.5 13.3 0.3
57	T.19193	80	T.19725	82	0.80275	T.99468	3	20 27.0 26.7 0.7
58	T.19273	80	T.19807	82	0.80193	T.99466	2	30 40.5 40.0 1.0
59	T.19353	80	T.19889	82	0.80111	T.99464	1	40 54.0 53.3 1.3
60	T.19433		T.19971		0.80029	T.99462	0	50 67.5 66.7 1.7
	log cos	d	log cot	c. d.	log tan	log sin	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos		p. p.			
									82	81	80
0	T.19433	80	T.19971	82	0.80029	T.99462	60	6	8.2	8.1	8.0
1	T.19513	79	T.20053	81	0.79947	T.99460	59	7	9.6	9.5	9.3
2	T.19592	80	T.20134	82	0.79866	T.99458	58	8	10.9	10.8	10.7
3	T.19672	79	T.20215	81	0.79784	T.99456	57	9	12.3	12.2	12.0
4	T.19751	79	T.20297	81	0.79703	T.99454	56	10	13.7	13.5	13.3
5	T.19830	79	T.20378	81	0.79622	T.99452	55	20	27.3	27.0	26.7
6	T.19909	79	T.20459	81	0.79541	T.99450	54	30	41.0	40.5	40.0
7	T.19988	79	T.20540	81	0.79460	T.99448	53	40	54.7	54.0	53.3
8	T.20067	78	T.20621	80	0.79379	T.99446	52	50	68.3	67.5	66.7
9	T.20145	78	T.20701	81	0.79299	T.99444	51				
10	T.20223	78	T.20782	81	0.79218	T.99442	50				
		79		80							
11	T.20302	78	T.20862	80	0.79138	T.99440	49		79	78	
12	T.20380	78	T.20942	80	0.79058	T.99438	48	6	7.9	7.8	
13	T.20458	77	T.21022	80	0.78978	T.99436	47	7	9.2	9.1	
14	T.20535	78	T.21102	80	0.78898	T.99434	46	8	10.5	10.4	
15	T.20613	78	T.21182	80	0.78818	T.99432	45	9	11.9	11.7	
16	T.20691	77	T.21261	79	0.78739	T.99429	44	10	13.2	13.0	
17	T.20768	77	T.21341	80	0.78659	T.99427	43	20	26.3	26.0	
18	T.20845	77	T.21420	79	0.78580	T.99425	42	30	39.5	39.0	
19	T.20922	77	T.21499	79	0.78501	T.99423	41	40	52.7	52.0	
20	T.20999	77	T.21578	79	0.78422	T.99421	40	50	65.8	65.0	
		77		79							
21	T.21076	77	T.21657	79	0.78343	T.99419	39		77	76	
22	T.21153	76	T.21736	78	0.78264	T.99417	38	6	7.7	7.6	
23	T.21229	77	T.21814	79	0.78186	T.99415	37	7	9.0	8.9	
24	T.21306	76	T.21893	78	0.78107	T.99413	36	8	10.3	10.1	
25	T.21382	76	T.21971	78	0.78029	T.99411	35	9	11.6	11.4	
26	T.21458	76	T.22049	78	0.77951	T.99409	34	10	12.8	12.7	
27	T.21534	76	T.22127	78	0.77873	T.99407	33	20	25.7	25.3	
28	T.21610	75	T.22205	78	0.77795	T.99404	32	30	38.5	38.0	
29	T.21685	76	T.22283	78	0.77717	T.99402	31	40	51.3	50.7	
30	T.21761	75	T.22361	78	0.77639	T.99400	30	50	64.2	63.3	
		76		77							
31	T.21836	75	T.22438	78	0.77562	T.99398	29		75	74	
32	T.21912	75	T.22516	77	0.77484	T.99396	28	6	7.5	7.4	
33	T.21987	75	T.22593	77	0.77407	T.99394	27	7	8.8	8.6	
34	T.22062	75	T.22670	77	0.77330	T.99392	26	8	10.0	9.9	
35	T.22137	74	T.22747	77	0.77253	T.99390	25	9	11.3	11.1	
36	T.22211	75	T.22824	77	0.77176	T.99388	24	10	12.5	12.3	
37	T.22286	75	T.22901	76	0.77099	T.99385	23	20	25.0	24.7	
38	T.22361	74	T.22977	77	0.77023	T.99383	22	30	37.5	37.0	
39	T.22435	74	T.23054	76	0.76946	T.99381	21	40	50.0	49.3	
40	T.22509	74	T.23130	76	0.76870	T.99379	20	50	62.5	61.7	
		74		77							
41	T.22583	74	T.23206	77	0.76794	T.99377	19		73	72	
42	T.22657	74	T.23283	76	0.76717	T.99375	18	6	7.3	7.2	
43	T.22731	74	T.23359	76	0.76641	T.99372	17	7	8.5	8.4	
44	T.22805	73	T.23435	76	0.76565	T.99370	16	8	9.7	9.6	
45	T.22878	74	T.23510	75	0.76490	T.99368	15	9	11.0	10.8	
46	T.22952	73	T.23586	76	0.76414	T.99366	14	10	12.2	12.0	
47	T.23025	73	T.23661	75	0.76339	T.99364	13	20	24.3	24.0	
48	T.23098	73	T.23737	76	0.76263	T.99362	12	30	36.5	36.0	
49	T.23171	73	T.23812	75	0.76188	T.99359	11	40	48.7	48.0	
50	T.23244	73	T.23887	75	0.76113	T.99357	10	50	60.8	60.0	
		73		75							
51	T.23317	73	T.23962	75	0.76038	T.99355	9		71	3	2
52	T.23390	72	T.24037	75	0.75963	T.99353	8	6	7.1	0.3	0.2
53	T.23462	73	.24112	75	0.75888	T.99351	7	7	8.3	0.4	0.2
54	T.23535	72	T.24186	74	0.75814	T.99348	6	8	9.5	0.4	0.3
55	T.23607	72	T.24261	75	0.75739	T.99346	5	9	10.7	0.5	0.3
56	T.23679	73	T.24335	74	0.75665	T.99344	4	10	11.8	0.5	0.3
57	T.23752	71	T.24410	75	0.75590	T.99342	3	20	23.7	1.0	0.7
58	T.23823	72	T.24484	74	0.75516	T.99340	2	30	35.5	1.5	1.0
59	T.23895	72	T.24558	74	0.75442	T.99337	1	40	47.3	2.0	1.3
60	T.23967	72	T.24632	74	0.75368	T.99335	0	50	59.2	2.5	1.7
	log cos	d	log cot	c. d.	log tan	log sin	′	p. p.			

′	log sin	d	log tan	c. d.	log cot	log cos		p. p.		
									74	73
0	T.23067	72	T.24632	74	0.75368	T.99335	60			
1	T.24039	71	T.24706	73	0.75294	T.99333	59	6	7.4	7.3
2	T.24110	71	T.24779	74	0.75221	T.99331	58	7	8.6	8.5
3	T.24181	72	T.24853	73	0.75147	T.99328	57	8	9.9	9.7
4	T.24253	71	T.24926	74	0.75074	T.99326	56	9	11.1	11.0
5	T.24324	71	T.25000	73	0.75000	T.99324	55	10	12.3	12.2
6	T.24395	71	T.25073	73	0.74927	T.99322	54	20	24.7	24.3
7	T.24466	70	T.25146	73	0.74854	T.99319	53	30	37.0	36.5
8	T.24536	71	T.25219	73	0.74781	T.99317	52	40	49.3	48.7
9	T.24607	70	T.25292	73	0.74708	T.99315	51	50	61.7	60.8
10	T.24677	71	T.25365	72	0.74635	T.99313	50			
									72	71
11	T.24748	70	T.25437	73	0.74563	T.99310	49	6	7.2	7.1
12	T.24818	70	T.25510	72	0.74490	T.99308	48	7	8.4	8.3
13	T.24888	70	T.25582	73	0.74418	T.99306	47	8	9.6	9.5
14	T.24958	70	T.25655	72	0.74345	T.99304	46	9	10.8	10.7
15	T.25028	70	T.25727	72	0.74273	T.99301	45	10	12.0	11.8
16	T.25098	70	T.25799	72	0.74201	T.99299	44	20	24.0	23.7
17	T.25168	69	T.25871	72	0.74129	T.99297	43	30	36.0	35.5
18	T.25237	70	T.25943	72	0.74057	T.99294	42	40	48.0	47.3
19	T.25307	69	T.26015	71	0.73985	T.99292	41	50	60.0	59.2
20	T.25376	69	T.26086	72	0.73914	T.99290	40			
									70	69
21	T.25445	69	T.26158	71	0.73842	T.99288	39	6	7.0	6.9
22	T.25514	69	T.26229	72	0.73771	T.99285	38	7	8.2	8.1
23	T.25583	69	T.26301	71	0.73699	T.99283	37	8	9.3	9.2
24	T.25652	69	T.26372	71	0.73628	T.99281	36	9	10.5	10.4
25	T.25721	69	T.26443	71	0.73557	T.99278	35	10	11.7	11.5
26	T.25790	68	T.26514	71	0.73486	T.99276	34	20	23.3	23.0
27	T.25858	69	T.26585	70	0.73415	T.99274	33	30	35.0	34.5
28	T.25927	68	T.26655	71	0.73345	T.99271	32	40	46.7	46.0
29	T.25995	68	T.26726	71	0.73274	T.99269	31	50	58.3	57.5
30	T.26063	68	T.26797	70	0.73203	T.99267	30			
									68	67
31	T.26131	68	T.26867	70	0.73133	T.99264	29	6	6.8	6.7
32	T.26199	68	T.26937	71	0.73063	T.99262	28	7	7.9	7.8
33	T.26267	68	T.27008	70	0.72992	T.99260	27	8	9.1	8.9
34	T.26335	68	T.27078	70	0.72922	T.99257	26	9	10.2	10.1
35	T.26403	67	T.27148	70	0.72852	T.99255	25	10	11.3	11.2
36	T.26470	68	T.27218	70	0.72782	T.99252	24	20	22.7	22.3
37	T.26538	67	T.27288	69	0.72712	T.99250	23	30	34.0	33.5
38	T.26605	67	T.27357	70	0.72643	T.99248	22	40	45.3	44.7
39	T.26672	67	T.27427	69	0.72573	T.99245	21	50	56.7	55.8
40	T.26739	67	T.27496	70	0.72504	T.99243	20			
									66	65
41	T.26806	67	T.27566	69	0.72434	T.99241	19	6	6.6	6.5
42	T.26873	67	T.27635	69	0.72365	T.99238	18	7	7.7	7.6
43	T.26940	67	T.27704	69	0.72296	T.99236	17	8	8.8	8.7
44	T.27007	66	T.27773	69	0.72227	T.99233	16	9	9.9	9.8
45	T.27073	67	T.27842	69	0.72158	T.99231	15	10	11.0	10.8
46	T.27140	66	T.27911	69	0.72089	T.99229	14	20	22.0	21.7
47	T.27206	67	T.27980	69	0.72020	T.99226	13	30	33.0	32.5
48	T.27273	66	T.28049	68	0.71951	T.99224	12	40	44.0	43.3
49	T.27339	66	T.28117	69	0.71883	T.99221	11	50	55.0	54.2
50	T.27405	66	T.28186	68	0.71814	T.99219	10			
									3	2
51	T.27471	66	T.28254	69	0.71746	T.99217	9	6	0.3	0.2
52	T.27537	65	T.28323	68	0.71677	T.99214	8	7	0.4	0.2
53	T.27602	66	T.28391	68	0.71609	T.99212	7	8	0.4	0.3
54	T.27668	66	T.28459	68	0.71541	T.99209	6	9	0.5	0.3
55	T.27734	65	T.28527	68	0.71473	T.99207	5	10	0.5	0.3
56	T.27799	65	T.28595	67	0.71405	T.99204	4	20	1.0	0.7
57	T.27864	66	T.28662	68	0.71338	T.99202	3	30	1.5	1.0
58	T.27930	65	T.28730	68	0.71270	T.99200	2	40	2.0	1.3
59	T.27995	65	T.28798	67	0.71202	T.99197	1	50	2.5	1.7
60	T.28060		T.28865		0.71135	T.99195	0			
	log cos	d	log cot	c. d.	log tan	log sin	′	p. p.		

′	log sin	d	log tan	c. d.	log cot	log cos		p. p.		
0	Ī.28060		Ī.28865		0.71135	Ī.99105	60		68	67
1	Ī.28125	65	Ī.28933	68	0.71067	Ī.99102	59	6	6.8	6.7
2	Ī.28190	65	Ī.29000	67	0.71000	Ī.99100	58	7	7.9	7.8
3	Ī.28254	64	Ī.29067	67	0.70933	Ī.99187	57	8	9.1	8.9
4	Ī.28319	65	Ī.29134	67	0.70866	Ī.99185	56	9	10.2	10.1
5	Ī.28384	64	Ī.29201	67	0.70799	Ī.99182	55	10	11.3	11.2
6	Ī.28448	64	Ī.29268	67	0.70732	Ī.99180	54	20	22.7	22.3
7	Ī.28512	65	Ī.29335	67	0.70665	Ī.99177	53	30	34.0	33.5
8	Ī.28577	64	Ī.29402	66	0.70598	Ī.99175	52	40	45.3	44.7
9	Ī.28641	64	Ī.29468	67	0.70532	Ī.99172	51	50	56.7	55.8
10	Ī.28705		Ī.29535		0.70465	Ī.99170	50			
		64		66					66	65
11	Ī.28769		Ī.29601		0.70399	Ī.99167	49	6	6.6	6.5
12	Ī.28833	64	Ī.29668	67	0.70332	Ī.99165	48	7	7.7	7.6
13	Ī.28896	63	Ī.29734	66	0.70266	Ī.99162	47	8	8.8	8.7
14	Ī.28960	64	Ī.29800	66	0.70200	Ī.99160	46	9	9.9	9.8
15	Ī.29024	64	Ī.29866	66	0.70134	Ī.99157	45	10	11.0	10.8
16	Ī.29087	63	Ī.29932	66	0.70068	Ī.99155	44	20	22.0	21.7
17	Ī.29150	63	Ī.29998	66	0.70002	Ī.99152	43	30	33.0	32.5
18	Ī.29214	64	Ī.30064	66	0.69936	Ī.99150	42	40	44.0	43.3
19	Ī.29277	63	Ī.30130	66	0.69870	Ī.99147	41	50	55.0	54.2
20	Ī.29340	63	Ī.30195	65	0.69805	Ī.99145	40			
				66					64	63
21	Ī.29403	63	Ī.30261	65	0.69739	Ī.99142	39	6	6.4	6.3
22	Ī.29466	63	Ī.30326	65	0.69674	Ī.99140	38	7	7.5	7.4
23	Ī.29529	62	Ī.30391	66	0.69609	Ī.99137	37	8	8.5	8.4
24	Ī.29591	63	Ī.30457	65	0.69543	Ī.99135	36	9	9.6	9.5
25	Ī.29654	62	Ī.30522	65	0.69478	Ī.99132	35	10	10.7	10.5
26	Ī.29716	63	Ī.30587	65	0.69413	Ī.99130	34	20	21.3	21.0
27	Ī.29779	62	Ī.30652	65	0.69348	Ī.99127	33	30	32.0	31.5
28	Ī.29841	62	Ī.30717	65	0.69283	Ī.99124	32	40	42.7	42.0
29	Ī.29903	63	Ī.30782	65	0.69218	Ī.99122	31	50	53.3	52.5
30	Ī.29966		Ī.30846	64	0.69154	Ī.99119	30			
		62		65					62	61
31	Ī.30028		Ī.30911		0.69089	Ī.99117	29	6	6.2	6.1
32	Ī.30090	62	Ī.30975	64	0.69025	Ī.99114	28	7	7.2	7.1
33	Ī.30151	61	Ī.31040	65	0.68960	Ī.99112	27	8	8.3	8.1
34	Ī.30213	62	Ī.31104	64	0.68896	Ī.99109	26	9	9.3	9.2
35	Ī.30275	62	Ī.31168	64	0.68832	Ī.99106	25	10	10.3	10.2
36	Ī.30336	61	Ī.31233	65	0.68767	Ī.99104	24	20	20.7	20.3
37	Ī.30398	62	Ī.31297	64	0.68703	Ī.99101	23	30	31.0	30.5
38	Ī.30459	61	Ī.31361	64	0.68639	Ī.99099	22	40	41.3	40.7
39	Ī.30521	62	Ī.31425	64	0.68575	Ī.99096	21	50	51.7	50.8
40	Ī.30582	61	Ī.31489	64	0.68511	Ī.99093	20			
		61		63					60	59
41	Ī.30643		Ī.31552		0.68448	Ī.99091	19	6	6.0	5.9
42	Ī.30704	61	Ī.31616	64	0.68384	Ī.99088	18	7	7.0	6.9
43	Ī.30765	61	Ī.31679	63	0.68321	Ī.99086	17	8	8.0	7.9
44	Ī.30826	61	Ī.31743	64	0.68257	Ī.99083	16	9	9.0	8.9
45	Ī.30887	61	Ī.31806	63	0.68194	Ī.99080	15	10	10.0	9.8
46	Ī.30947	60	Ī.31870	64	0.68130	Ī.99078	14	20	20.0	19.7
47	Ī.31008	61	Ī.31933	63	0.68067	Ī.99075	13	30	30.0	29.5
48	Ī.31068	60	Ī.31996	63	0.68004	Ī.99072	12	40	40.0	39.3
49	Ī.31129	61	Ī.32059	63	0.67941	Ī.99070	11	50	50.0	49.2
50	Ī.31189	60	Ī.32122	63	0.67878	Ī.99067	10			
		61		63						
51	Ī.31250		Ī.32185		0.67815	Ī.99064	9		3	2
52	Ī.31310	60	Ī.32248	63	0.67752	Ī.99062	8	6	0.3	0.2
53	Ī.31370	60	Ī.32311	63	0.67689	Ī.99059	7	7	0.4	0.2
54	Ī.31430	60	Ī.32373	62	0.67627	Ī.99056	6	8	0.4	0.3
55	Ī.31490	60	Ī.32436	63	0.67564	Ī.99054	5	9	0.5	0.3
56	Ī.31549	59	Ī.32498	62	0.67502	Ī.99051	4	10	0.5	0.3
57	Ī.31609	60	Ī.32561	63	0.67439	Ī.99048	3	20	1.0	0.7
58	Ī.31669	60	Ī.32623	62	0.67377	Ī.99046	2	30	1.5	1.0
59	Ī.31728	59	Ī.32685	62	0.67315	Ī.99043	1	40	2.0	1.3
60	Ī.31788	60	Ī.32747	62	0.67253	Ī.99040	0	50	2.5	1.7
	log cos	d	log cot	c. d.	log tan	log sin	′	p. p.		

′	log sin	d	log tan	c. d.	log cot	log cos		p. p.
0	1.31788		1.32747		0.67253	1.99040	60	
1	1.31847	59	1.32810	63	0.67190	1.99038	59	63 | 62
2	1.31907	60	1.32872	62	0.67128	1.99035	58	6 | 6.3 | 6.2
3	1.31966	59	1.32933	61	0.67067	1.99032	57	7 | 7.4 | 7.2
4	1.32025	59	1.32995	62	0.67005	1.99030	56	8 | 8.4 | 8.3
5	1.32084	59	1.33057	62	0.66943	1.99027	55	9 | 9.5 | 9.3
6	1.32143	59	1.33119	61	0.66881	1.99024	54	10 | 10.5 | 10.3
7	1.32202	59	1.33180	62	0.66820	1.99022	53	20 | 21.0 | 20.7
8	1.32261	58	1.33242	61	0.66758	1.99019	52	30 | 31.5 | 31.0
9	1.32319	59	1.33303	62	0.66697	1.99016	51	40 | 42.0 | 41.3
10	1.32378		1.33365		0.66635	1.99013	50	50 | 52.5 | 51.7
		59		61				
11	1.32437		1.33426		0.66574	1.99011	49	61 | 60
12	1.32495	58	1.33487	61	0.66513	1.99008	48	6 | 6.1 | 6.0
13	1.32553	58	1.33548	61	0.66452	1.99005	47	7 | 7.1 | 7.0
14	1.32612	59	1.33609	61	0.66391	1.99002	46	8 | 8.1 | 8.0
15	1.32670	58	1.33670	61	0.66330	1.99000	45	9 | 9.2 | 9.0
16	1.32728	58	1.33731	61	0.66269	1.98997	44	10 | 10.2 | 10.0
17	1.32786	58	1.33792	61	0.66208	1.98994	43	20 | 20.3 | 20.0
18	1.32844	58	1.33853	61	0.66147	1.98991	42	30 | 30.5 | 30.0
19	1.32902	58	1.33913	60	0.66087	1.98989	41	40 | 40.7 | 40.0
20	1.32960		1.33974	61	0.66026	1.98986	40	50 | 50.8 | 50.0
		58		60				
21	1.33018		1.34034		0.65966	1.98983	39	59
22	1.33075	57	1.34095	61	0.65905	1.98980	38	6 | 5.9
23	1.33133	58	1.34155	60	0.65845	1.98978	37	7 | 6.9
24	1.33190	57	1.34215	60	0.65785	1.98975	36	8 | 7.9
25	1.33248	58	1.34276	61	0.65724	1.98972	35	9 | 8.9
26	1.33305	57	1.34336	60	0.65664	1.98969	34	10 | 9.8
27	1.33362	57	1.34396	60	0.65604	1.98967	33	20 | 19.7
28	1.33420	58	1.34456	60	0.65544	1.98964	32	30 | 29.5
29	1.33477	57	1.34516	60	0.65484	1.98961	31	40 | 39.3
30	1.33534	57	1.34576	60	0.65424	1.98958	30	50 | 49.2
31	1.33591		1.34635	59	0.65365	1.98955	29	58 | 57
32	1.33647	56	1.34695	60	0.65305	1.98953	28	6 | 5.8 | 5.7
33	1.33704	57	1.34755	60	0.65245	1.98950	27	7 | 6.8 | 6.7
34	1.33761	57	1.34814	59	0.65186	1.98947	26	8 | 7.7 | 7.6
35	1.33818	57	1.34874	60	0.65126	1.98944	25	9 | 8.7 | 8.6
36	1.33874	56	1.34933	59	0.65067	1.98941	24	10 | 9.7 | 9.5
37	1.33931	57	1.34992	59	0.65008	1.98938	23	20 | 19.3 | 19.0
38	1.33987	56	1.35051	59	0.64949	1.98936	22	30 | 29.0 | 28.5
39	1.34043	56	1.35111	60	0.64889	1.98933	21	40 | 38.7 | 38.0
40	1.34100	57	1.35170	59	0.64830	1.98930	20	50 | 48.3 | 47.5
41	1.34156	56	1.35229	59	0.64771	1.98927	19	56 | 55
42	1.34212	56	1.35288	59	0.64712	1.98924	18	6 | 5.6 | 5.5
43	1.34268	56	1.35347	59	0.64653	1.98921	17	7 | 6.5 | 6.4
44	1.34324	56	1.35405	58	0.64595	1.98919	16	8 | 7.5 | 7.3
45	1.34380	56	1.35464	59	0.64536	1.98916	15	9 | 8.4 | 8.3
46	1.34436	56	1.35523	59	0.64477	1.98913	14	10 | 9.3 | 9.2
47	1.34491	55	1.35581	58	0.64419	1.98910	13	20 | 18.7 | 18.3
48	1.34547	56	1.35640	59	0.64360	1.98907	12	30 | 28.0 | 27.5
49	1.34602	55	1.35698	58	0.64302	1.98904	11	40 | 37.3 | 36.7
50	1.34658	56	1.35757	59	0.64243	1.98901	10	50 | 46.7 | 45.8
		55		58				
51	1.34713		1.35815		0.64185	1.98898	9	3 | 2
52	1.34769	56	1.35873	58	0.64127	1.98896	8	6 | 0.3 | 0.2
53	1.34824	55	1.35931	58	0.64069	1.98893	7	7 | 0.4 | 0.2
54	1.34879	55	1.35989	58	0.64011	1.98890	6	8 | 0.4 | 0.3
55	1.34934	55	1.36047	58	0.63953	1.98887	5	9 | 0.5 | 0.3
56	1.34989	55	1.36105	58	0.63895	1.98884	4	10 | 0.5 | 0.3
57	1.35044	55	1.36163	58	0.63837	1.98881	3	20 | 1.0 | 0.7
58	1.35099	55	1.36221	58	0.63779	1.98878	2	30 | 1.5 | 1.0
59	1.35154	55	1.36279	58	0.63721	1.98875	1	40 | 2.0 | 1.3
60	1.35209	55	1.36336	57	0.63664	1.98872	0	50 | 2.5 | 1.7
	log cos	d	log cot	c. d.	log tan	log sin	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos		p. p.
0	Ī.35209		Ī.36336		0.63664	Ī.98872	60	
1	Ī.35263	54	Ī.36394	58	0.63606	Ī.98869	59	58 57
2	Ī.35318	55	Ī.36452	58	0.63548	Ī.98867	58	6 5.8 5.7
3	Ī.35373	55	Ī.36509	57	0.63491	Ī.98864	57	7 6.8 6.7
4	Ī.35427	54	Ī.36566	57	0.63434	Ī.98861	56	8 7.7 7.6
5	Ī.35481	54	Ī.36624	58	0.63376	Ī.98858	55	9 8.7 8.6
6	Ī.35536	55	Ī.36681	57	0.63319	Ī.98855	54	10 9.7 9.5
7	Ī.35590	54	Ī.36738	57	0.63262	Ī.98852	53	20 19.3 19.0
8	Ī.35644	54	Ī.36795	57	0.63205	Ī.98849	52	30 29.0 28.5
9	Ī.35698	54	Ī.36852	57	0.63148	Ī.98846	51	40 38.7 38.0
10	Ī.35752	54	Ī.36909	57	0.63091	Ī.98843	50	50 48.3 47.5
11	Ī.35806	54	Ī.36966	57	0.63034	Ī.98840	49	58 56 55
12	Ī.35860	54	Ī.37023	57	0.62977	Ī.98837	48	6 5.6 5.5
13	Ī.35914	54	Ī.37080	57	0.62920	Ī.98834	47	7 6.5 6.4
14	Ī.35968	54	Ī.37137	57	0.62863	Ī.98831	46	8 7.5 7.3
15	Ī.36022	53	Ī.37193	56	0.62807	Ī.98828	45	9 8.4 8.3
16	Ī.36075	54	Ī.37250	57	0.62750	Ī.98825	44	10 9.3 9.2
17	Ī.36129	53	Ī.37306	56	0.62694	Ī.98822	43	20 18.7 18.3
18	Ī.36182	54	Ī.37363	57	0.62637	Ī.98819	42	30 28.0 27.5
19	Ī.36236	53	Ī.37419	56	0.62581	Ī.98816	41	40 37.3 36.7
20	Ī.36289	53	Ī.37476	57	0.62524	Ī.98813	40	50 46.7 45.8
21	Ī.36342	53	Ī.37532	56	0.62468	Ī.98810	39	54
22	Ī.36395	53	Ī.37588	56	0.62412	Ī.98807	38	6 5.4
23	Ī.36449	54	Ī.37644	56	0.62356	Ī.98804	37	7 6.3
24	Ī.36502	53	Ī.37700	56	0.62300	Ī.98801	36	8 7.2
25	Ī.36555	53	Ī.37756	56	0.62244	Ī.98798	35	9 8.1
26	Ī.36608	53	Ī.37812	56	0.62188	Ī.98795	34	10 9.0
27	Ī.36660	52	Ī.37868	56	0.62132	Ī.98792	33	20 18.0
28	Ī.36713	53	Ī.37924	56	0.62076	Ī.98789	32	30 27.0
29	Ī.36766	53	Ī.37980	56	0.62020	Ī.98786	31	40 36.0
30	Ī.36819	53	Ī.38035	55	0.61965	Ī.98783	30	50 45.0
31	Ī.36871	52	Ī.38091	56	0.61909	Ī.98780	29	53 52
32	Ī.36924	53	Ī.38147	56	0.61853	Ī.98777	28	6 5.3 5.2
33	Ī.36976	52	Ī.38202	55	0.61798	Ī.98774	27	7 6.2 6.1
34	Ī.37028	52	Ī.38257	55	0.61743	Ī.98771	26	8 7.1 6.9
35	Ī.37081	53	Ī.38313	56	0.61687	Ī.98768	25	9 8.0 7.8
36	Ī.37133	52	Ī.38368	55	0.61632	Ī.98765	24	10 8.8 8.7
37	Ī.37185	52	Ī.38423	55	0.61577	Ī.98762	23	20 17.7 17.3
38	Ī.37237	52	Ī.38479	56	0.61521	Ī.98759	22	30 26.5 26.0
39	Ī.37289	52	Ī.38534	55	0.61466	Ī.98756	21	40 35.3 34.7
40	Ī.37341	52	Ī.38589	55	0.61411	Ī.98753	20	50 44.2 43.3
41	Ī.37393	52	Ī.38644	55	0.61356	Ī.98750	19	51 4
42	Ī.37445	52	Ī.38699	55	0.61301	Ī.98746	18	6 5.1 0.4
43	Ī.37497	52	Ī.38754	55	0.61246	Ī.98743	17	7 6.0 0.5
44	Ī.37549	52	Ī.38808	54	0.61192	Ī.98740	16	8 6.8 0.5
45	Ī.37600	51	Ī.38863	55	0.61137	Ī.98737	15	9 7.7 0.6
46	Ī.37652	52	Ī.38918	55	0.61082	Ī.98734	14	10 8.5 0.7
47	Ī.37703	51	Ī.38972	54	0.61028	Ī.98731	13	20 17.0 1.3
48	Ī.37755	52	Ī.39027	55	0.60973	Ī.98728	12	30 25.5 2.0
49	Ī.37806	51	Ī.39082	55	0.60918	Ī.98725	11	40 34.0 2.7
50	Ī.37858	52	Ī.39136	54	0.60864	Ī.98722	10	50 42.5 3.3
51	Ī.37909	51	Ī.39190	54	0.60810	Ī.98719	9	3 2
52	Ī.37960	51	Ī.39245	55	0.60755	Ī.98715	8	6 0.3 0.2
53	Ī.38011	51	Ī.39299	54	0.60701	Ī.98712	7	7 0.4 0.2
54	Ī.38062	51	Ī.39353	54	0.60647	Ī.98709	6	8 0.4 0.3
55	Ī.38113	51	Ī.39407	54	0.60593	Ī.98706	5	9 0.5 0.3
56	Ī.38164	51	Ī.39461	54	0.60539	Ī.98703	4	10 0.5 0.3
57	Ī.38215	51	Ī.39515	54	0.60485	Ī.98700	3	20 1.0 0.7
58	Ī.38266	51	Ī.39569	54	0.60431	Ī.98697	2	30 1.5 1.0
59	Ī.38317	51	Ī.39623	54	0.60377	Ī.98694	1	40 2.0 1.3
60	Ī.38368	51	Ī.39677	54	0.60323	Ī.98690	0	50 2.5 1.7
	log cos	d	log cot	c. d.	log tan	log sin	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.38368		T.39677		0.60323	T.98690		60	
1	T.38418	50	T.39731	54	0.60269	T.98687	3	59	
2	T.38469	51	T.39785	54	0.60215	T.98684	3	58	
3	T.38519	50	T.39838	53	0.60162	T.98681	3	57	54 | 53
4	T.38570	51	T.39892	54	0.60108	T.98678	3	56	6 | 5.4 | 5.3
5	T.38620	50	T.39945	53	0.60055	T.98675	3	55	7 | 6.3 | 6.2
6	T.38670	50	T.39999	54	0.60001	T.98671	4	54	8 | 7.2 | 7.1
7	T.38721	51	T.40052	53	0.59948	T.98668	3	53	9 | 8.1 | 8.0
8	T.38771	50	T.40106	54	0.59894	T.98665	3	52	10 | 9.0 | 8.8
9	T.38821	50	T.40159	53	0.59841	T.98662	3	51	20 | 18.0 | 17.7
10	T.38871	50	T.40212	53	0.59788	T.98659	3	50	30 | 27.0 | 26.5
11	T.38921		T.40266	54	0.59734	T.98656	3	49	40 | 36.0 | 35.3
12	T.38971	50	T.40319	53	0.59681	T.98652	4	48	50 | 45.0 | 44.2
13	T.39021	50	T.40372	53	0.59628	T.98649	3	47	
14	T.39071	50	T.40425	53	0.59575	T.98646	3	46	52 | 51
15	T.39121	50	T.40478	53	0.59522	T.98643	3	45	6 | 5.2 | 5.1
16	T.39170	49	T.40531	53	0.59469	T.98640	3	44	7 | 6.1 | 6.0
17	T.39220	50	T.40584	53	0.59416	T.98636	4	43	8 | 6.9 | 6.8
18	T.39270	50	T.40636	52	0.59364	T.98633	3	42	9 | 7.8 | 7.7
19	T.39319	49	T.40689	53	0.59311	T.98630	3	41	10 | 8.7 | 8.5
20	T.39369	50	T.40742	53	0.59258	T.98627	3	40	20 | 17.3 | 17.0
21	T.39418	49	T.40795	53	0.59205	T.98623	4	39	30 | 26.0 | 25.5
22	T.39467	49	T.40847	52	0.59153	T.98620	3	38	40 | 34.7 | 34.0
23	T.39517	50	T.40900	53	0.59100	T.98617	3	37	50 | 43.3 | 42.5
24	T.39566	49	T.40952	52	0.59048	T.98614	3	36	
25	T.39615	49	T.41005	53	0.58995	T.98610	4	35	
26	T.39664	49	T.41057	52	0.58943	T.98607	3	34	50 | 49
27	T.39713	49	T.41109	52	0.58891	T.98604	3	33	6 | 5.0 | 4.9
28	T.39762	49	T.41161	52	0.58839	T.98601	3	32	7 | 5.8 | 5.7
29	T.39811	49	T.41214	53	0.58786	T.98597	4	31	8 | 6.7 | 6.5
30	T.39860	49	T.41266	52	0.58734	T.98594	3	30	9 | 7.5 | 7.4
31	T.39909		T.41318	52	0.58682	T.98591	3	29	10 | 8.3 | 8.2
32	T.39958	49	T.41370	52	0.58630	T.98588	3	28	20 | 16.7 | 16.3
33	T.40006	48	T.41422	52	0.58578	T.98584	4	27	30 | 25.0 | 24.5
34	T.40055	49	T.41474	52	0.58526	T.98581	3	26	40 | 33.3 | 32.7
35	T.40103	48	T.41526	52	0.58474	T.98578	3	25	50 | 41.7 | 40.8
36	T.40152	49	T.41578	52	0.58422	T.98574	4	24	
37	T.40200	48	T.41629	51	0.58371	T.98571	3	23	
38	T.40249	49	T.41681	52	0.58319	T.98568	3	22	48 | 47
39	T.40297	48	T.41733	52	0.58267	T.98565	3	21	6 | 4.8 | 4.7
40	T.40346	49	T.41784	51	0.58216	T.98561	4	20	7 | 5.6 | 5.5
41	T.40394	48	T.41836	52	0.58164	T.98558	3	19	8 | 6.4 | 6.3
42	T.40442	48	T.41887	51	0.58113	T.98555	3	18	9 | 7.2 | 7.1
43	T.40490	48	T.41939	52	0.58061	T.98551	4	17	10 | 8.0 | 7.8
44	T.40538	48	T.41990	51	0.58010	T.98548	3	16	20 | 16.0 | 15.7
45	T.40586	48	T.42041	51	0.57959	T.98545	3	15	30 | 24.0 | 23.5
46	T.40634	48	T.42093	52	0.57907	T.98541	4	14	40 | 32.0 | 31.3
47	T.40682	48	T.42144	51	0.57856	T.98538	3	13	50 | 40.0 | 39.2
48	T.40730	48	T.42195	51	0.57805	T.98535	3	12	
49	T.40778	47	T.42246	51	0.57754	T.98531	4	11	
50	T.40825	48	T.42297	51	0.57703	T.98528	3	10	4 | 3
51	T.40873	48	T.42348	51	0.57652	T.98525	3	9	6 | 0.4 | 0.3
52	T.40921	47	T.42399	51	0.57601	T.98521	4	8	7 | 0.5 | 0.4
53	T.40968	48	T.42450	51	0.57550	T.98518	3	7	8 | 0.5 | 0.4
54	T.41016	47	T.42501	51	0.57499	T.98515	3	6	9 | 0.6 | 0.5
55	T.41063	48	T.42552	51	0.57448	T.98511	4	5	10 | 0.7 | 0.5
56	T.41111	47	T.42603	51	0.57397	T.98508	3	4	20 | 1.3 | 1.0
57	T.41158	47	T.42653	50	0.57347	T.98505	3	3	30 | 2.0 | 1.5
58	T.41205	47	T.42704	51	0.57296	T.98501	4	2	40 | 2.7 | 2.0
59	T.41252	47	T.42755	51	0.57245	T.98498	3	1	50 | 3.3 | 2.5
60	T.41300	48	T.42805	50	0.57195	T.98494	4	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.41300		T.42805		0.57195	T.98494		60	
1	T.41347	47	T.42856	51	0.57144	T.98491	3	59	
2	T.41394	47	T.42906	50	0.57094	T.98488	3	58	
3	T.41441	47	T.42957	51	0.57043	T.98484	4	57	51 50
4	T.41488	47	T.43007	50	0.56993	T.98481	3	56	6 5.1 5.0
5	T.41535	47	T.43057	50	0.56943	T.98477	4	55	7 6.0 5.8
6	T.41582	46	T.43108	51	0.56892	T.98474	3	54	8 6.8 6.7
7	T.41628	47	T.43158	50	0.56842	T.98471	3	53	9 7.7 7.5
8	T.41675	47	T.43208	50	0.56792	T.98467	4	52	10 8.5 8.3
9	T.41722	46	T.43258	50	0.56742	T.98464	3	51	20 17.0 16.7
10	T.41768		T.43308	50	0.56692	T.98460	4	50	30 25.5 25.0
		47					3		40 34.0 33.3
11	T.41815		T.43358		0.56642	T.98457		49	50 42.5 41.7
12	T.41861	46	T.43408	50	0.56592	T.98453	4	48	
13	T.41908	47	T.43458	50	0.56542	T.98450	3	47	
14	T.41954	46	T.43508	50	0.56492	T.98447	3	46	49 48
15	T.42001	47	T.43558	50	0.56442	T.98443	4	45	6 4.9 4.8
16	T.42047	46	T.43607	49	0.56393	T.98440	3	44	7 5.7 5.6
17	T.42093	46	T.43657	50	0.56343	T.98436	4	43	8 6.5 6.4
18	T.42140	47	T.43707	50	0.56293	T.98433	3	42	9 7.4 7.2
19	T.42186	46	T.43756	49	0.56244	T.98429	4	41	10 8.2 8.0
20	T.42232	46	T.43806	50	0.56194	T.98426	3	40	20 16.3 16.0
				49			4		30 24.5 24.0
21	T.42278		T.43855		0.56145	T.98422		39	40 32.7 32.0
22	T.42324	46	T.43905	50	0.56095	T.98419	3	38	50 40.8 40.0
23	T.42370	46	T.43954	49	0.56046	T.98415	4	37	
24	T.42416	46	T.44004	50	0.55996	T.98412	3	36	
25	T.42461	45	T.44053	49	0.55947	T.98409	3	35	
26	T.42507	46	T.44102	49	0.55898	T.98405	4	34	47 46
27	T.42553	46	T.44151	49	0.55849	T.98402	3	33	6 4.7 4.6
28	T.42599	46	T.44201	50	0.55799	T.98398	4	32	7 5.5 5.4
29	T.42644	45	T.44250	49	0.55750	T.98395	3	31	8 6.3 6.1
30	T.42690	46	T.44299	49	0.55701	T.98391	4	30	9 7.1 6.9
		45		49			3		10 7.8 7.7
31	T.42735		T.44348		0.55652	T.98388		29	20 15.7 15.3
32	T.42781	46	T.44397	49	0.55603	T.98384	4	28	30 23.5 23.0
33	T.42826	45	T.44446	49	0.55554	T.98381	3	27	40 31.3 30.7
34	T.42872	46	T.44495	49	0.55505	T.98377	4	26	50 39.2 38.3
35	T.42917	45	T.44544	49	0.55456	T.98373	4	25	
36	T.42962	45	T.44592	48	0.55408	T.98370	3	24	
37	T.43008	46	T.44641	49	0.55359	T.98366	4	23	
38	T.43053	45	T.44690	49	0.55310	T.98363	3	22	45 44
39	T.43098	45	T.44738	48	0.55262	T.98359	4	21	6 4.5 4.4
40	T.43143	45	T.44787	49	0.55213	T.98356	3	20	7 5.3 5.1
		45		49			4		8 6.0 5.9
41	T.43188		T.44836		0.55164	T.98352		19	9 6.8 6.6
42	T.43233	45	T.44884	48	0.55116	T.98349	3	18	10 7.5 7.3
43	T.43278	45	T.44933	49	0.55067	T.98345	4	17	20 15.0 14.7
44	T.43323	45	T.44981	48	0.55019	T.98342	3	16	30 22.5 22.0
45	T.43367	44	T.45029	48	0.54971	T.98338	4	15	40 30.0 29.3
46	T.43412	45	T.45078	49	0.54922	T.98334	4	14	50 37.5 36.7
47	T.43457	45	T.45126	48	0.54874	T.98331	3	13	
48	T.43502	45	T.45174	48	0.54826	T.98327	4	12	
49	T.43546	44	T.45222	48	0.54778	T.98324	3	11	
50	T.43591	45	T.45271	49	0.54729	T.98320	4	10	4 3
		44		48			3		6 0.4 0.3
51	T.43635		T.45319		0.54681	T.98317		9	7 0.5 0.4
52	T.43680	45	T.45367	48	0.54633	T.98313	4	8	8 0.5 0.4
53	T.43724	44	T.45415	48	0.54585	T.98309	4	7	9 0.6 0.5
54	T.43769	45	T.45463	48	0.54537	T.98306	3	6	10 0.7 0.5
55	T.43813	44	T.45511	48	0.54489	T.98302	4	5	20 1.3 1.0
56	T.43857	44	T.45559	48	0.54441	T.98299	3	4	30 2.0 1.5
57	T.43901	44	T.45606	47	0.54394	T.98295	4	3	40 2.7 2.0
58	T.43946	45	T.45654	48	0.54346	T.98291	4	2	50 3.3 2.5
59	T.43990	44	T.45702	48	0.54298	T.98288	3	1	
60	T.44034	44	T.45750	48	0.54250	T.98284	4	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.44034		T.45750		0.54250	T.98284		60	
1	T.44078	44	T.45797	47	0.54203	T.98281	3	59	
2	T.44122	44	T.45845	48	0.54155	T.98277	4	58	48 47
3	T.44166	44	T.45892	47	0.54108	T.98273	4	57	6 4.8 4.7
4	T.44210	44	T.45940	48	0.54060	T.98270	3	56	7 5.6 5.5
5	T.44253	43	T.45987	47	0.54013	T.98266	4	55	8 6.4 6.3
6	T.44297	44	T.46035	48	0.53965	T.98262	4	54	9 7.2 7.1
7	T.44341	44	T.46082	47	0.53918	T.98259	3	53	10 8.0 7.8
8	T.44385	44	T.46130	48	0.53870	T.98255	4	52	20 16.0 15.7
9	T.44428	43	T.46177	47	0.53823	T.98251	4	51	30 24.0 23.5
10	T.44472	44	T.46224	47	0.53776	T.98248	3	50	40 32.0 31.3
		44		47			4		50 40.0 39.2
11	T.44516		T.46271		0.53729	T.98244		49	
12	T.44559	43	T.46319	48	0.53681	T.98240	4	48	
13	T.44602	43	T.46366	47	0.53634	T.98237	3	47	
14	T.44646	44	T.46413	47	0.53587	T.98233	4	46	46 45
15	T.44689	43	T.46460	47	0.53540	T.98229	4	45	6 4.6 4.5
16	T.44733	44	T.46507	47	0.53493	T.98226	3	44	7 5.4 5.3
17	T.44776	43	T.46554	47	0.53446	T.98222	4	43	8 6.1 6.0
18	T.44819	43	T.46601	47	0.53399	T.98218	4	42	9 6.9 6.8
19	T.44862	43	T.46648	47	0.53352	T.98215	3	41	10 7.7 7.5
20	T.44905	43	T.46694	46	0.53306	T.98211	4	40	20 15.3 15.0
				47			4		30 23.0 22.5
21	T.44948		T.46741		0.53259	T.98207		39	40 30.7 30.0
22	T.44992	44	T.46788	47	0.53212	T.98204	3	38	50 38.3 37.5
23	T.45035	43	T.46835	47	0.53165	T.98200	4	37	
24	T.45077	42	T.46881	46	0.53119	T.98196	4	36	
25	T.45120	43	T.46928	47	0.53072	T.98192	4	35	
26	T.45163	43	T.46975	47	0.53025	T.98189	3	34	44 43
27	T.45206	43	T.47021	46	0.52979	T.98185	4	33	6 4.4 4.3
28	T.45249	43	T.47068	47	0.52932	T.98181	4	32	7 5.1 5.0
29	T.45292	43	T.47114	46	0.52886	T.98177	4	31	8 5.9 5.7
30	T.45334	42	T.47160	46	0.52840	T.98174	3	30	9 6.6 6.5
		43		47			4		10 7.3 7.2
31	T.45377		T.47207		0.52793	T.98170		29	20 14.7 14.3
32	T.45419	42	T.47253	46	0.52747	T.98166	4	28	30 22.0 21.5
33	T.45462	43	T.47299	46	0.52701	T.98162	4	27	40 29.3 28.7
34	T.45504	42	T.47346	47	0.52654	T.98159	3	26	50 36.7 35.8
35	T.45547	43	T.47392	46	0.52608	T.98155	4	25	
36	T.45589	42	T.47438	46	0.52562	T.98151	4	24	
37	T.45632	43	T.47484	46	0.52516	T.98147	4	23	
38	T.45674	42	T.47530	46	0.52470	T.98144	3	22	42 41
39	T.45716	42	T.47576	46	0.52424	T.98140	4	21	6 4.2 4.1
40	T.45758	42	T.47622	46	0.52378	T.98136	4	20	7 4.9 4.8
		43		46			4		8 5.6 5.5
41	T.45801		T.47668		0.52332	T.98132		19	9 6.3 6.2
42	T.45843	42	T.47714	46	0.52286	T.98129	3	18	10 7.0 6.8
43	T.45885	42	T.47760	46	0.52240	T.98125	4	17	20 14.0 13.7
44	T.45927	42	T.47806	46	0.52194	T.98121	4	16	30 21.0 20.5
45	T.45969	42	T.47852	46	0.52148	T.98117	4	15	40 28.0 27.3
46	T.46011	42	T.47897	45	0.52103	T.98113	4	14	50 35.0 34.2
47	T.46053	42	T.47943	46	0.52057	T.98110	3	13	
48	T.46095	41	T.47989	46	0.52011	T.98106	4	12	
49	T.46136	42	T.48035	46	0.51965	T.98102	4	11	
50	T.46178	42	T.48080	45	0.51920	T.98098	4	10	4 3
				46			4		6 0.4 0.3
51	T.46220		T.48126		0.51874	T.98094		9	7 0.5 0.4
52	T.46262	42	T.48171	45	0.51829	T.98090	4	8	8 0.5 0.4
53	T.46303	41	T.48217	46	0.51783	T.98087	3	7	9 0.6 0.5
54	T.46345	42	T.48262	45	0.51738	T.98083	4	6	10 0.7 0.5
55	T.46386	41	T.48307	45	0.51693	T.98079	4	5	20 1.3 1.0
56	T.46428	42	T.48353	46	0.51647	T.98075	4	4	30 2.0 1.5
57	T.46469	41	T.48398	45	0.51602	T.98071	4	3	40 2.7 2.0
58	T.46511	42	T.48443	45	0.51557	T.98067	4	2	50 3.3 2.5
59	T.46552	41	T.48489	46	0.51511	T.98063	4	1	
60	T.46594	42	T.48534	45	0.51466	T.98060	3	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.46594		T.48534		0.51466	T.98060		60	
1	T.46635	41	T.48579	45	0.51421	T.98056	4	59	
2	T.46676	41	T.48624	45	0.51376	T.98052	4	58	45 44
3	T.46717	41	T.48669	45	0.51331	T.98048	4	57	6 4.5 4.4
4	T.46758	41	T.48714	45	0.51286	T.98044	4	56	7 5.3 5.1
5	T.46800	42	T.48759	45	0.51241	T.98040	4	55	8 6.0 5.9
6	T.46841	41	T.48804	45	0.51196	T.98036	4	54	9 6.8 6.6
7	T.46882	41	T.48849	45	0.51151	T.98032	4	53	10 7.5 7.3
8	T.46923	41	T.48894	45	0.51106	T.98029	3	52	20 15.0 14.7
9	T.46964	41	T.48939	45	0.51061	T.98025	4	51	30 22.5 22.0
10	T.47005	41	T.48984	45	0.51016	T.98021	4	50	40 30.0 29.3
		40		45			4		50 37.5 36.7
11	T.47045		T.49029		0.50971	T.98017		49	
12	T.47086	41	T.49073	44	0.50927	T.98013	4	48	
13	T.47127	41	T.49118	45	0.50882	T.98009	4	47	
14	T.47168	41	T.49163	45	0.50837	T.98005	4	46	43
15	T.47209	41	T.49207	44	0.50793	T.98001	4	45	6 4.3
16	T.47249	40	T.49252	45	0.50748	T.97997	4	44	7 5.0
17	T.47290	41	T.49296	44	0.50704	T.97993	4	43	8 5.7
18	T.47330	40	T.49341	45	0.50659	T.97989	4	42	9 6.5
19	T.47371	41	T.49385	44	0.50615	T.97986	3	41	10 7.2
20	T.47411	40	T.49430	45	0.50570	T.97982	4	40	20 14.3
		41		44			4		30 21.5
21	T.47452		T.49474		0.50526	T.97978		39	40 28.7
22	T.47492	40	T.49519	45	0.50481	T.97974	4	38	50 35.8
23	T.47533	41	T.49563	44	0.50437	T.97970	4	37	
24	T.47573	40	T.49607	44	0.50393	T.97966	4	36	
25	T.47613	40	T.49652	45	0.50348	T.97962	4	35	
26	T.47654	41	T.49696	44	0.50304	T.97958	4	34	42 41
27	T.47694	40	T.49740	44	0.50260	T.97954	4	33	6 4.2 4.1
28	T.47734	40	T.49784	44	0.50216	T.97950	4	32	7 4.9 4.8
29	T.47774	40	T.49828	44	0.50172	T.97946	4	31	8 5.6 5.5
30	T.47814	40	T.49872	44	0.50128	T.97942	4	30	9 6.3 6.2
		40		44			4		10 7.0 6.8
31	T.47854		T.49916		0.50084	T.97938		29	20 14.0 13.7
32	T.47894	40	T.49960	44	0.50040	T.97934	4	28	30 21.0 20.5
33	T.47934	40	T.50004	44	0.49996	T.97930	4	27	40 28.0 27.3
34	T.47974	40	T.50048	44	0.49952	T.97926	4	26	50 35.0 34.2
35	T.48014	40	T.50092	44	0.49908	T.97922	4	25	
36	T.48054	40	T.50136	44	0.49864	T.97918	4	24	
37	T.48094	40	T.50180	44	0.49820	T.97914	4	23	
38	T.48133	39	T.50223	43	0.49777	T.97910	4	22	40 39
39	T.48173	40	T.50267	44	0.49733	T.97906	4	21	6 4.0 3.9
40	T.48213	40	T.50311	44	0.49689	T.97902	4	20	7 4.7 4.6
		39		44			4		8 5.3 5.2
41	T.48252		T.50355		0.49645	T.97898		19	9 6.0 5.9
42	T.48292	40	T.50398	43	0.49602	T.97894	4	18	10 6.7 6.5
43	T.48332	40	T.50442	44	0.49558	T.97890	4	17	20 13.3 13.0
44	T.48371	39	T.50485	43	0.49515	T.97886	4	16	30 20.0 19.5
45	T.48411	40	T.50529	44	0.49471	T.97882	4	15	40 26.7 26.0
46	T.48450	39	T.50572	43	0.49428	T.97878	4	14	50 33.3 32.5
47	T.48490	40	T.50616	44	0.49384	T.97874	4	13	
48	T.48529	39	T.50659	43	0.49341	T.97870	4	12	
49	T.48568	39	T.50703	44	0.49297	T.97866	4	11	
50	T.48607	39	T.50746	43	0.49254	T.97861	5	10	5 4 3
		40		43			4		6 0.5 0.4 0.3
51	T.48647		T.50789		0.49211	T.97857		9	7 0.6 0.5 0.4
52	T.48686	39	T.50833	44	0.49167	T.97853	4	8	8 0.7 0.5 0.4
53	T.48725	39	T.50876	43	0.49124	T.97849	4	7	9 0.8 0.6 0.5
54	T.48764	39	T.50919	43	0.49081	T.97845	4	6	10 0.8 0.7 0.5
55	T.48803	39	T.50962	43	0.49038	T.97841	4	5	20 1.7 1.3 1.0
56	T.48842	39	T.51005	43	0.48995	T.97837	4	4	30 2.5 2.0 1.5
57	T.48881	39	T.51048	43	0.48952	T.97833	4	3	40 3.3 2.7 2.0
58	T.48920	39	T.51092	44	0.48908	T.97829	4	2	50 4.2 3.3 2.5
59	T.48959	39	T.51135	43	0.48865	T.97825	4	1	
60	T.48998	39	T.51178	43	0.48822	T.97821	4	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

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′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	1.48998		1.51178		0.48822	1.97821		60	
1	1.49037	39	1.51221	43	0.48779	1.97817	4	59	
2	1.49076	39	1.51264	43	0.48736	1.97812	5	58	
3	1.49115	39	1.51306	42	0.48694	1.97808	4	57	43 | 42
4	1.49153	38	1.51349	43	0.48651	1.97804	4	56	6 | 4.3 | 4.2
5	1.49192	39	1.51392	43	0.48608	1.97800	4	55	7 | 5.0 | 4.9
6	1.49231	39	1.51435	43	0.48565	1.97796	4	54	8 | 5.7 | 5.6
7	1.49269	38	1.51478	43	0.48522	1.97792	4	53	9 | 6.5 | 6.3
8	1.49308	39	1.51520	42	0.48480	1.97788	4	52	10 | 7.2 | 7.0
9	1.49347	39	1.51563	43	0.48437	1.97784	4	51	20 | 14.3 | 14.0
10	1.49385	38	1.51606	43	0.48394	1.97779	5	50	30 | 21.5 | 21.0
		39		42			4		40 | 28.7 | 28.0
11	1.49424		1.51648		0.48352	1.97775	4	49	50 | 35.8 | 35.0
12	1.49462	38	1.51691	43	0.48309	1.97771	4	48	
13	1.49500	38	1.51734	43	0.48266	1.97767	4	47	
14	1.49539	39	1.51776	42	0.48224	1.97763	4	46	
15	1.49577	38	1.51819	43	0.48181	1.97759	4	45	41
16	1.49615	38	1.51861	42	0.48139	1.97754	5	44	6 | 4.1
17	1.49654	39	1.51903	42	0.48097	1.97750	4	43	7 | 4.8
18	1.49692	38	1.51946	43	0.48054	1.97746	4	42	8 | 5.5
19	1.49730	38	1.51988	42	0.48012	1.97742	4	41	9 | 6.2
20	1.49768	38	1.52031	43	0.47969	1.97738	4	40	10 | 6.8
		38		42			4		20 | 13.7
21	1.49806		1.52073		0.47927	1.97734	5	39	30 | 20.5
22	1.49844	38	1.52115	42	0.47885	1.97729	4	38	40 | 27.3
23	1.49882	38	1.52157	42	0.47843	1.97725	4	37	50 | 34.1
24	1.49920	38	1.52200	43	0.47800	1.97721	4	36	
25	1.49958	38	1.52242	42	0.47758	1.97717	4	35	
26	1.49996	38	1.52284	42	0.47716	1.97713	4	34	39 | 38
27	1.50034	38	1.52326	42	0.47674	1.97708	5	33	6 | 3.9 | 3.8
28	1.50072	38	1.52368	42	0.47632	1.97704	4	32	7 | 4.6 | 4.4
29	1.50110	38	1.52410	42	0.47590	1.97700	4	31	8 | 5.2 | 5.1
30	1.50148	37	1.52452	42	0.47548	1.97696	4	30	9 | 5.9 | 5.7
							5		10 | 6.5 | 6.3
31	1.50185	38	1.52494	42	0.47506	1.97691	4	29	20 | 13.0 | 12.7
32	1.50223	38	1.52536	42	0.47464	1.97687	4	28	30 | 19.5 | 19.0
33	1.50261	37	1.52578	42	0.47422	1.97683	4	27	40 | 26.0 | 25.3
34	1.50298	38	1.52620	42	0.47380	1.97679	4	26	50 | 32.5 | 31.7
35	1.50336	38	1.52661	41	0.47339	1.97674	5	25	
36	1.50374	37	1.52703	42	0.47297	1.97670	4	24	
37	1.50411	38	1.52745	42	0.47255	1.97666	4	23	
38	1.50449	37	1.52787	42	0.47213	1.97662	4	22	37 | 36
39	1.50486	37	1.52829	42	0.47171	1.97657	5	21	6 | 3.7 | 3.6
40	1.50523	38	1.52870	41	0.47130	1.97653	4	20	7 | 4.3 | 4.2
				42			4		8 | 4.9 | 4.8
41	1.50561		1.52912		0.47088	1.97649		19	9 | 5.6 | 5.4
42	1.50598	37	1.52953	41	0.47047	1.97645	4	18	10 | 6.2 | 6.0
43	1.50635	37	1.52995	42	0.47005	1.97640	5	17	20 | 12.3 | 12.0
44	1.50673	38	1.53037	42	0.46963	1.97636	4	16	30 | 18.5 | 18.0
45	1.50710	37	1.53078	41	0.46922	1.97632	4	15	40 | 24.7 | 24.0
46	1.50747	37	1.53120	42	0.46880	1.97628	4	14	50 | 30.8 | 30.0
47	1.50784	37	1.53161	41	0.46839	1.97623	5	13	
48	1.50821	37	1.53202	41	0.46798	1.97619	4	12	
49	1.50858	37	1.53244	42	0.46756	1.97615	4	11	
50	1.50896	38	1.53285	41	0.46715	1.97610	5	10	5 | 4
		37		42			4		6 | 0.5 | 0.4
51	1.50933		1.53327		0.46673	1.97606		9	7 | 0.6 | 0.5
52	1.50970	37	1.53368	41	0.46632	1.97602	4	8	8 | 0.7 | 0.5
53	1.51007	37	1.53409	41	0.46591	1.97597	5	7	9 | 0.8 | 0.6
54	1.51043	36	1.53450	41	0.46550	1.97593	4	6	10 | 0.8 | 0.7
55	1.51080	37	1.53492	42	0.46508	1.97589	4	5	20 | 1.7 | 1.3
56	1.51117	37	1.53533	41	0.46467	1.97584	5	4	30 | 2.5 | 2.0
57	1.51154	37	1.53574	41	0.46426	1.97580	4	3	40 | 3.3 | 2.7
58	1.51191	37	1.53615	41	0.46385	1.97576	4	2	50 | 4.2 | 3.3
59	1.51227	36	1.53656	41	0.46344	1.97571	5	1	
60	1.51264	37	1.53697	41	0.46303	1.97567	4	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		′
0	T.51264		T.53697		0.46303	T.97567		60	0
1	T.51301	37	T.53738	41	0.46262	T.97563	4	59	1
2	T.51338	37	T.53779	41	0.46221	T.97558	5	58	2
3	T.51374	36	T.53820	41	0.46180	T.97554	4	57	3
4	T.51411	37	T.53861	41	0.46139	T.97550	4	56	4
5	T.51447	36	T.53902	41	0.46098	T.97545	5	55	5
6	T.51484	37	T.53943	41	0.46057	T.97541	4	54	6
7	T.51520	36	T.53984	41	0.46016	T.97536	5	53	7
8	T.51557	37	T.54025	40	0.45975	T.97532	4	52	8
9	T.51593	36	T.54065	41	0.45935	T.97528	4	51	9
10	T.51629	36	T.54106	41	0.45894	T.97523	5	50	10
		37					4		
11	T.51666		T.54147	40	0.45853	T.97519	4	49	11
12	T.51702	36	T.54187	41	0.45813	T.97515	5	48	12
13	T.51738	36	T.54228	41	0.45772	T.97510	4	47	13
14	T.51774	36	T.54269	40	0.45731	T.97506	5	46	14
15	T.51811	37	T.54309	41	0.45691	T.97501	4	45	15
16	T.51847	36	T.54350	40	0.45650	T.97497	5	44	16
17	T.51883	36	T.54390	41	0.45610	T.97492	4	43	17
18	T.51919	36	T.54431	40	0.45569	T.97488	4	42	18
19	T.51955	36	T.54471	41	0.45529	T.97484	4	41	19
20	T.51991	36	T.54512	40	0.45488	T.97479	5	40	20
		36					4		
21	T.52027		T.54552	41	0.45448	T.97475	5	39	21
22	T.52063	36	T.54593	40	0.45407	T.97470	4	38	22
23	T.52099	36	T.54633	40	0.45367	T.97466	4	37	23
24	T.52135	36	T.54673	41	0.45327	T.97461	5	36	24
25	T.52171	36	T.54714	40	0.45286	T.97457	4	35	25
26	T.52207	36	T.54754	40	0.45246	T.97453	4	34	26
27	T.52242	35	T.54794	41	0.45206	T.97448	5	33	27
28	T.52278	36	T.54835	40	0.45165	T.97444	4	32	28
29	T.52314	36	T.54875	40	0.45125	T.97439	5	31	29
30	T.52350	36	T.54915	40	0.45085	T.97435	4	30	30
		35					5		
31	T.52385		T.54955	40	0.45045	T.97430	4	29	31
32	T.52421	36	T.54995	40	0.45005	T.97426	5	28	32
33	T.52456	35	T.55035	40	0.44965	T.97421	4	27	33
34	T.52492	36	T.55075	40	0.44925	T.97417	4	26	34
35	T.52527	35	T.55115	40	0.44885	T.97412	5	25	35
36	T.52563	36	T.55155	40	0.44845	T.97408	4	24	36
37	T.52598	35	T.55195	40	0.44805	T.97403	5	23	37
38	T.52634	36	T.55235	40	0.44765	T.97399	4	22	38
39	T.52669	35	T.55275	40	0.44725	T.97394	5	21	39
40	T.52705	36	T.55315	40	0.44685	T.97390	4	20	40
		35					5		
41	T.52740		T.55355	40	0.44645	T.97385	4	19	41
42	T.52775	35	T.55395	39	0.44605	T.97381	5	18	42
43	T.52811	36	T.55434	40	0.44566	T.97376	4	17	43
44	T.52846	35	T.55474	40	0.44526	T.97372	5	16	44
45	T.52881	35	T.55514	40	0.44486	T.97367	4	15	45
46	T.52916	35	T.55554	39	0.44446	T.97363	5	14	46
47	T.52951	35	T.55593	40	0.44407	T.97358	5	13	47
48	T.52986	35	T.55633	40	0.44367	T.97353	4	12	48
49	T.53021	35	T.55673	39	0.44327	T.97349	5	11	49
50	T.53056	35	T.55712	40	0.44288	T.97344	5	10	50
		36					4		
51	T.53092		T.55752	39	0.44248	T.97340	5	9	51
52	T.53126	34	T.55791	40	0.44209	T.97335	4	8	52
53	T.53161	35	T.55831	39	0.44169	T.97331	5	7	53
54	T.53196	35	T.55870	40	0.44130	T.97326	4	6	54
55	T.53231	35	T.55910	39	0.44090	T.97322	5	5	55
56	T.53266	35	T.55949	40	0.44051	T.97317	5	4	56
57	T.53301	35	T.55989	39	0.44011	T.97312	4	3	57
58	T.53336	35	T.56028	39	0.43972	T.97308	5	2	58
59	T.53370	34	T.56067	40	0.43933	T.97303	4	1	59
60	T.53405	35	T.56107		0.43893	T.97299		0	60
	log cos	d	log cot	c. d.	log tan	log sin	d	′	

p. p.

	41	40
6	4.1	4.0
7	4.8	4.7
8	5.5	5.3
9	6.2	6.0
10	6.8	6.7
20	13.7	13.3
30	20.5	20.0
40	27.3	26.7
50	34.2	33.3

	39
6	3.9
7	4.6
8	5.2
9	5.9
10	6.5
20	13.0
30	19.5
40	26.0
50	32.5

	37	36
6	3.7	3.6
7	4.3	4.2
8	4.9	4.8
9	5.6	5.4
10	6.2	6.0
20	12.3	12.0
30	18.5	18.0
40	24.7	24.0
50	30.8	30.0

	35	34
6	3.5	3.4
7	4.1	4.0
8	4.7	4.5
9	5.3	5.1
10	5.8	5.7
20	11.7	11.3
30	17.5	17.0
40	23.3	22.7
50	29.2	28.3

	5	4
6	0.5	0.4
7	0.6	0.5
8	0.7	0.5
9	0.8	0.6
10	0.8	0.7
20	1.7	1.3
30	2.5	2.0
40	3.3	2.7
50	4.2	3.3

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.53405		T.56107		0.43893	T.97299		60	
1	T.53440	35	T.56146	39	0.43854	T.97294	5	59	
2	T.53475	35	T.56185	39	0.43815	T.97289	5	58	
3	T.53509	34	T.56224	39	0.43776	T.97285	4	57	40 39
4	T.53544	35	T.56264	40	0.43736	T.97280	5	56	6 4.0 3.9
5	T.53578	34	T.56303	39	0.43697	T.97276	4	55	7 4.7 4.6
6	T.53613	35	T.56342	39	0.43658	T.97271	5	54	8 5.3 5.2
7	T.53647	34	T.56381	39	0.43619	T.97266	5	53	9 6.0 5.9
8	T.53682	35	T.56420	39	0.43580	T.97262	4	52	10 6.7 6.5
9	T.53716	34	T.56459	39	0.43541	T.97257	5	51	20 13.3 13.0
10	T.53751	35	T.56498	39	0.43502	T.97252	5	50	30 20.0 19.5
		34					4		40 26.7 26.0
11	T.53785		T.56537		0.43463	T.97248		49	50 33.3 32.5
12	T.53819	34	T.56576	39	0.43424	T.97243	5	48	
13	T.53854	35	T.56615	39	0.43385	T.97238	5	47	
14	T.53888	34	T.56654	39	0.43346	T.97234	4	46	38 37
15	T.53922	34	T.56693	39	0.43307	T.97229	5	45	6 3.8 3.7
16	T.53957	35	T.56732	39	0.43268	T.97224	5	44	7 4.4 4.3
17	T.53991	34	T.56771	39	0.43229	T.97220	4	43	8 5.1 4.9
18	T.54025	34	T.56810	39	0.43190	T.97215	5	42	9 5.7 5.6
19	T.54059	34	T.56849	39	0.43151	T.97210	5	41	10 6.3 6.2
20	T.54093	34	T.56887	38	0.43113	T.97206	4	40	20 12.7 12.3
		34		39			5		30 19.0 18.5
21	T.54127		T.56926		0.43074	T.97201		39	40 25.3 24.7
22	T.54161	34	T.56965	39	0.43035	T.97196	5	38	50 31.7 30.8
23	T.54195	34	T.57004	39	0.42996	T.97192	4	37	
24	T.54229	34	T.57042	38	0.42958	T.97187	5	36	
25	T.54263	34	T.57081	39	0.42919	T.97182	5	35	
26	T.54297	34	T.57120	39	0.42880	T.97178	4	34	35
27	T.54331	34	T.57158	38	0.42842	T.97173	5	33	6 3.5
28	T.54365	34	T.57197	39	0.42803	T.97168	5	32	7 4.1
29	T.54399	34	T.57235	38	0.42765	T.97163	5	31	8 4.7
30	T.54433	34	T.57274	39	0.42726	T.97159	4	30	9 5.3
		33		38			5		10 5.8
31	T.54466		T.57312		0.42688	T.97154		29	20 11.7
32	T.54500	34	T.57351	39	0.42649	T.97149	5	28	30 17.5
33	T.54534	34	T.57389	38	0.42611	T.97145	4	27	40 23.3
34	T.54567	33	T.57428	39	0.42572	T.97140	5	26	50 29.2
35	T.54601	34	T.57466	38	0.42534	T.97135	5	25	
36	T.54635	34	T.57504	38	0.42496	T.97130	5	24	
37	T.54668	33	T.57543	39	0.42457	T.97126	4	23	
38	T.54702	34	T.57581	38	0.42419	T.97121	5	22	34 33
39	T.54735	33	T.57619	38	0.42381	T.97116	5	21	6 3.4 3.3
40	T.54769	34	T.57658	39	0.42342	T.97111	5	20	7 4.0 3.9
		33		38			4		8 4.5 4.4
41	T.54802		T.57696		0.42304	T.97107		19	9 5.1 5.0
42	T.54836	34	T.57734	38	0.42266	T.97102	5	18	10 5.7 5.5
43	T.54869	33	T.57772	38	0.42228	T.97097	5	17	20 11.3 11.0
44	T.54903	34	T.57810	38	0.42190	T.97092	5	16	30 17.0 16.5
45	T.54936	33	T.57849	39	0.42151	T.97087	5	15	40 22.7 22.0
46	T.54969	33	T.57887	38	0.42113	T.97083	4	14	50 28.3 27.5
47	T.55003	34	T.57925	38	0.42075	T.97078	5	13	
48	T.55036	33	T.57963	38	0.42037	T.97073	5	12	
49	T.55069	33	T.58001	38	0.41999	T.97068	5	11	
50	T.55102	33	T.58039	38	0.41961	T.97063	5	10	5 4
		34					4		6 0.5 0.4
51	T.55136		T.58077		0.41923	T.97059		9	7 0.6 0.5
52	T.55169	33	T.58115	38	0.41885	T.97054	5	8	8 0.7 0.5
53	T.55202	33	T.58153	38	0.41847	T.97049	5	7	9 0.8 0.6
54	T.55235	33	T.58191	38	0.41809	T.97044	5	6	10 0.8 0.7
55	T.55268	33	T.58229	38	0.41771	T.97039	5	5	20 1.7 1.3
56	T.55301	33	T.58267	38	0.41733	T.97035	4	4	30 2.5 2.0
57	T.55334	33	T.58304	37	0.41696	T.97030	5	3	40 3.3 2.7
58	T.55367	33	T.58342	38	0.41658	T.97025	5	2	50 4.2 3.3
59	T.55400	33	T.58380	38	0.41620	T.97020	5	1	
60	T.55433	33	T.58418	38	0.41582	T.97015	5	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.55433		T.58418		0.41582	T.97015		60	
		33		37			5		
1	T.55466		T.58455		0.41545	T.97010		59	
		33		38			5		
2	T.55499		T.58493		0.41507	T.97005		58	
		33		38			4		38 37
3	T.55532		T.58531		0.41469	T.97001		57	
		32		38			5		6 3.8 3.7
4	T.55564		T.58569		0.41431	T.96996		56	
		33		37			5		7 4.4 4.3
5	T.55597		T.58606		0.41394	T.96991		55	
		33		38			5		8 5.1 4.9
6	T.55630		T.58644		0.41356	T.96986		54	
		33		37			5		9 5.7 5.6
7	T.55663		T.58681		0.41319	T.96981		53	
		32		38			5		10 6.3 6.2
8	T.55695		T.58719		0.41281	T.96976		52	
		33		38			5		20 12.7 12.3
9	T.55728		T.58757		0.41243	T.96971		51	
		33		37			5		30 19.0 18.5
10	T.55761		T.58794		0.41206	T.96966		50	
		32		38			4		40 25.3 24.7
									50 31.7 30.8
11	T.55793		T.58832		0.41168	T.96962		49	
		33		37			5		
12	T.55826		T.58869		0.41131	T.96957		48	
		32		38			5		
13	T.55858		T.58907		0.41093	T.96952		47	
		33		37			5		36 33
14	T.55891		T.58944		0.41056	T.96947		46	
		32		37			5		6 3.6 3.3
15	T.55923		T.58981		0.41019	T.96942		45	
		33		38			5		7 4.2 3.9
16	T.55956		T.59019		0.40981	T.96937		44	
		32		37			5		8 4.8 4.4
17	T.55988		T.59056		0.40944	T.96932		43	
		33		38			5		9 5.4 5.0
18	T.56021		T.59094		0.40906	T.96927		42	
		32		37			5		10 6.0 5.5
19	T.56053		T.59131		0.40869	T.96922		41	
		32		37			5		20 12.0 11.0
20	T.56085		T.59168		0.40832	T.96917		40	
		33		37			5		30 18.0 16.5
21	T.56118		T.59205		0.40795	T.96912		39	40 24.0 22.0
		32		38			5		50 30.0 27.5
22	T.56150		T.59243		0.40757	T.96907		38	
		32		37			4		
23	T.56182		T.59280		0.40720	T.96903		37	
		33		37			5		
24	T.56215		T.59317		0.40683	T.96898		36	
		32		37			5		
25	T.56247		T.59354		0.40646	T.96893		35	
		32		37			5		
26	T.56279		T.59391		0.40609	T.96888		34	32
		32		38			5		
27	T.56311		T.59429		0.40571	T.96883		33	6 3.2
		32		37			5		
28	T.56343		T.59466		0.40534	T.96878		32	7 3.7
		32		37			5		
29	T.56375		T.59503		0.40497	T.96873		31	8 4.3
		33		37			5		
30	T.56408		T.59540		0.40460	T.96868		30	9 4.8
		32		37			5		10 5.3
31	T.56440		T.59577		0.40423	T.96863		29	20 10.7
		32		37			5		
32	T.56472		T.59614		0.40386	T.96858		28	30 16.0
		32		37			5		
33	T.56504		T.59651		0.40349	T.96853		27	40 21.3
		32		37			5		
34	T.56536		T.59688		0.40312	T.96848		26	50 26.7
		32		37			5		
35	T.56568		T.59725		0.40275	T.96843		25	
		31		37			5		
36	T.56599		T.59762		0.40238	T.96838		24	
		32		37			5		
37	T.56631		T.59799		0.40201	T.96833		23	31 6
		32		36			5		
38	T.56663		T.59835		0.40165	T.96828		22	
		32		37			5		6 3.1 0.6
39	T.56695		T.59872		0.40128	T.96823		21	
		32		37			5		7 3.6 0.7
40	T.56727		T.59909		0.40091	T.96818		20	
		32		37			5		8 4.1 0.8
41	T.56759		T.59946		0.40054	T.96813		19	9 4.7 0.9
		31		37			5		
42	T.56790		T.59983		0.40017	T.96808		18	10 5.2 1.0
		32		36			5		
43	T.56822		T.60019		0.39981	T.96803		17	20 10.3 2.0
		32		37			5		
44	T.56854		T.60056		0.39944	T.96798		16	30 15.5 3.0
		32		37			5		
45	T.56886		T.60093		0.39907	T.96793		15	40 20.7 4.0
		31		37			5		
46	T.56917		T.60130		0.39870	T.96788		14	50 25.8 5.0
		32		36			5		
47	T.56949		T.60166		0.39834	T.96783		13	
		31		37			5		
48	T.56980		T.60203		0.39797	T.96778		12	
		32		37			6		
49	T.57012		T.60240		0.39760	T.96772		11	
		32		36			5		
50	T.57044		T.60276		0.39724	T.96767		10	5 4
		31		37			5		6 0.5 0.4
51	T.57075		T.60313		0.39687	T.96762		9	7 0.6 0.5
		32		36			5		
52	T.57107		T.60349		0.39651	T.96757		8	8 0.7 0.5
		31		37			5		
53	T.57138		T.60386		0.39614	T.96752		7	9 0.8 0.6
		31		36			5		
54	T.57169		T.60422		0.39578	T.96747		6	10 0.8 0.7
		32		37			5		
55	T.57201		T.60459		0.39541	T.96742		5	20 1.7 1.3
		31		36			5		
56	T.57232		T.60495		0.39505	T.96737		4	30 2.5 2.0
		32		37			5		
57	T.57264		T.60532		0.39468	T.96732		3	40 3.3 2.7
		31		36			5		
58	T.57295		T.60568		0.39432	T.96727		2	50 4.2 3.3
		31		37			5		
59	T.57326		T.60605		0.39395	T.96722		1	
		32		36			5		
60	T.57358		T.60641		c.39359	T.96717		0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.57188	31	T.60641	36	0.39359	T.96717	6	60	
1	T.57389	31	T.60677	37	0.39323	T.96711	5	59	
2	T.57420	31	T.60714	36	0.39286	T.96706	5	58	37 | 36
3	T.57451	31	T.60750	36	0.39250	T.96701	5	57	6 | 3.7 | 3.6
4	T.57482	32	T.60786	37	0.39214	T.96696	5	56	7 | 4.3 | 4.2
5	T.57514	31	T.60823	36	0.39177	T.96691	5	55	8 | 4.9 | 4.8
6	T.57545	31	T.60859	36	0.39141	T.96686	5	54	9 | 5.6 | 5.4
7	T.57576	31	T.60895	36	0.39105	T.96681	5	53	10 | 6.2 | 6.0
8	T.57607	31	T.60931	36	0.39069	T.96676	5	52	20 | 12.3 | 12.0
9	T.57638	31	T.60967	37	0.39033	T.96670	6	51	30 | 18.5 | 18.0
10	T.57669	31	T.61004	36	0.38996	T.96665	5	50	40 | 24.7 | 24.0
11	T.57700	31	T.61040	36	0.38960	T.96660	5	49	50 | 30.8 | 30.0
12	T.57731	31	T.61076	36	0.38924	T.96655	5	48	
13	T.57762	31	T.61112	36	0.38888	T.96650	5	47	
14	T.57793	31	T.61148	36	0.38852	T.96645	5	46	35
15	T.57824	31	T.61184	36	0.38816	T.96640	6	45	6 | 3.5
16	T.57855	30	T.61220	36	0.38780	T.96634	5	44	7 | 4.1
17	T.57885	31	T.61256	36	0.38744	T.96629	5	43	8 | 4.7
18	T.57916	31	T.61292	36	0.38708	T.96624	5	42	9 | 5.3
19	T.57947	31	T.61328	36	0.38672	T.96619	5	41	10 | 5.8
20	T.57978	30	T.61364	36	0.38636	T.96614	5	40	20 | 11.7
21	T.58008	31	T.61400	36	0.38600	T.96608	6	39	30 | 17.5
22	T.58039	31	T.61436	36	0.38564	T.96603	5	38	40 | 23.3
23	T.58070	31	T.61472	36	0.38528	T.96598	5	37	50 | 29.2
24	T.58101	30	T.61508	36	0.38492	T.96593	5	36	
25	T.58131	31	T.61544	36	0.38456	T.96588	5	35	
26	T.58162	30	T.61579	35	0.38421	T.96582	6	34	32 | 31
27	T.58192	31	T.61615	36	0.38385	T.96577	5	33	6 | 3.2 | 3.1
28	T.58223	30	T.61651	36	0.38349	T.96572	5	32	7 | 3.7 | 3.6
29	T.58253	31	T.61687	36	0.38313	T.96567	5	31	8 | 4.3 | 4.1
30	T.58284	30	T.61722	35	0.38278	T.96562	5	30	9 | 4.8 | 4.7
31	T.58314	31	T.61758	36	0.38242	T.96556	6	29	10 | 5.3 | 5.2
32	T.58345	30	T.61794	36	0.38206	T.96551	5	28	20 | 10.7 | 10.3
33	T.58375	31	T.61830	36	0.38170	T.96546	5	27	30 | 16.0 | 15.5
34	T.58406	30	T.61865	35	0.38135	T.96541	5	26	40 | 21.3 | 20.7
35	T.58436	31	T.61901	36	0.38099	T.96535	6	25	50 | 26.7 | 25.8
36	T.58467	30	T.61936	35	0.38064	T.96530	5	24	
37	T.58497	30	T.61972	36	0.38028	T.96525	5	23	
38	T.58527	30	T.62008	36	0 37992	T.96520	5	22	30 | 29
39	T.58557	30	T.62043	35	0.37957	T.96514	6	21	6 | 3.0 | 2.9
40	T.58588	31	T.62079	36	0.37921	T.96509	5	20	7 | 3.5 | 3.4
41	T.58618	30	T.62114	35	0.37886	T.96504	5	19	8 | 4.0 | 3.9
42	T.58648	30	T.62150	36	0.37850	T.96498	6	18	9 | 4.5 | 4.4
43	T.58678	30	T.62185	35	0.37815	T.96493	5	17	10 | 5.0 | 4.8
44	T.58709	31	T.62221	36	0.37779	T.96488	5	16	20 | 10.0 | 9.7
45	T.58739	30	T.62256	35	0.37744	T.96483	5	15	30 | 15.0 | 14.5
46	T.58769	30	T.62292	36	0.37708	T.96477	6	14	40 | 20.0 | 19.3
47	T.58799	30	T.62327	35	0.37673	T.96472	5	13	50 | 25.0 | 24.2
48	T.58829	30	T.62362	35	0.37638	T.96467	5	12	
49	T.58859	30	T.62398	36	0.37602	T.96461	6	11	
50	T.58889	30	T.62433	35	0.37567	T.96456	5	10	6 | 5
51	T.58919	30	T.62468	35	0.37532	T.96451	5	9	6 | 0.6 | 0.5
52	T.58949	30	T.62504	36	0.37496	T.96445	6	8	7 | 0.7 | 0.6
53	T.58979	30	T.62539	35	0.37461	T.96440	5	7	8 | 0.8 | 0.7
54	T.59009	30	T.62574	35	0.37426	T.96435	5	6	9 | 0 9 | 0.8
55	T.59039	30	T.62609	35	0.37391	T.96429	6	5	10 | 1.0 | 0.8
56	T.59069	30	T.62645	36	0.37355	T.96424	5	4	20 | 2.0 | 1.7
57	T.59098	29	T.62680	35	0.37320	T.96419	5	3	30 | 3.0 | 2.5
58	T.59128	30	T.62715	35	0.37285	T.96413	6	2	40 | 4.0 | 3.3
59	T.59158	30	T.62750	35	0.37250	T.96408	5	1	50 | 5.0 | 4.2
60	T.59188	30	T.62785	35	0.37215	T.96403	5	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.59188		T.62785		0.37215	T.96403		60	
1	T.59218	30	T.62820	35	0.37180	T.96397	6	59	
2	T.59247	29	T.62855	35	0.37145	T.96392	5	58	
3	T.59277	30	T.62890	35	0.37110	T.96387	5	57	36 35
4	T.59307	29	T.62926	30	0.37074	T.96381	6	56	6 3.6 3.5
5	T.59336	30	T.62961	35	0.37039	T.96376	5	55	7 4.2 4.1
6	T.59366	30	T.62996	35	0.37004	T.96370	6	54	8 4.8 4.7
7	T.59396	29	T.63031	35	0.36969	T.96365	5	53	9 5.4 5.3
8	T.59425	30	T.63066	35	0.36934	T.96360	5	52	10 6.0 5.8
9	T.59455	29	T.63101	35	0.36899	T.96354	6	51	20 12.0 11.7
10	T.59484		T.63135	34	0.36865	T.96349	5	50	30 18.0 17.5
		30		35			6		40 24.0 23.3
11	T.59514		T.63170		0.36830	T.96343		49	50 30.0 29.2
12	T.59543	29	T.63205	35	0.36795	T.96338	5	48	
13	T.59573	30	T.63240	35	0.36760	T.96333	5	47	
14	T.59602	29	T.63275	35	0.36725	T.96327	6	46	34
15	T.59632	30	T.63310	35	0.36690	T.96322	5	45	6 3.4
16	T.59661	29	T.63345	35	0.36655	T.96316	6	44	7 4.0
17	T.59690	29	T.63379	34	0.36621	T.96311	5	43	8 4.5
18	T.59720	30	T.63414	35	0.36586	T.96305	6	42	9 5.1
19	T.59749	29	T.63449	35	0.36551	T.96300	5	41	10 5.7
20	T.59778	29	T.63484	35	0.36516	T.96294	6	40	20 11.3
		30		35			5		30 17.0
21	T.59808		T.63519		0.36481	T.96289		39	40 22.7
22	T.59837	29	T.63553	34	0.36447	T.96284	5	38	50 28.3
23	T.59866	29	T.63588	35	0.36412	T.96278	6	37	
24	T.59895	29	T.63623	35	0.36377	T.96273	5	36	
25	T.59924	29	T.63657	34	0.36343	T.96267	6	35	
26	T.59954	30	T.63692	35	0.36308	T.96262	5	34	30 29
27	T.59983	29	T.63726	34	0.36274	T.96256	6	33	6 3.0 2.9
28	T.60012	29	T.63761	35	0.36239	T.96251	5	32	7 3.5 3.4
29	T.60041	29	T.63796	35	0.36204	T.96245	6	31	8 4.0 3.9
30	T.60070	29	T.63830	34	0.36170	T.96240	5	30	9 4.5 4.4
				35			6		10 5.0 4.8
31	T.60099		T.63865		0.36135	T.96234		29	20 10.0 9.7
32	T.60128	29	T.63899	34	0.36101	T.96229	5	28	30 15.0 14.5
33	T.60157	29	T.63934	35	0.36066	T.96223	6	27	40 20.0 19 3
34	T.60186	29	T.63968	34	0.36032	T.96218	5	26	50 25.0 24.2
35	T.60215	29	T.64003	35	0.35997	T.96212	6	25	
36	T.60244	29	T.64037	34	0.35963	T.96207	5	24	
37	T.60273	29	T.64072	35	0.35928	T.96201	6	23	
38	T.60302	29	T.64106	34	0.35894	T.96196	5	22	28
39	T.60331	28	T.64140	34	0.35860	T.96190	6	21	6 2.8
40	T.60359	29	T.64175	35	0.35825	T.96185	5	20	7 3.3
				34			6		8 3.7
41	T.60388		T.64209		0.35791	T.96179		19	9 4.2
42	T.60417	29	T.64243	34	0.35757	T.96174	5	18	10 4.7
43	T.60446	29	T.64278	35	0.35722	T.96168	6	17	20 9.3
44	T.60474	28	T.64312	34	0.35688	T.96162	6	16	30 14.0
45	T.60503	29	T.64346	34	0.35654	T.96157	5	15	40 18.7
46	T.60532	29	T.64381	35	0.35619	T.96151	6	14	50 23.3
47	T.60561	29	T.64415	34	0.35585	T.96146	5	13	
48	T.60589	28	T.64449	34	0.35551	T.96140	6	12	
49	T.60618	29	T.64483	34	0.35517	T.96135	5	11	
50	T.60646	28	T.64517	34	0.35483	T.96129	6	10	6 5
		29		35			6		6 0.6 0.5
51	T.60675		T.64552		0.35448	T.96123		9	7 0.7 0.6
52	T.60704	29	T.64586	34	0.35414	T.96118	5	8	8 0.8 0.7
53	T.60732	28	T.64620	34	0.35380	T.96112	6	7	9 0.9 0.8
54	T.60761	29	T.64654	34	0.35346	T.96107	6	6	10 1.0 0.8
55	T.60789	28	T.64688	34	0.35312	T.96101	6	5	20 2.0 1.7
56	T.60818	29	T.64722	34	0.35278	T.96095	6	4	30 3.0 2.5
57	T.60846	28	T.64756	34	0.35244	T.96090	5	3	40 4.0 3.3
58	T.60875	29	T.64790	34	0.35210	T.96084	6	2	50 5.0 4.2
59	T.60903	28	T.64824	34	0.35176	T.96079	5	1	
60	T.60931	28	T.64858	34	0.35142	T.96073	6	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.60931		T.64858		0.35142	T.96073		60	
1	T.60960	29	T.64892	34	0.35108	T.96067	6	59	
2	T.60988	28	T.64926	34	0.35074	T.96062	5	58	34 | 33
3	T.61016	28	T.64960	34	0.35040	T.96056	6	57	6 | 3.4 | 3.3
4	T.61045	29	T.64994	34	0.35006	T.96050	6	56	7 | 4.0 | 3.9
5	T.61073	28	T.65028	34	0.34972	T.96045	5	55	8 | 4.5 | 4.4
6	T.61101	28	T.65062	34	0.34938	T.96039	6	54	9 | 5.1 | 5.0
7	T.61129	28	T.65096	34	0.34904	T.96034	5	53	10 | 5.7 | 5.5
8	T.61158	29	T.65130	34	0.34870	T.96028	6	52	20 | 11.3 | 11.0
9	T.61186	28	T.65164	34	0.34836	T.96022	6	51	30 | 17.0 | 16.5
10	T.61214	28	T.65197	33	0.34803	T.96017	5	50	40 | 22.7 | 22.0
11	T.61242		T.65231	34	0.34769	T.96011	6	49	50 | 28.3 | 27.5
12	T.61270	28	T.65265	34	0.34735	T.96005	6	48	
13	T.61298	28	T.65299	34	0.34701	T.96000	5	47	
14	T.61326	28	T.65333	34	0.34667	T.95994	6	46	29
15	T.61354	28	T.65366	33	0.34634	T.95988	6	45	6 | 2.9
16	T.61382	28	T.65400	34	0.34600	T.95982	6	44	7 | 3.4
17	T.61411	29	T.65434	34	0.34566	T.95977	5	43	8 | 3.9
18	T.61438	27	T.65467	33	0.34533	T.95971	6	42	9 | 4.4
19	T.61466	28	T.65501	34	0.34499	T.95965	6	41	10 | 4.8
20	T.61494	28	T.65535	34	0.34465	T.95960	5	40	20 | 9.7
21	T.61522	28	T.65568	33	0.34432	T.95954	6	39	30 | 14.5
22	T.61550	28	T.65602	34	0.34398	T.95948	6	38	40 | 19.3
23	T.61578	28	T.65636	34	0.34364	T.95942	6	37	50 | 24.2
24	T.61606	28	T.65669	33	0.34331	T.95937	5	36	
25	T.61634	28	T.65703	34	0.34297	T.95931	6	35	28
26	T.61662	28	T.65736	33	0.34264	T.95925	6	34	6 | 2.8
27	T.61689	27	T.65770	34	0.34230	T.95920	5	33	7 | 3.3
28	T.61717	28	T.65803	33	0.34197	T.95914	6	32	8 | 3.7
29	T.61745	28	T.65837	34	0.34163	T.95908	6	31	9 | 4.2
30	T.61773	28	T.65870	33	0.34130	T.95902	6	30	10 | 4.7
31	T.61800	27	T.65904	34	0.34096	T.95897	5	29	20 | 9.3
32	T.61828	28	T.65937	33	0.34063	T.95891	6	28	30 | 14.0
33	T.61856	28	T.65971	34	0.34029	T.95885	6	27	40 | 18.7
34	T.61883	27	T.66004	33	0.33996	T.95879	6	26	50 | 23.3
35	T.61911	28	T.66038	34	0.33962	T.95873	6	25	
36	T.61939	28	T.66071	33	0.33929	T.95868	5	24	
37	T.61966	27	T.66104	33	0.33896	T.95862	6	23	27
38	T.61994	28	T.66138	34	0.33862	T.95856	6	22	6 | 2.7
39	T.62021	27	T.66171	33	0.33829	T.95850	6	21	7 | 3.2
40	T.62049	28	T.66204	33	0.33796	T.95844	6	20	8 | 3.6
41	T.62076	27	T.66238	34	0.33762	T.95839	5	19	9 | 4.1
42	T.62104	28	T.66271	33	0.33729	T.95833	6	18	10 | 4.5
43	T.62131	27	T.66304	33	0.33696	T.95827	6	17	20 | 9.0
44	T.62159	28	T.66337	33	0.33663	T.95821	6	16	30 | 13.5
45	T.62186	27	T.66371	34	0.33629	T.95815	6	15	40 | 18.0
46	T.62214	28	T.66404	33	0.33596	T.95810	5	14	50 | 22.5
47	T.62241	27	T.66437	33	0.33563	T.95804	6	13	
48	T.62268	27	T.66470	33	0.33530	T.95798	6	12	
49	T.62296	28	T.66503	33	0.33497	T.95792	6	11	
50	T.62323	27	T.66537	34	0.33463	T.95786	6	10	6 | 5
51	T.62350	27	T.66570	33	0.33430	T.95780	6	9	6 | 0.6 | 0.5
52	T.62377	27	T.66603	33	0.33397	T.95775	5	8	7 | 0.7 | 0.6
53	T.62405	28	T.66636	33	0.33364	T.95769	6	7	8 | 0.8 | 0.7
54	T.62432	27	T.66669	33	0.33331	T.95763	6	6	9 | 0.9 | 0.8
55	T.62459	27	T.66702	33	0.33298	T.95757	6	5	10 | 1.0 | 0.8
56	T.62486	27	T.66735	33	0.33265	T.95751	6	4	20 | 2.0 | 1.7
57	T.62513	27	T.66768	33	0.33232	T.95745	6	3	30 | 3.0 | 2.5
58	T.62541	28	T.66801	33	0.33199	T.95739	6	2	40 | 4.0 | 3.3
59	T.62568	27	T.66834	33	0.33166	T.95733	6	1	50 | 5.0 | 4.2
60	T.62595	27	T.66867	33	0.33133	T.95728	5	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.62595	27	T.66867	33	0.33133	T.95728	6	60	
1	T.62622	27	T.66900	33	0.33100	T.95722	6	59	
2	T.62649	27	T.66933	33	0.33067	T.95716	6	58	33 | 32
3	T.62676	27	T.66966	33	0.33034	T.95710	6	57	6 | 3.3 | 3.2
4	T.62703	27	T.66999	33	0.33001	T.95704	6	56	7 | 3.9 | 3.7
5	T.62730	27	T.67032	33	0.32968	T.95698	6	55	8 | 4.4 | 4.3
6	T.62757	27	T.67065	33	0.32935	T.95692	6	54	9 | 5.0 | 4.8
7	T.62784	27	T.67098	33	0.32902	T.95686	6	53	10 | 5.5 | 5.3
8	T.62811	27	T.67131	32	0.32869	T.95680	6	52	20 | 11.0 | 10.7
9	T.62838	27	T.67163	33	0.32837	T.95674	6	51	30 | 16.5 | 16.0
10	T.62865	27	T.67196	33	0.32804	T.95668	6	50	40 | 22.0 | 21.3
11	T.62892	26	T.67229	33	0.32771	T.95663	5	49	50 | 27.5 | 26.7
12	T.62918	27	T.67262	33	0.32738	T.95657	6	48	
13	T.62945	27	T.67295	32	0.32705	T.95651	6	47	
14	T.62972	27	T.67327	33	0.32673	T.95645	6	46	27
15	T.62999	27	T.67360	33	0.32640	T.95639	6	45	6 | 2.7
16	T.63026	26	T.67393	33	0.32607	T.95633	6	44	7 | 3.2
17	T.63052	27	T.67426	32	0.32574	T.95627	6	43	8 | 3.6
18	T.63079	27	T.67458	33	0.32542	T.95621	6	42	9 | 4.1
19	T.63106	27	T.67491	33	0.32509	T.95615	6	41	10 | 4.5
20	T.63133	26	T.67524	32	0.32476	T.95609	6	40	20 | 9.0
21	T.63159	27	T.67556	33	0.32444	T.95603	6	39	30 | 13.5
22	T.63186	27	T.67589	33	0.32411	T.95597	6	38	40 | 18.0
23	T.63213	26	T.67622	32	0.32378	T.95591	6	37	50 | 22.5
24	T.63239	27	T.67654	33	0.32346	T.95585	6	36	
25	T.63266	26	T.67687	32	0.32313	T.95579	6	35	
26	T.63292	27	T.67719	33	0.32281	T.95573	6	34	26
27	T.63319	26	T.67752	33	0.32248	T.95567	6	33	6 | 2.6
28	T.63345	27	T.67785	32	0.32215	T.95561	6	32	7 | 3.0
29	T.63372	26	T.67817	33	0.32183	T.95555	6	31	8 | 3.5
30	T.63398	27	T.67850	32	0.32150	T.95549	6	30	9 | 3.9
31	T.63425	26	T.67882	33	0.32118	T.95543	6	29	10 | 4.3
32	T.63451	27	T.67915	32	0.32085	T.95537	6	28	20 | 8.7
33	T.63478	26	T.67947	33	0.32053	T.95531	6	27	30 | 13.0
34	T.63504	27	T.67980	32	0.32020	T.95525	6	26	40 | 17.3
35	T.63531	26	T.68012	32	0.31988	T.95519	6	25	50 | 21.7
36	T.63557	26	T.68044	33	0.31956	T.95513	6	24	
37	T.63583	27	T.68077	32	0.31923	T.95507	6	23	
38	T.63610	26	T.68109	33	0.31891	T.95500	7	22	7
39	T.63636	26	T.68142	32	0.31858	T.95494	6	21	6 | 0.7
40	T.63662	27	T.68174	32	0.31826	T.95488	6	20	7 | 0.8
41	T.63689	26	T.68206	33	0.31794	T.95482	6	19	8 | 0.9
42	T.63715	26	T.68239	32	0.31761	T.95476	6	18	9 | 1.1
43	T.63741	26	T.68271	32	0.31729	T.95470	6	17	10 | 1.2
44	T.63767	27	T.68303	33	0.31697	T.95464	6	16	20 | 2.3
45	T.63794	26	T.68336	32	0.31664	T.95458	6	15	30 | 3.5
46	T.63820	26	T.68368	32	0.31632	T.95452	6	14	40 | 4.7
47	T.63846	26	T.68400	32	0.31600	T.95446	6	13	50 | 5.8
48	T.63872	26	T.68432	33	0.31568	T.95440	6	12	
49	T.63898	26	T.68465	32	0.31535	T.95434	6	11	
50	T.63924	26	T.68497	32	0.31503	T.95427	7	10	8 | 5
51	T.63950	26	T.68529	32	0.31471	T.95421	6	9	6 | 0.6 | 0.5
52	T.63976	26	T.68561	32	0.31439	T.95415	6	8	7 | 0.7 | 0.6
53	T.64002	26	T.68593	32	0.31407	T.95409	6	7	8 | 0.8 | 0.7
54	T.64028	26	T.68626	33	0.31374	T.95403	6	6	9 | 0.9 | 0.8
55	T.64054	26	T.68658	32	0.31342	T.95397	6	5	10 | 1.0 | 0.8
56	T.64080	26	T.68690	32	0.31310	T.95391	6	4	20 | 2.0 | 1.7
57	T.64106	26	T.68722	32	0.31278	T.95384	7	3	30 | 3.0 | 2.5
58	T.64132	26	T.68754	32	0.31246	T.95378	6	2	40 | 4.0 | 3.3
59	T.64158	26	T.68786	32	0.31214	T.95372	6	1	50 | 5.0 | 4 2
60	T.64184		T.68818		0.31182	T.95366	6	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.64184	26	T.68818	32	0.31182	T.95366	6	60	
1	T.64210	26	T.68850	32	0.31150	T.95360	6	59	
2	T.64236	26	T.68882	32	0.31118	T.95354	6	58	32 31
3	T.64262	26	T.68914	32	0.31086	T.95348	7	57	6 3.2 3.1
4	T.64288	26	T.68946	32	0.31054	T.95341	6	56	7 3.7 3.6
5	T.64313	25	T.68978	32	0.31022	T.95335	6	55	8 4.3 4.1
6	T.64339	26	T.69010	32	0.30990	T.95329	6	54	9 4.8 4.7
7	T.64365	26	T.69042	32	0.30958	T.95323	6	53	10 5.3 5.2
8	T.64391	26	T.69074	32	0.30926	T.95317	6	52	20 10.7 10.3
9	T.64417	25	T.69106	32	0.30894	T.95310	7	51	30 16.0 15.5
10	T.64442	26	T.69138	32	0.30862	T.95304	6	50	40 21.3 20.7
11	T.64468	26	T.69170	32	0.30830	T.95298	6	49	50 26.7 25.8
12	T.64494	25	T.69202	32	0.30798	T.95292	6	48	
13	T.64519	26	T.69234	32	0.30766	T.95286	6	47	
14	T.64545	26	T.69266	32	0.30734	T.95279	7	46	28
15	T.64571	25	T.69298	32	0.30702	T.95273	6	45	6 2.6
16	T.64596	26	T.69329	31	0.30671	T.95267	6	44	7 3.0
17	T.64622	25	T.69361	32	0.30639	T.95261	6	43	8 3.5
18	T.64647	26	T.69393	32	0.30607	T.95254	7	42	9 3.9
19	T.64673	25	T.69425	32	0.30575	T.95248	6	41	10 4.3
20	T.64698	26	T.69457	32	0.30543	T.95242	6	40	20 8.7
21	T.64724	25	T.69488	31	0.30512	T.95236	6	39	30 13.0
22	T.64749	26	T.69520	32	0.30480	T.95229	7	38	40 17.3
23	T.64775	25	T.69552	32	0.30448	T.95223	6	37	50 21.7
24	T.64800	26	T.69584	32	0.30416	T.95217	6	36	
25	T.64826	25	T.69615	31	0.30385	T.95211	6	35	
26	T.64851	26	T.69647	32	0.30353	T.95204	7	34	25
27	T.64877	25	T.69679	32	0.30321	T.95198	6	33	6 2.5
28	T.64902	25	T.69710	31	0.30290	T.95192	6	32	7 2.9
29	T.64927	26	T.69742	32	0.30258	T.95185	7	31	8 3.3
30	T.64953	25	T.69774	32	0.30226	T.95179	6	30	9 3.8
31	T.64978	25	T.69805	31	0.30195	T.95173	6	29	10 4.2
32	T.65003	26	T.69837	32	0.30163	T.95167	6	28	20 8.3
33	T.65029	25	T.69868	31	0.30132	T.95160	7	27	30 12.5
34	T.65054	25	T.69900	32	0.30100	T.95154	6	26	40 16.7
35	T.65079	25	T.69932	32	0.30068	T.95148	6	25	50 20.8
36	T.65104	26	T.69963	31	0.30037	T.95141	7	24	
37	T.65130	25	T.69995	32	0.30005	T.95135	6	23	
38	T.65155	25	T.70026	31	0.29974	T.95129	6	22	24
39	T.65180	25	T.70058	32	0.29942	T.95122	7	21	6 2.4
40	T.65205	25	T.70089	31	0.29911	T.95116	6	20	7 2.8
41	T.65230	25	T.70121	32	0.29879	T.95110	6	19	8 3.2
42	T.65255	26	T.70152	31	0.29848	T.95103	7	18	9 3.6
43	T.65281	25	T.70184	32	0.29816	T.95097	6	17	10 4.0
44	T.65306	25	T.70215	31	0.29785	T.95090	7	16	20 8.0
45	T.65331	25	T.70247	32	0.29753	T.95084	6	15	30 12.0
46	T.65356	25	T.70278	31	0.29722	T.95078	6	14	40 16.0
47	T.65381	25	T.70309	31	0.29691	T.95071	7	13	50 20.0
48	T.65406	25	T.70341	32	0.29659	T.95065	6	12	
49	T.65431	25	T.70372	31	0.29628	T.95059	6	11	
50	T.65456	25	T.70404	32	0.29596	T.95052	7	10	7 6
51	T.65481	25	T.70435	31	0.29565	T.95046	6	9	6 0.7 0.6
52	T.65506	25	T.70466	31	0.29534	T.95039	7	8	7 0.8 0.7
53	T.65531	25	T.70498	32	0.29502	T.95033	6	7	8 0.9 0.8
54	T.65556	24	T.70529	31	0.29471	T.95027	6	6	9 1.1 0.9
55	T.65580	25	T.70560	31	0.29440	T.95020	7	5	10 1.2 1.0
56	T.65605	25	T.70592	32	0.29408	T.95014	6	4	20 2.3 2.0
57	T.65630	25	T.70623	31	0.29377	T.95007	6	3	30 3.5 3.0
58	T.65655	25	T.70654	31	0.29346	T.95001	7	2	40 4.7 4.0
59	T.65680	25	T.70685	31	0.29315	T.94995	6	1	50 5.8 5.0
60	T.65705	25	T.70717	32	0.29283	T.94988	7	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.65705		T.70717		0.29283	T.94988		60	
1	T.65729	24	T.70748	31	0.29252	T.94982	6	59	
2	T.65754	25	T.70779	31	0.29221	T.94975	7	58	
3	T.65779	25	T.70810	31	0.29190	T.94969	6	57	32 31
4	T.65804	25	T.70841	31	0.29159	T.94962	7	56	6 3.2 3.1
5	T.65828	24	T.70873	32	0.29127	T.94956	6	55	7 3.7 3.6
6	T.65853	25	T.70904	31	0.29096	T.94949	7	54	8 4.3 4.1
7	T.65878	25	T.70935	31	0.29065	T.94943	6	53	9 4.8 4.7
8	T.65902	24	T.70966	31	0.29034	T.94936	7	52	10 5.3 5.2
9	T.65927	25	T.70997	31	0.29003	T.94930	6	51	20 10.7 10.3
10	T.65952	25	T.71028	31	0.28972	T.94923	7	50	30 16.0 15.5
		24					6		40 21.3 20.7
11	T.65976		T.71059	31	0.28941	T.94917	6	49	50 26.7 25.8
12	T.66001	25	T.71090	31	0.28910	T.94911	7	48	
13	T.66025	24	T.71121	32	0.28879	T.94904	6	47	
14	T.66050	25	T.71153	31	0.28847	T.94898	7	46	30
15	T.66075	25	T.71184	31	0.28816	T.94891	6	45	6 3.0
16	T.66099	24	T.71215	31	0.28785	T.94885	7	44	7 3.5
17	T.66124	25	T.71246	31	0.28754	T.94878	7	43	8 4.0
18	T.66148	24	T.71277	31	0.28723	T.94871	6	42	9 4.5
19	T.66173	25	T.71308	31	0.28692	T.94865	7	41	10 5.0
20	T.66197	24	T.71339	31	0.28661	T.94858	6	40	20 10.0
		24							30 15.0
21	T.66221		T.71370	31	0.28630	T.94852	7	39	40 20.0
22	T.66246	25	T.71401	30	0.28599	T.94845	6	38	50 25.0
23	T.66270	24	T.71431	31	0.28569	T.94839	7	37	
24	T.66295	25	T.71462	31	0.28538	T.94832	6	36	
25	T.66319	24	T.71493	31	0.28507	T.94826	7	35	
26	T.66343	24	T.71524	31	0.28476	T.94819	6	34	25 24
27	T.66368	25	T.71555	31	0.28445	T.94813	7	33	6 2.5 2.4
28	T.66392	24	T.71586	31	0.28414	T.94806	7	32	7 2.9 2.8
29	T.66416	24	T.71617	31	0.28383	T.94799	6	31	8 3.3 3.2
30	T.66441	25	T.71648	31	0.28352	T.94793	7	30	9 3.8 3.6
		24					7		10 4.2 4.0
31	T.66465		T.71679	30	0.28321	T.94786	6	29	20 8.3 8.0
32	T.66489	24	T.71709	31	0.28291	T.94780	7	28	30 12.5 12.0
33	T.66513	24	T.71740	31	0.28260	T.94773	6	27	40 16.7 16.0
34	T.66537	24	T.71771	31	0.28229	T.94767	7	26	50 20.8 20.0
35	T.66562	25	T.71802	31	0.28198	T.94760	7	25	
36	T.66586	24	T.71833	30	0.28167	T.94753	6	24	
37	T.66610	24	T.71863	31	0.28137	T.94747	7	23	
38	T.66634	24	T.71894	31	0.28106	T.94740	7	22	23
39	T.66658	24	T.71925	30	0.28075	T.94734	6	21	6 2.3
40	T.66682	24	T.71955	31	0.28045	T.94727	7	20	7 2.7
		24					7		8 3.1
41	T.66706		T.71986	31	0.28014	T.94720	6	19	9 3.5
42	T.66731	25	T.72017	31	0.27983	T.94714	7	18	10 3.8
43	T.66755	24	T.72048	30	0.27952	T.94707	7	17	20 7.7
44	T.66779	24	T.72078	31	0.27922	T.94700	6	16	30 11.5
45	T.66803	24	T.72109	31	0.27891	T.94694	7	15	40 15.3
46	T.66827	24	T.72140	30	0.27860	T.94687	7	14	50 19.2
47	T.66851	24	T.72170	31	0.27830	T.94680	6	13	
48	T.66875	24	T.72201	30	0.27799	T.94674	7	12	
49	T.66899	24	T.72231	31	0.27769	T.94667	7	11	
50	T.66922	23	T.72262	31	0.27738	T.94660	7	10	
		24					6		7 8
51	T.66946		T.72293	30	0.27707	T.94654	7	9	6 0.7 0.6
52	T.66970	24	T.72323	31	0.27677	T.94647	7	8	7 0.8 0.7
53	T.66994	24	T.72354	30	0.27646	T.94640	6	7	8 0.9 0.8
54	T.67018	24	T.72384	31	0.27616	T.94634	7	6	9 1.1 0.9
55	T.67042	24	T.72415	30	0.27585	T.94627	7	5	10 1.2 1.0
56	T.67066	24	T.72445	31	0.27555	T.94620	7	4	20 2.3 2.0
57	T.67090	24	T.72476	30	0.27524	T.94614	6	3	30 3.5 3.0
58	T.67113	23	T.72506	31	0.27494	T.94607	7	2	40 4.7 4.0
59	T.67137	24	T.72537	30	0.27463	T.94600	7	1	50 5.8 5.0
60	T.67161	24	T.72567	30	0.27433	T.94593	7	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.67161	24	T.72567	31	0.27433	T.94593	6	60	
1	T.67185	23	T.72598	30	0.27402	T.94587	7	59	
2	T.67208	24	T.72628	31	0.27372	T.94580	7	58	31 | 30
3	T.67232	24	T.72659	30	0.27341	T.94573	6	57	6 | 3.1 | 3.0
4	T.67256	24	T.72689	31	0.27311	T.94567	7	56	7 | 3.6 | 3.5
5	T.67280	24	T.72720	30	0.27280	T.94560	7	55	8 | 4.1 | 4.0
6	T.67303	23	T.72750	30	0.27250	T.94553	7	54	9 | 4.7 | 4.5
7	T.67327	24	T.72780	31	0.27220	T.94546	6	53	10 | 5.2 | 5.0
8	T.67350	23	T.72811	30	0.27189	T.94540	7	52	20 | 10.3 | 10.0
9	T.67374	24	T.72841	31	0.27159	T.94533	7	51	30 | 15.5 | 15.0
10	T.67398	24	T.72872	30	0.27128	T.94526	7	50	40 | 20.7 | 20.0
		23							50 | 25.8 | 25.0
11	T.67421	24	T.72902	30	0.27098	T.94519	6	49	
12	T.67445	23	T.72932	31	0.27068	T.94513	7	48	
13	T.67468	24	T.72963	30	0.27037	T.94506	7	47	
14	T.67492	23	T.72993	30	0.27007	T.94499	7	46	29
15	T.67515	24	T.73023	31	0.26977	T.94492	7	45	6 | 2.9
16	T.67539	23	T.73054	30	0.26946	T.94485	6	44	7 | 3.4
17	T.67562	24	T.73084	30	0.26916	T.94479	7	43	8 | 3.9
18	T.67586	23	T.73114	30	0.26886	T.94472	7	42	9 | 4.4
19	T.67609	24	T.73144	31	0.26856	T.94465	7	41	10 | 4.8
20	T.67633	23	T.73175	30	0.26825	T.94458	7	40	20 | 9.7
21	T.67656	24	T.73205	30	0.26795	T.94451	6	39	30 | 14.5
22	T.67680	23	T.73235	30	0.26765	T.94445	7	38	40 | 19.3
23	T.67703	23	T.73265	30	0.26735	T.94438	7	37	50 | 24.2
24	T.67726	24	T.73295	31	0.26705	T.94431	7	36	
25	T.67750	23	T.73326	30	0.26674	T.94424	7	35	
26	T.67773	23	T.73356	30	0.26644	T.94417	7	34	24 | 23
27	T.67796	24	T.73386	30	0.26614	T.94410	6	33	6 | 2.4 | 2.3
28	T.67820	23	T.73416	30	0.26584	T.94404	7	32	7 | 2.8 | 2.7
29	T.67843	23	T.73446	30	0.26554	T.94397	7	31	8 | 3.2 | 3.1
30	T.67866	23	T.73476	30	0.26524	T.94390	7	30	9 | 3.6 | 3.5
		24		31			7		10 | 4.0 | 3.8
31	T.67890	23	T.73507	30	0.26493	T.94383	7	29	20 | 8.0 | 7.7
32	T.67913	23	T.73537	30	0.26463	T.94376	7	28	30 | 12.0 | 11.5
33	T.67936	23	T.73567	30	0.26433	T.94369	7	27	40 | 16.0 | 15.3
34	T.67959	23	T.73597	30	0.26403	T.94362	7	26	50 | 20.0 | 19.2
35	T.67982	24	T.73627	30	0.26373	T.94355	7	25	
36	T.68006	23	T.73657	30	0.26343	T.94349	6	24	
37	T.68029	23	T.73687	30	0.26313	T.94342	7	23	
38	T.68052	23	T.73717	30	0.26283	T.94335	7	22	22
39	T.68075	23	T.73747	30	0.26253	T.94328	7	21	6 | 2.2
40	T.68098	23	T.73777	30	0.26223	T.94321	7	20	7 | 2.6
		23							8 | 2.9
41	T.68121	23	T.73807	30	0.26193	T.94314	7	19	9 | 3.3
42	T.68144	23	T.73837	30	0.26163	T.94307	7	18	10 | 3.7
43	T.68167	23	T.73867	30	0.26133	T.94300	7	17	20 | 7.3
44	T.68190	23	T.73897	30	0.26103	T.94293	7	16	30 | 11.0
45	T.68213	24	T.73927	30	0.26073	T.94286	7	15	40 | 14.7
46	T.68237	23	T.73957	30	0.26043	T.94279	7	14	50 | 18.3
47	T.68260	23	T.73987	30	0.26013	T.94273	6	13	
48	T.68283	22	T.74017	30	0.25983	T.94266	7	12	
49	T.68305	23	T.74047	30	0.25953	T.94259	7	11	
50	T.68328	23	T.74077	30	0.25923	T.94252	7	10	7 | 6
51	T.68351	23	T.74107	30	0.25893	T.94245	7	9	6 | 0.7 | 0.6
52	T.68374	23	T.74137	29	0.25863	T.94238	7	8	7 | 0.8 | 0.7
53	T.68397	23	T.74166	30	0.25834	T.94231	7	7	8 | 0.9 | 0.8
54	T.68420	23	T.74196	30	0.25804	T.94224	7	6	9 | 1.1 | 0.9
55	T.68443	23	T.74226	30	0.25774	T.94217	7	5	10 | 1.2 | 1.0
56	T.68466	23	T.74256	30	0.25744	T.94210	7	4	20 | 2.3 | 2.0
57	T.68489	23	T.74286	30	0.25714	T.94203	7	3	30 | 3.5 | 3.0
58	T.68512	23	T.74316	30	0.25684	T.94196	7	2	40 | 4.7 | 4.0
59	T.68534	22	T.74345	29	0.25655	T.94189	7	1	50 | 5.8 | 5.0
60	T.68557	23	T.74375	30	0.25625	T.94182	7	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.68557		T.74375		0.25625	T.94182		60	
1	T.68580	23	T.74405	30	0.25595	T.94175	7	59	
2	T.68603	23	T.74435	30	0.25565	T.94168	7	58	30
3	T.68625	22	T.74465	29	0.25535	T.94161	7	57	6 3.0
4	T.68648	23	T.74494	30	0.25506	T.94154	7	56	7 3.5
5	T.68671	23	T.74524	30	0.25476	T.94147	7	55	8 4.0
6	T.68694	23	T.74554	29	0.25446	T.94140	7	54	9 4.5
7	T.68716	22	T.74583	30	0.25417	T.94133	7	53	10 5.0
8	T.68739	23	T.74613	30	0.25387	T.94126	7	52	20 10.0
9	T.68762	23	T.74643	30	0.25357	T.94119	7	51	30 15.0
10	T.68784	22	T.74673	29	0.25327	T.94112	7	50	40 20.0
		23					7		50 25.0
11	T.68807		T.74702	30	0.25298	T.94105	7	49	
12	T.68829	22	T.74732	30	0.25268	T.94098	8	48	
13	T.68852	23	T.74762	30	0.25238	T.94090	7	47	
14	T.68875	23	T.74791	29	0.25209	T.94083	7	46	29
15	T.68897	22	T.74821	30	0.25179	T.94076	7	45	6 2.9
16	T.68920	23	T.74851	30	0.25149	T.94069	7	44	7 3.4
17	T.68942	22	T.74880	29	0.25120	T.94062	7	43	8 3.9
18	T.68965	23	T.74910	30	0.25090	T.94055	7	42	9 4.4
19	T.68987	22	T.74939	29	0.25061	T.94048	7	41	10 4.8
20	T.69010	23	T.74969	30	0.25031	T.94041	7	40	20 9.7
		22		29			7		30 14.5
21	T.69032		T.74998	30	0.25002	T.94034	7	39	40 19.3
22	T.69055	23	T.75028	30	0.24972	T.94027	7	38	50 24.2
23	T.69077	22	T.75058	29	0.24942	T.94020	8	37	
24	T.69100	23	T.75087	30	0.24913	T.94012	7	36	
25	T.69122	22	T.75117	29	0.24883	T.94005	7	35	
26	T.69144	22	T.75146	30	0.24854	T.93998	7	34	28
27	T.69167	23	T.75176	29	0.24824	T.93991	7	33	6 2.3
28	T.69189	22	T.75205	30	0.24795	T.93984	7	32	7 2.7
29	T.69212	23	T.75235	29	0.24765	T.93977	7	31	8 3.1
30	T.69234	22	T.75264	30	0.24736	T.93970	7	30	9 3.5
		22					8		10 3.8
31	T.69256		T.75294	29	0.24706	T.93963	7	29	20 7.7
32	T.69279	23	T.75323	30	0.24677	T.93955	7	28	30 11.5
33	T.69301	22	T.75353	29	0.24647	T.93948	7	27	40 15.3
34	T.69323	22	T.75382	29	0.24618	T.93941	7	26	50 19.2
35	T.69345	22	T.75411	30	0.24589	T.93934	7	25	
36	T.69368	23	T.75441	29	0.24559	T.93927	7	24	
37	T.69390	22	T.75470	30	0.24530	T.93920	8	23	
38	T.69412	22	T.75500	29	0.24500	T.93912	7	22	22
39	T.69434	22	T.75529	29	0.24471	T.93905	7	21	6 2.2
40	T.69456	22	T.75558	30	0.24442	T.93898	7	20	7 2.6
		23					7		8 2.9
41	T.69479		T.75588	29	0.24412	T.93891	7	19	9 3.3
42	T.69501	22	T.75617	30	0.24383	T.93884	8	18	10 3.7
43	T.69523	22	T.75647	29	0.24353	T.93876	7	17	20 7.3
44	T.69545	22	T.75676	29	0.24324	T.93869	7	16	30 11.0
45	T.69567	22	T.75705	30	0.24295	T.93862	7	15	40 14.7
46	T.69589	22	T.75735	29	0.24265	T.93855	8	14	50 18.3
47	T.69611	22	T.75764	29	0.24236	T.93847	7	13	
48	T.69633	22	T.75793	29	0.24207	T.93840	7	12	
49	T.69655	22	T.75822	30	0.24178	T.93833	7	11	
50	T.69677	22	T.75852.	29	0.24148	T.93826	7	10	8 7
51	T.69699		T.75881	29	0.24119	T.93819	8	9	6 0.8 0.7
52	T.69721	22	T.75910	29	0.24090	T.93811	7	8	7 0.9 0.8
53	T.69743	22	T.75939	30	0.24061	T.93804	7	7	8 1.1 0.9
54	T.69765	22	T.75969	29	0.24031	T.93797	8	6	9 1.2 1.1
55	T.69787	22	T.75998	29	0.24002	T.93789	7	5	10 1.3 1.2
56	T.69809	22	T.76027	29	0.23973	T.93782	7	4	20 2.7 2.3
57	T.69831	22	T.76056	29	0.23944	T.93775	7	3	30 4.0 3.5
58	T.69853	22	T.76086	30	0.23914	T.93768	8	2	40 5.3 4.7
59	T.69875	22	T.76115	29	0.23885	T.93760	7	1	50 6.7 5.8
60	T.69897	22	T.76144	29	0.23856	T.93753	7	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

60°

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.69897	22	T.76144	29	0.23856	T.93753	7	60	
1	T.69919	22	T.76173	29	0.23827	T.93746	8	59	
2	T.69941	22	T.76202	29	0.23798	T.93738		58	30 | 29
3	T.69963	21	T.76231	30	0.23769	T.93731	7	57	6 | 3.0 | 2.9
4	T.69984	22	T.76261	29	0.23739	T.93724	7	56	7 | 3.5 | 3.4
5	T.70006	22	T.76290	29	0.23710	T.93717	8	55	8 | 4.0 | 3.9
6	T.70028	22	T.76319	29	0.23681	T.93709	7	54	9 | 4.5 | 4.4
7	T.70050	22	T.76348	29	0.23652	T.93702	7	53	10 | 5.0 | 4.8
8	T.70072	21	T.76377	29	0.23623	T.93695	8	52	20 | 10.0 | 9.7
9	T.70093	22	T.76406	29	0.23594	T.93687	7	51	30 | 15.0 | 14.5
10	T.70115	22	T.76435	29	0.23565	T.93680	7	50	40 | 20.0 | 19.3
11	T.70137	22	T.76464	29	0.23536	T.93673	8	49	50 | 25.0 | 24.2
12	T.70159	21	T.76493	29	0.23507	T.93665	7	48	
13	T.70180	22	T.76522	29	0.23478	T.93658	8	47	
14	T.70202	22	T.76551	29	0.23449	T.93650	7	46	28
15	T.70224	21	T.76580	29	0.23420	T.93643	7	45	6 | 2.8
16	T.70245	22	T.76609	30	0.23391	T.93636	8	44	7 | 3.3
17	T.70267	21	T.76639	29	0.23361	T.93628	7	43	8 | 3.7
18	T.70288	22	T.76668	29	0.23332	T.93621	7	42	9 | 4.2
19	T.70310	22	T.76697	28	0.23303	T.93614	8	41	10 | 4.7
20	T.70332	21	T.76725	29	0.23275	T.93606	7	40	20 | 9.3
21	T.70353	22	T.76754	29	0.23246	T.93599	8	39	30 | 14.0
22	T.70375	21	T.76783	29	0.23217	T.93591	7	38	40 | 18.7
23	T.70396	22	T.76812	29	0.23188	T.93584	7	37	50 | 23.3
24	T.70418	21	T.76841	29	0.23159	T.93577	8	36	
25	T.70439	22	T.76870	29	0.23130	T.93569	7	35	
26	T.70461	21	T.76899	29	0.23101	T.93562	8	34	22
27	T.70482	22	T.76928	29	0.23072	T.93554	7	33	6 | 2.2
28	T.70504	21	T.76957	29	0.23043	T.93547	8	32	7 | 2.6
29	T.70525	22	T.76986	29	0.23014	T.93539	7	31	8 | 2.9
30	T.70547	21	T.77015	29	0.22985	T.93532	7	30	9 | 3.3
31	T.70568	22	T.77044	29	0.22956	T.93525	8	29	10 | 3.7
32	T.70590	21	T.77073	28	0.22927	T.93517	7	28	20 | 7.3
33	T.70611	22	T.77101	29	0.22899	T.93510	8	27	30 | 11.0
34	T.70633	21	T.77130	29	0.22870	T.93502	7	26	40 | 14.7
35	T.70654	21	T.77159	29	0.22841	T.93495	8	25	50 | 18.3
36	T.70675	22	T.77188	29	0.22812	T.93487	7	24	
37	T.70697	21	T.77217	29	0.22783	T.93480	8	23	
38	T.70718	21	T.77246	28	0.22754	T.93472	7	22	21
39	T.70739	22	T.77274	29	0.22726	T.93465	8	21	6 | 2.1
40	T.70761	21	T.77303	29	0.22697	T.93457	7	20	7 | 2.5
41	T.70782	21	T.77332	29	0.22668	T.93450	8	19	8 | 2.8
42	T.70803	21	T.77361	29	0.22639	T.93442	7	18	9 | 3.2
43	T.70824	22	T.77390	28	0.22610	T.93435	8	17	10 | 3.5
44	T.70846	21	T.77418	29	0.22582	T.93427	7	16	20 | 7.0
45	T.70867	21	T.77447	29	0.22553	T.93420	8	15	30 | 10.5
46	T.70888	21	T.77476	29	0.22524	T.93412	7	14	40 | 14.0
47	T.70909	22	T.77505	28	0.22495	T.93405	8	13	50 | 17.5
48	T.70931	21	T.77533	29	0.22467	T.93397	7	12	
49	T.70952	21	T.77562	29	0.22438	T.93390	8	11	
50	T.70973	21	T.77591	28	0.22409	T.93382	7	10	8 | 7
51	T.70994	21	T.77619	29	0.22381	T.93375	8	9	6 | 0.8 | 0.7
52	T.71015	21	T.77648	29	0.22352	T.93367	7	8	7 | 0.9 | 0.8
53	T.71036	22	T.77677	29	0.22323	T.93360	8	7	8 | 1.1 | 0.9
54	T.71058	21	T.77706	28	0.22294	T.93352	8	6	9 | 1.2 | 1.1
55	T.71079	21	T.77734	29	0.22266	T.93344	7	5	10 | 1.3 | 1.2
56	T.71100	21	T.77763	28	0.22237	T.93337	8	4	20 | 2.7 | 2.3
57	T.71121	21	T.77791	29	0.22209	T.93329	7	3	30 | 4.0 | 3.5
58	T.71142	21	T.77820	29	0.22180	T.93322	8	2	40 | 5.3 | 4.7
59	T.71163	21	T.77849	28	0.22151	T.93314	7	1	50 | 6.7 | 5.8
60	T.71184		T.77877		0.22123	T.93307		0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	1̄.71184	21	1̄.77877	29	0.22123	1̄.93307	8	60	
1	1̄.71205	21	1̄.77906	29	0.22094	1̄.93299	8	59	
2	1̄.71226	21	1̄.77935	28	0.22065	1̄.93291	7	58	29
3	1̄.71247	21	1̄.77963	29	0.22037	1̄.93284	8	57	6 | 2.9
4	1̄.71268	21	1̄.77992	28	0.22008	1̄.93276	7	56	7 | 3.4
5	1̄.71289	21	1̄.78020	29	0.21980	1̄.93269	8	55	8 | 3.9
6	1̄.71310	21	1̄.78049	28	0.21951	1̄.93261	8	54	9 | 4.4
7	1̄.71331	21	1̄.78077	29	0.21923	1̄.93253	7	53	10 | 4.8
8	1̄.71352	21	1̄.78106	29	0.21894	1̄.93246	8	52	20 | 9.7
9	1̄.71373	20	1̄.78135	28	0.21865	1̄.93238	8	51	30 | 14.5
10	1̄.71393	21	1̄.78163	29	0.21837	1̄.93230	7	50	40 | 19.3
									50 | 24.2
11	1̄.71414	21	1̄.78192	28	0.21808	1̄.93223	8	49	
12	1̄.71435	21	1̄.78220	29	0.21780	1̄.93215	8	48	
13	1̄.71456	21	1̄.78249	28	0.21751	1̄.93207	7	47	
14	1̄.71477	21	1̄.78277	29	0.21723	1̄.93200	8	46	28
15	1̄.71498	21	1̄.78306	28	0.21694	1̄.93192	8	45	6 | 2.8
16	1̄.71519	20	1̄.78334	29	0.21666	1̄.93184	7	44	7 | 3.3
17	1̄.71539	21	1̄.78363	28	0.21637	1̄.93177	8	43	8 | 3.7
18	1̄.71560	21	1̄.78391	28	0.21609	1̄.93169	8	42	9 | 4.2
19	1̄.71581	21	1̄.78419	29	0.21581	1̄.93161	7	41	10 | 4.7
20	1̄.71602	20	1̄.78448	28	0.21552	1̄.93154	8	40	20 | 9.3
									30 | 14.0
21	1̄.71622	21	1̄.78476	29	0.21524	1̄.93146	8	39	40 | 18.7
22	1̄.71643	21	1̄.78505	28	0.21495	1̄.93138	7	38	50 | 23.3
23	1̄.71664	21	1̄.78533	29	0.21467	1̄.93131	8	37	
24	1̄.71685	20	1̄.78562	28	0.21438	1̄.93123	8	36	
25	1̄.71705	21	1̄.78590	28	0.21410	1̄.93115	7	35	
26	1̄.71726	21	1̄.78618	29	0.21382	1̄.93108	8	34	21
27	1̄.71747	20	1̄.78647	28	0.21353	1̄.93100	8	33	6 | 2.1
28	1̄.71767	21	1̄.78675	29	0.21325	1̄.93092	8	32	7 | 2.5
29	1̄.71788	21	1̄.78704	28	0.21296	1̄.93084	7	31	8 | 2.8
30	1̄.71809	20	1̄.78732	28	0.21268	1̄.93077	8	30	9 | 3.2
									10 | 3.5
31	1̄.71829	21	1̄.78760	29	0.21240	1̄.93069	8	29	20 | 7.0
32	1̄.71850	20	1̄.78789	28	0.21211	1̄.93061	8	28	30 | 10.5
33	1̄.71870	21	1̄.78817	28	0.21183	1̄.93053	7	27	40 | 14.0
34	1̄.71891	20	1̄.78845	29	0.21155	1̄.93046	8	26	50 | 17.5
35	1̄.71911	21	1̄.78874	28	0.21126	1̄.93038	8	25	
36	1̄.71932	20	1̄.78902	28	0.21098	1̄.93030	8	24	
37	1̄.71952	21	1̄.78930	29	0.21070	1̄.93022	8	23	
38	1̄.71973	21	1̄.78959	28	0.21041	1̄.93014	7	22	20
39	1̄.71994	20	1̄.78987	28	0.21013	1̄.93007	8	21	6 | 2.0
40	1̄.72014	20	1̄.79015	28	0.20985	1̄.92999	8	20	7 | 2.3
									8 | 2.7
41	1̄.72034	21	1̄.79043	29	0.20957	1̄.92991	8	19	9 | 3.0
42	1̄.72055	20	1̄.79072	28	0.20928	1̄.92983	7	18	10 | 3.3
43	1̄.72075	21	1̄.79100	28	0.20900	1̄.92976	8	17	20 | 6.7
44	1̄.72096	20	1̄.79128	28	0.20872	1̄.92968	8	16	30 | 10.0
45	1̄.72116	21	1̄.79156	29	0.20844	1̄.92960	8	15	40 | 13.3
46	1̄.72137	20	1̄.79185	28	0.20815	1̄.92952	8	14	50 | 16.7
47	1̄.72157	20	1̄.79213	28	0.20787	1̄.92944	8	13	
48	1̄.72177	21	1̄.79241	28	0.20759	1̄.92936	7	12	
49	1̄.72198	20	1̄.79269	28	0.20731	1̄.92929	8	11	
50	1̄.72218	20	1̄.79297	29	0.20703	1̄.92921	8	10	8 | 7
									6 | 0.8 | 0.7
51	1̄.72238	21	1̄.79326	28	0.20674	1̄.92913	8	9	7 | 0.9 | 0.8
52	1̄.72259	20	1̄.79354	28	0.20646	1̄.92905	8	8	8 | 1.1 | 0.9
53	1̄.72279	20	1̄.79382	28	0.20618	1̄.92897	8	7	9 | 1.2 | 1.1
54	1̄.72299	21	1̄.79410	28	0.20590	1̄.92889	8	6	10 | 1.3 | 1.2
55	1̄.72320	20	1̄.79438	28	0.20562	1̄.92881	8	5	20 | 2.7 | 2.3
56	1̄.72340	20	1̄.79466	29	0.20534	1̄.92874	7	4	30 | 4.0 | 3.5
57	1̄.72360	21	1̄.79495	28	0.20505	1̄.92866	8	3	40 | 5.3 | 4.7
58	1̄.72381	20	1̄.79523	28	0.20477	1̄.92858	8	2	50 | 6.7 | 5.8
59	1̄.72401	20	1̄.79551	28	0.20449	1̄.92850	8	1	
60	1̄.72421		1̄.79579		0.20421	1̄.92842	8	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.69897	22	T.76144	29	0.23856	T.93753	7	60	
1	T.69919	22	T.76173	29	0.23827	T.93746	8	59	
2	T.69941	22	T.76202	29	0.23798	T.93738	7	58	30 | 29
3	T.69963	21	T.76231	30	0.23769	T.93731	7	57	6 | 3.0 | 2.9
4	T.69984	22	T.76261	29	0.23739	T.93724	7	56	7 | 3.5 | 3.4
5	T.70006	22	T.76290	29	0.23710	T.93717	8	55	8 | 4.0 | 3.9
6	T.70028	22	T.76319	29	0.23681	T.93709	7	54	9 | 4.5 | 4.4
7	T.70050	22	T.76348	29	0.23652	T.93702	7	53	10 | 5.0 | 4.8
8	T.70072	21	T.76377	29	0.23623	T.93695	8	52	20 | 10.0 | 9.7
9	T.70093	22	T.76406	29	0.23594	T.93687	7	51	30 | 15.0 | 14.5
10	T.70115	22	T.76435	29	0.23565	T.93680	7	50	40 | 20.0 | 19.3
11	T.70137	22	T.76464	29	0.23536	T.93673	8	49	50 | 25.0 | 24.2
12	T.70159	21	T.76493	29	0.23507	T.93665	7	48	
13	T.70180	22	T.76522	29	0.23478	T.93658	8	47	
14	T.70202	22	T.76551	29	0.23449	T.93650	7	46	28
15	T.70224	21	T.76580	29	0.23420	T.93643	7	45	6 | 2.8
16	T.70245	22	T.76609	30	0.23391	T.93636	8	44	7 | 3.3
17	T.70267	21	T.76639	29	0.23361	T.93628	7	43	8 | 3.7
18	T.70288	22	T.76668	29	0.23332	T.93621	7	42	9 | 4.2
19	T.70310	22	T.76697	28	0.23303	T.93614	8	41	10 | 4.7
20	T.70332	21	T.76725	29	0.23275	T.93606	7	40	20 | 9.3
21	T.70353	22	T.76754	29	0.23246	T.93599	8	39	30 | 14.0
22	T.70375	21	T.76783	29	0.23217	T.93591	7	38	40 | 18.7
23	T.70396	22	T.76812	29	0.23188	T.93584	7	37	50 | 23.3
24	T.70418	21	T.76841	29	0.23159	T.93577	8	36	
25	T.70439	22	T.76870	29	0.23130	T.93569	7	35	
26	T.70461	21	T.76899	29	0.23101	T.93562	8	34	22
27	T.70482	22	T.76928	29	0.23072	T.93554	7	33	6 | 2.2
28	T.70504	21	T.76957	29	0.23043	T.93547	8	32	7 | 2.6
29	T.70525	22	T.76986	29	0.23014	T.93539	7	31	8 | 2.9
30	T.70547	21	T.77015	29	0.22985	T.93532	7	30	9 | 3.3
31	T.70568	22	T.77044	29	0.22956	T.93525	8	29	10 | 3.7
32	T.70590	21	T.77073	28	0.22927	T.93517	7	28	20 | 7.3
33	T.70611	22	T.77101	29	0.22899	T.93510	8	27	30 | 11.0
34	T.70633	21	T.77130	29	0.22870	T.93502	7	26	40 | 14.7
35	T.70654	21	T.77159	29	0.22841	T.93495	8	25	50 | 18.3
36	T.70675	22	T.77188	29	0.22812	T.93487	7	24	
37	T.70697	21	T.77217	29	0.22783	T.93480	8	23	
38	T.70718	21	T.77246	28	0.22754	T.93472	7	22	21
39	T.70739	22	T.77274	29	0.22726	T.93465	8	21	6 | 2.1
40	T.70761	21	T.77303	29	0.22697	T.93457	7	20	7 | 2.5
41	T.70782	21	T.77332	29	0.22668	T.93450	8	19	8 | 2.8
42	T.70803	21	T.77361	29	0.22639	T.93442	7	18	9 | 3.2
43	T.70824	22	T.77390	28	0.22610	T.93435	8	17	10 | 3.5
44	T.70846	21	T.77418	29	0.22582	T.93427	7	16	20 | 7.0
45	T.70867	21	T.77447	29	0.22553	T.93420	8	15	30 | 10.5
46	T.70888	21	T.77476	29	0.22524	T.93412	7	14	40 | 14.0
47	T.70909	22	T.77505	28	0.22495	T.93405	8	13	50 | 17.5
48	T.70931	21	T.77533	29	0.22467	T.93397	7	12	
49	T.70952	21	T.77562	29	0.22438	T.93390	7	11	
50	T.70973	21	T.77591	28	0.22409	T.93382	8	10	8 | 7
51	T.70994	21	T.77619	29	0.22381	T.93375	7	9	6 | 0.8 | 0.7
52	T.71015	21	T.77648	29	0.22352	T.93367	8	8	7 | 0.9 | 0.8
53	T.71036	22	T.77677	29	0.22323	T.93360	7	7	8 | 1.1 | 0.9
54	T.71058	21	T.77706	28	0.22294	T.93352	8	6	9 | 1.2 | 1.1
55	T.71079	21	T.77734	29	0.22266	T.93344	7	5	10 | 1.3 | 1.2
56	T.71100	21	T.77763	28	0.22237	T.93337	8	4	20 | 2.7 | 2.3
57	T.71121	21	T.77791	29	0.22209	T.93329	7	3	30 | 4.0 | 3.5
58	T.71142	21	T.77820	29	0.22180	T.93322	8	2	40 | 5.3 | 4.7
59	T.71163	21	T.77849	28	0.22151	T.93314	7	1	50 | 6.7 | 5.8
60	T.71184		T.77877		0.22123	T.93307	7	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.73611		T.81252		0.18748	T.92359		60	
1	T.73630	19	T.81279	27	0.18721	T.92351	8	59	
2	T.73650	20	T.81307	28	0.18693	T.92343	8	58	
3	T.73669	19	T.81335	28	0.18665	T.92335	8	57	28 | 27
4	T.73689	20	T.81362	27	0.18638	T.92326	9	56	6 | 2.8 | 2.7
5	T.73708	19	T.81390	28	0.18610	T.92318	8	55	7 | 3.3 | 3.2
6	T.73727	19	T.81418	28	0.18582	T.92310	8	54	8 | 3.7 | 3.6
7	T.73747	20	T.81445	27	0.18555	T.92302	8	53	9 | 4.2 | 4.1
8	T.73766	19	T.81473	28	0.18527	T.92293	9	52	10 | 4.7 | 4.5
9	T.73785	19	T.81500	27	0.18500	T.92285	8	51	20 | 9.3 | 9.0
10	T.73805	20	T.81528	28	0.18472	T.92277	8	50	30 | 14.0 | 13.5
		19		28			8		40 | 18.7 | 18.0
11	T.73824		T.81556		0.18444	T.92269		49	50 | 23.3 | 22.5
12	T.73843	19	T.81583	27	0.18417	T.92260	9	48	
13	T.73863	20	T.81611	28	0.18389	T.92252	8	47	
14	T.73882	19	T.81638	27	0.18362	T.92244	8	46	20
15	T.73901	19	T.81666	28	0.18334	T.92235	9	45	6 | 2.0
16	T.73921	20	T.81693	27	0.18307	T.92227	8	44	7 | 2.3
17	T.73940	19	T.81721	28	0.18279	T.92219	8	43	8 | 2.7
18	T.73959	19	T.81748	27	0.18252	T.92211	8	42	9 | 3.0
19	T.73978	19	T.81776	28	0.18224	T.92202	9	41	10 | 3.3
20	T.73997	19	T.81803	27	0.18197	T.92194	8	40	20 | 6.7
		20		28			8		30 | 10.0
21	T.74017		T.81831		0.18169	T.92186		39	40 | 13.3
22	T.74036	19	T.81858	27	0.18142	T.92177	9	38	50 | 16.7
23	T.74055	19	T.81886	28	0.18114	T.92169	8	37	
24	T.74074	19	T.81913	27	0.18087	T.92161	8	36	
25	T.74093	19	T.81941	28	0.18059	T.92152	9	35	
26	T.74113	20	T.81968	27	0.18032	T.92144	8	34	19
27	T.74132	19	T.81996	28	0.18004	T.92136	8	33	6 | 1.9
28	T.74151	19	T.82023	27	0.17977	T.92127	9	32	7 | 2.2
29	T.74170	19	T.82051	28	0.17949	T.92119	8	31	8 | 2.5
30	T.74189	19	T.82078	27	0.17922	T.92111	8	30	9 | 2.9
		19		28			9		10 | 3.2
31	T.74208		T.82106		0.17894	T.92102		29	20 | 6.3
32	T.74227	19	T.82133	27	0.17867	T.92094	8	28	30 | 9.5
33	T.74246	19	T.82161	28	0.17839	T.92086	8	27	40 | 12.7
34	T.74265	19	T.82188	27	0.17812	T.92077	9	26	50 | 15.8
35	T.74284	19	T.82215	27	0.17785	T.92069	8	25	
36	T.74303	19	T.82243	28	0.17757	T.92060	9	24	
37	T.74322	19	T.82270	27	0.17730	T.92052	8	23	
38	T.74341	19	T.82298	28	0.17702	T.92044	8	22	18
39	T.74360	19	T.82325	27	0.17675	T.92035	9	21	6 | 1.8
40	T.74379	19	T.82352	27	0.17648	T.92027	8	20	7 | 2.1
		19		28			9		8 | 2.4
41	T.74398		T.82380		0.17620	T.92018		19	9 | 2.7
42	T.74417	19	T.82407	27	0.17593	T.92010	8	18	10 | 3.0
43	T.74436	19	T.82435	28	0.17565	T.92002	8	17	20 | 6.0
44	T.74455	19	T.82462	27	0.17538	T.91993	9	16	30 | 9.0
45	T.74474	19	T.82489	27	0.17511	T.91985	8	15	40 | 12.0
46	T.74493	19	T.82517	28	0.17483	T.91976	9	14	50 | 15.0
47	T.74512	19	T.82544	27	0.17456	T.91968	8	13	
48	T.74531	18	T.82571	27	0.17429	T.91959	9	12	
49	T.74549	19	T.82599	28	0.17401	T.91951	8	11	
50	T.74568	19	T.82626	27	0.17374	T.91942	9	10	9 | 8
		19		27			8		6 | 0.9 | 0.8
51	T.74587		T.82653		0.17347	T.91934		9	7 | 1.1 | 0.9
52	T.74606	19	T.82681	28	0.17319	T.91925	9	8	8 | 1.2 | 1.1
53	T.74625	19	T.82708	27	0.17292	T.91917	8	7	9 | 1.4 | 1.2
54	T.74644	19	T.82735	27	0.17265	T.91908	9	6	10 | 1.5 | 1.3
55	T.74662	18	T.82762	27	0.17238	T.91900	8	5	20 | 3.0 | 2.7
56	T.74681	19	T.82790	28	0.17210	T.91891	9	4	30 | 4.5 | 4.0
57	T.74700	19	T.82817	27	0.17183	T.91883	8	3	40 | 6.0 | 5.3
58	T.74719	19	T.82844	27	0.17156	T.91874	9	2	50 | 7.5 | 6.7
59	T.74737	18	T.82871	27	0.17129	T.91866	8	1	
60	T.74756	19	T.82899	28	0.17101	T.91857	9	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

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′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.74756		T.82899		0.17101	T.91857		60	
		19		27			8		
1	T.74775		T.82926		0.17074	T.91849		59	
		19		27			9		
2	T.74794		T.82953		0.17047	T.91840		58	28 | 27
		18		27			8		
3	T.74812		T.82980		0.17020	T.91832		57	6 | 2.8 | 2.7
		19		28			9		
4	T.74831		T.83008		0.16992	T.91823		56	7 | 3.3 | 3.2
		19		27			8		
5	T.74850		T.83035		0.16965	T.91815		55	8 | 3.7 | 3.6
		18		27			9		
6	T.74868		T.83062		0.16938	T.91806		54	9 | 4.2 | 4.1
		19		27			9		
7	T.74887		T.83089		0.16911	T.91798		53	10 | 4.7 | 4.5
		19		28			8		
8	T.74906		T.83117		0.16883	T.91789		52	20 | 9.3 | 9.0
		18		27			9		
9	T.74924		T.83144		0.16856	T.91781		51	30 | 14.0 | 13.5
		19		27			9		
10	T.74943		T.83171		0.16829	T.91772		50	40 | 18.7 | 18.0
		18		27			9		50 | 23.3 | 22.5
11	T.74961		T.83198		0.16802	T.91763		49	
		19		27			8		
12	T.74980		T.83225		0.16775	T.91755		48	
		19		27			9		
13	T.74999		T.83252		0.16748	T.91746		47	
		18		28			8		
14	T.75017		T.83280		0.16720	T.91738		46	26
		19		27			9		
15	T.75036		T.83307		0.16693	T.91729		45	6 | 2.6
		18		27			9		
16	T.75054		T.83334		0.16666	T.91720		44	7 | 3.0
		19		27			8		
17	T.75073		T.83361		0.16639	T.91712		43	8 | 3.5
		18		27			9		
18	T.75091		T.83388		0.16612	T.91703		42	9 | 3.9
		19		27			8		
19	T.75110		T.83415		0.16585	T.91695		41	10 | 4.3
		18		27			9		
20	T.75128		T.83442		0.16558	T.91686		40	20 | 8.7
		19		28			9		30 | 13.0
21	T.75147		T.83470		0.16530	T.91677		39	40 | 17.3
		18		27			8		
22	T.75165		T.83497		0.16503	T.91669		38	50 | 21.7
		19		27			9		
23	T.75184		T.83524		0.16476	T.91660		37	
		18		27			9		
24	T.75202		T.83551		0.16449	T.91651		36	
		19		27			8		
25	T.75221		T.83578		0.16422	T.91643		35	
		18		27			9		
26	T.75239		T.83605		0.16395	T.91634		34	19
		19		27			9		
27	T.75258		T.83632		0.16368	T.91625		33	6 | 1.9
		18		27			8		
28	T.75276		T.83659		0.16341	T.91617		32	7 | 2.2
		18		27			9		
29	T.75294		T.83686		0.16314	T.91608		31	8 | 2.5
		19		27			9		
30	T.75313		T.83713		0.16287	T.91599		30	9 | 2.9
		18		27			8		10 | 3.2
31	T.75331		T.83740		0.16260	T.91591		29	20 | 6.3
		19		28			9		
32	T.75350		T.83768		0.16232	T.91582		28	30 | 9.5
		18		27			9		
33	T.75368		T.83795		0.16205	T.91573		27	40 | 12.7
		18		27			8		
34	T.75386		T.83822		0.16178	T.91565		26	50 | 15.8
		19		27			9		
35	T.75405		T.83849		0.16151	T.91556		25	
		18		27			9		
36	T.75423		T.83876		0.16124	T.91547		24	
		18		27			9		
37	T.75441		T.83903		0.16097	T.91538		23	
		18		27			8		
38	T.75459		T.83930		0.16070	T.91530		22	18
		19		27			9		
39	T.75478		T.83957		0.16043	T.91521		21	6 | 1.8
		18		27			9		
40	T.75496		T.83984		0.16016	T.91512		20	7 | 2.1
		18		27			8		8 | 2.4
41	T.75514		T.84011		0.15989	T.91504		19	9 | 2.7
		19		27			9		
42	T.75533		T.84038		0.15962	T.91495		18	10 | 3.0
		18		27			9		
43	T.75551		T.84065		0.15935	T.91486		17	20 | 6.0
		18		27			9		
44	T.75569		T.84092		0.15908	T.91477		16	30 | 9.0
		18		27			8		
45	T.75587		T.84119		0.15881	T.91469		15	40 | 12.0
		18		27			9		
46	T.75605		T.84146		0.15854	T.91460		14	50 | 15.0
		19		27			9		
47	T.75624		T.84173		0.15827	T.91451		13	
		18		27			9		
48	T.75642		T.84200		0.15800	T.91442		12	
		18		27			9		
49	T.75660		T.84227		0.15773	T.91433		11	
		18		27			8		
50	T.75678		T.84254		0.15746	T.91425		10	
		18		26			9		9 | 8
51	T.75696		T.84280		0.15720	T.91416		9	6 | 0.9 | 0.8
		18		27			9		
52	T.75714		T.84307		0.15693	T.91407		8	7 | 1.1 | 0.9
		19		27			9		
53	T.75733		T.84334		0.15666	T.91398		7	8 | 1.2 | 1.1
		18		27			9		
54	T.75751		T.84361		0.15639	T.91389		6	9 | 1.4 | 1.2
		18		27			8		
55	T.75769		T.84388		0.15612	T.91381		5	10 | 1.5 | 1.3
		18		27			9		
56	T.75787		T.84415		0.15585	T.91372		4	20 | 3.0 | 2.7
		18		27			9		
57	T.75805		T.84442		0.15558	T.91363		3	30 | 4.5 | 4.0
		18		27			9		
58	T.75823		T.84469		0.15531	T.91354		2	40 | 6.0 | 5.3
		18		27			9		
59	T.75841		T.84496		0.15504	T.91345		1	50 | 7.5 | 6.7
		18		27			9		
60	T.75859		T.84523		0.15477	T.91336		0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.75859	18	T.84523	27	0.15477	T.91336	8	60	
1	T.75877	18	T.84550	26	0.15450	T.91328	9	59	
2	T.75895	18	T.84576	27	0.15424	T.91319	9	58	27 | 26
3	T.75913	18	T.84603	27	0.15397	T.91310	9	57	6 | 2.7 | 2.6
4	T.75931	18	T.84630	27	0.15370	T.91301	9	56	7 | 3.2 | 3.0
5	T.75949	18	T.84657	27	0.15343	T.91292	9	55	8 | 3.6 | 3.5
6	T.75967	18	T.84684	27	0.15316	T.91283	9	54	9 | 4.1 | 3.9
7	T.75985	18	T.84711	27	0.15289	T.91274	9	53	10 | 4.5 | 4.3
8	T.76003	18	T.84738	27	0.15262	T.91266	8	52	20 | 9.0 | 8.7
9	T.76021	18	T.84764	26	0.15236	T.91257	9	51	30 | 13.5 | 13.0
10	T.76039	18	T.84791	27	0.15209	T.91248	9	50	40 | 18.0 | 17.3
11	T.76057	18	T.84818	27	0.15182	T.91239	9	49	50 | 22.5 | 21.7
12	T.76075	18	T.84845	27	0.15155	T.91230	9	48	
13	T.76093	18	T.84872	27	0.15128	T.91221	9	47	
14	T.76111	18	T.84899	27	0.15101	T.91212	9	46	18
15	T.76129	17	T.84925	26	0.15075	T.91203	9	45	6 | 1.8
16	T.76146	18	T.84952	27	0.15048	T.91194	9	44	7 | 2.1
17	T.76164	18	T.84979	27	0.15021	T.91185	9	43	8 | 2.4
18	T.76182	18	T.85006	27	0.14994	T.91176	9	42	9 | 2.7
19	T.76200	18	T.85033	27	0.14967	T.91167	9	41	10 | 3.0
20	T.76218	18	T.85059	26	0.14941	T.91158	9	40	20 | 6.0
21	T.76236	17	T.85086	27	0.14914	T.91149	8	39	30 | 9.0
22	T.76253	18	T.85113	27	0.14887	T.91141	9	38	40 | 12.0
23	T.76271	18	T.85140	27	0.14860	T.91132	9	37	50 | 15.0
24	T.76289	18	T.85166	26	0.14834	T.91123	9	36	
25	T.76307	17	T.85193	27	0.14807	T.91114	9	35	
26	T.76324	18	T.85220	27	0.14780	T.91105	9	34	17
27	T.76342	18	T.85247	27	0.14753	T.91096	9	33	6 | 1.7
28	T.76360	18	T.85273	26	0.14727	T.91087	9	32	7 | 2.0
29	T.76378	18	T.85300	27	0.14700	T.91078	9	31	8 | 2.3
30	T.76395	17	T.85327	27	0.14673	T.91069	9	30	9 | 2.6
31	T.76413	18	T.85354	27	0.14646	T.91060	9	29	10 | 2.8
32	T.76431	18	T.85380	26	0.14620	T.91051	9	28	20 | 5.7
33	T.76448	17	T.85407	27	0.14593	T.91042	9	27	30 | 8.5
34	T.76466	18	T.85434	27	0.14566	T.91033	9	26	40 | 11.3
35	T.76484	18	T.85460	26	0.14540	T.91023	10	25	50 | 14.2
36	T.76501	17	T.85487	27	0.14513	T.91014	9	24	
37	T.76519	18	T.85514	27	0.14486	T.91005	9	23	
38	T.76537	18	T.85540	26	0.14460	T.90996	9	22	10
39	T.76554	17	T.85567	27	0.14433	T.90987	9	21	6 | 1.0
40	T.76572	18	T.85594	27	0.14406	T.90978	9	20	7 | 1.2
41	T.76590	18	T.85620	26	0.14380	T.90969	9	19	8 | 1.3
42	T.76607	17	T.85647	27	0.14353	T.90960	9	18	9 | 1.5
43	T.76625	18	T.85674	27	0.14326	T.90951	9	17	10 | 1.7
44	T.76642	17	T.85700	26	0.14300	T.90942	9	16	20 | 3.3
45	T.76660	18	T.85727	27	0.14273	T.90933	9	15	30 | 5.0
46	T.76677	17	T.85754	27	0.14246	T.90924	9	14	40 | 6.7
47	T.76695	18	T.85780	26	0.14220	T.90915	9	13	50 | 8.3
48	T.76712	17	T.85807	27	0.14193	T.90906	9	12	
49	T.76730	18	T.85834	27	0.14166	T.90896	10	11	
50	T.76747	17	T.85860	26	0.14140	T.90887	9	10	9 | 8
51	T.76765	18	T.85887	27	0.14113	T.90878	9	9	6 | 0.9 | 0.8
52	T.76782	17	T.85913	26	0.14087	T.90869	9	8	7 | 1.1 | 0.9
53	T.76800	18	T.85940	27	0.14060	T.90860	9	7	8 | 1.2 | 1.1
54	T.76817	17	T.85967	27	0.14033	T.90851	9	6	9 | 1.4 | 1.2
55	T.76835	18	T.85993	26	0.14007	T.90842	9	5	10 | 1.5 | 1.3
56	T.76852	17	T.86020	27	0.13980	T.90832	10	4	20 | 3.0 | 2.7
57	T.76870	18	T.86046	26	0.13954	T.90823	9	3	30 | 4.5 | 4.0
58	T.76887	17	T.86073	27	0.13927	T.90814	9	2	40 | 6.0 | 5.3
59	T.76904	17	T.86100	27	0.13900	T.90805	9	1	50 | 7.5 | 6.7
60	T.76922	18	T.86126	26	0.13874	T.90796	9	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		′
0	T.76922	17	T.86126	27	0.13874	T.90796	9	60	0
1	T.76939	18	T.86153	26	0.13847	T.90787	10	59	1
2	T.76957	17	T.86179	27	0.13821	T.90777	9	58	2
3	T.76974	17	T.86206	26	0.13794	T.90768	9	57	3
4	T.76991	18	T.86232	27	0.13768	T.90759	9	56	4
5	T.77009	17	T.86259	26	0.13741	T.90749	9	55	5
6	T.77026	17	T.86285	27	0.13715	T.90741	10	54	6
7	T.77043	18	T.86312	26	0.13688	T.90731	9	53	7
8	T.77061	17	T.86338	27	0.13662	T.90722	9	52	8
9	T.77078	17	T.86365	27	0.13635	T.90713	9	51	9
10	T.77095	17	T.86392	26	0.13608	T.90704	10	50	10
11	T.77112	18	T.86418	27	0.13582	T.90694	9	49	11
12	T.77130	17	T.86445	26	0.13555	T.90685	9	48	12
13	T.77147	17	T.86471	27	0.13529	T.90676	9	47	13
14	T.77164	17	T.86498	26	0.13502	T.90667	10	46	14
15	T.77181	18	T.86524	27	0.13476	T.90657	9	45	15
16	T.77199	17	T.86551	26	0.13449	T.90648	9	44	16
17	T.77216	17	T.86577	26	0.13423	T.90639	9	43	17
18	T.77233	17	T.86603	27	0.13397	T.90630	10	42	18
19	T.77250	18	T.86630	26	0.13370	T.90620	9	41	19
20	T.77268	17	T.86656	27	0.13344	T.90611	9	40	20
21	T.77285	17	T.86683	26	0.13317	T.90602	10	39	21
22	T.77302	17	T.86709	27	0.13291	T.90592	9	38	22
23	T.77319	17	T.86736	26	0.13264	T.90583	9	37	23
24	T.77336	17	T.86762	27	0.13238	T.90574	9	36	24
25	T.77353	17	T.86789	26	0.13211	T.90565	10	35	25
26	T.77370	17	T.86815	27	0.13185	T.90555	9	34	26
27	T.77387	18	T.86842	26	0.13158	T.90546	9	33	27
28	T.77405	17	T.86868	26	0.13132	T.90537	10	32	28
29	T.77422	17	T.86894	27	0.13106	T.90527	9	31	29
30	T.77439	17	T.86921	26	0.13079	T.90518	9	30	30
31	T.77456	17	T.86947	27	0.13053	T.90509	10	29	31
32	T.77473	17	T.86974	26	0.13026	T.90499	9	28	32
33	T.77490	17	T.87000	27	0.13000	T.90490	10	27	33
34	T.77507	17	T.87027	26	0.12973	T.90480	9	26	34
35	T.77524	17	T.87053	26	0.12947	T.90471	9	25	35
36	T.77541	17	T.87079	27	0.12921	T.90462	10	24	36
37	T.77558	17	T.87106	26	0.12894	T.90452	9	23	37
38	T.77575	17	T.87132	26	0.12868	T.90443	9	22	38
39	T.77592	17	T.87158	27	0.12842	T.90434	10	21	39
40	T.77609	17	T.87185	26	0.12815	T.90424	9	20	40
41	T.77626	17	T.87211	27	0.12789	T.90415	10	19	41
42	T.77643	17	T.87238	26	0.12762	T.90405	9	18	42
43	T.77660	17	T.87264	26	0.12736	T.90396	10	17	43
44	T.77677	17	T.87290	27	0.12710	T.90386	9	16	44
45	T.77694	17	T.87317	26	0.12683	T.90377	9	15	45
46	T.77711	17	T.87343	26	0.12657	T.90368	10	14	46
47	T.77728	16	T.87369	27	0.12631	T.90358	9	13	47
48	T.77744	17	T.87396	26	0.12604	T.90349	10	12	48
49	T.77761	17	T.87422	26	0.12578	T.90339	9	11	49
50	T.77778	17	T.87448	27	0.12552	T.90330	10	10	50
51	T.77795	17	T.87475	26	0.12525	T.90320	9	9	51
52	T.77812	17	T.87501	26	0.12499	T.90311	10	8	52
53	T.77829	17	T.87527	27	0.12473	T.90301	9	7	53
54	T.77846	16	T.87554	26	0.12446	T.90292	10	6	54
55	T.77862	17	T.87580	26	0.12420	T.90282	9	5	55
56	T.77879	17	T.87606	27	0.12394	T.90273	10	4	56
57	T.77896	17	T.87633	26	0.12367	T.90263	9	3	57
58	T.77913	17	T.87659	26	0.12341	T.90254	10	2	58
59	T.77930	16	T.87685	26	0.12315	T.90244	9	1	59
60	T.77946		T.87711		0.12289	T.90235		0	60
	log cos	d	log cot	c. d.	log tan	log sin	d	′	

p. p.

	27	26
6	2.7	2.6
7	3.2	3.0
8	3.6	3.5
9	4.1	3.9
10	4.5	4.3
20	9.0	8.7
30	13.5	13.0
40	18.0	17.3
50	22.5	21.7

	18
6	1.8
7	2.1
8	2.4
9	2.7
10	3.0
20	6.0
30	9.0
40	12.0
50	15.0

	17
6	1.7
7	2.0
8	2.3
9	2.6
10	2.8
20	5.7
30	8.5
40	11.3
50	14.2

	16
6	1.6
7	1.9
8	2.1
9	2.4
10	2.7
20	5.3
30	8.0
40	10.7
50	13.3

	10	9
6	1.0	0.9
7	1.2	1.1
8	1.3	1.2
9	1.5	1.4
10	1.7	1.5
20	3.3	3.0
30	5.0	4.5
40	6.7	6.0
50	8.3	7.5

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.77946		T.87711		0.12289	T.90235		60	
1	T.77963	17	T.87738	27	0.12262	T.90225	10	59	
2	T.77980	17	T.87764	26	0.12236	T.90216	9	58	27
3	T.77997	17	T.87790	26	0.12210	T.90206	10	57	6 | 2.7
4	T.78013	16	T.87817	27	0.12183	T.90197	9	56	7 | 3.2
5	T.78030	17	T.87843	26	0.12157	T.90187	10	55	8 | 3.6
6	T.78047	17	T.87869	26	0.12131	T.90178	9	54	9 | 4.1
7	T.78063	16	T.87895	26	0.12105	T.90168	10	53	10 | 4.5
8	T.78080	17	T.87922	27	0.12078	T.90159	9	52	20 | 9.0
9	T.78097	17	T.87948	26	0.12052	T.90149	10	51	30 | 13.5
10	T.78113	16	T.87974	26	0.12026	T.90139	10	50	40 | 18.0
		17		26			9		50 | 22.5
11	T.78130		T.88000		0.12000	T.90130		49	
12	T.78147	17	T.88027	27	0.11973	T.90120	10	48	
13	T.78163	16	T.88053	26	0.11947	T.90111	9	47	
14	T.78180	17	T.88079	26	0.11921	T.90101	10	46	26
15	T.78197	17	T.88105	26	0.11895	T.90091	10	45	6 | 2.6
16	T.78213	16	T.88131	27	0.11869	T.90082	9	44	7 | 3.0
17	T.78230	17	T.88158	26	0.11842	T.90072	10	43	8 | 3.5
18	T.78246	16	T.88184	26	0.11816	T.90063	9	42	9 | 3.9
19	T.78263	17	T.88210	26	0.11790	T.90053	10	41	10 | 4.3
20	T.78280	17	T.88236	26	0.11764	T.90043	10	40	20 | 8.7
		16		26			9		30 | 13.0
21	T.78296		T.88262		0.11738	T.90034		39	40 | 17.3
22	T.78313	17	T.88289	27	0.11711	T.90024	10	38	50 | 21.7
23	T.78329	16	T.88315	26	0.11685	T.90014	10	37	
24	T.78346	17	T.88341	26	0.11659	T.90005	9	36	
25	T.78362	16	T.88367	26	0.11633	T.89995	10	35	
26	T.78379	17	T.88393	26	0.11607	T.89985	10	34	17
27	T.78395	16	T.88420	27	0.11580	T.89976	9	33	6 | 1.7
28	T.78412	17	T.88446	26	0.11554	T.89966	10	32	7 | 2.0
29	T.78428	16	T.88472	26	0.11528	T.89956	10	31	8 | 2.3
30	T.78445	17	T.88498	26	0.11502	T.89947	9	30	9 | 2.6
		16		26			10		10 | 2.8
31	T.78461		T.88524		0.11476	T.89937		29	20 | 5.7
32	T.78478	17	T.88550	26	0.11450	T.89927	10	28	30 | 8.5
33	T.78494	16	T.88577	27	0.11423	T.89918	9	27	40 | 11.3
34	T.78510	16	T.88603	26	0.11397	T.89908	10	26	50 | 14.2
35	T.78527	17	T.88629	26	0.11371	T.89898	10	25	
36	T.78543	16	T.88655	26	0.11345	T.89888	10	24	
37	T.78560	17	T.88681	26	0.11319	T.89879	9	23	
38	T.78576	16	T.88707	26	0.11293	T.89869	10	22	16
39	T.78592	16	T.88733	26	0.11267	T.89859	10	21	6 | 1.6
40	T.78609	17	T.88759	26	0.11241	T.89849	10	20	7 | 1.9
		16		27			9		8 | 2.1
41	T.78625		T.88786		0.11214	T.89840		19	9 | 2.4
42	T.78642	17	T.88812	26	0.11188	T.89830	10	18	10 | 2.7
43	T.78658	16	T.88838	26	0.11162	T.89820	10	17	20 | 5.3
44	T.78674	16	T.88864	26	0.11136	T.89810	10	16	30 | 8.0
45	T.78691	17	T.88890	26	0.11110	T.89801	9	15	40 | 10.7
46	T.78707	16	T.88916	26	0.11084	T.89791	10	14	50 | 13.3
47	T.78723	16	T.88942	26	0.11058	T.89781	10	13	
48	T.78739	16	T.88968	26	0.11032	T.89771	10	12	
49	T.78756	17	T.88994	26	0.11006	T.89761	10	11	
50	T.78772	16	T.89020	26	0.10980	T.89752	9	10	10 | 9
		16		26			10		6 | 1.0 | 0.9
51	T.78788		T.89046		0.10954	T.89742		9	7 | 1.2 | 1.1
52	T.78805	17	T.89073	27	0.10927	T.89732	10	8	8 | 1.3 | 1.2
53	T.78821	16	T.89099	26	0.10901	T.89722	10	7	9 | 1.5 | 1.4
54	T.78837	16	T.89125	26	0.10875	T.89712	10	6	10 | 1.7 | 1.5
55	T.78853	16	T.89151	26	0.10849	T.89702	10	5	20 | 3.3 | 3.0
56	T.78869	16	T.89177	26	0.10823	T.89693	9	4	30 | 5.0 | 4.5
57	T.78886	17	T.89203	26	0.10797	T.89683	10	3	40 | 6.7 | 6.0
58	T.78902	16	T.89229	26	0.10771	T.89673	10	2	50 | 8.3 | 7.5
59	T.78918	16	T.89255	26	0.10745	T.89663	10	1	
60	T.78934		T.89281		0.10719	T.89653	10	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.78934	16	T.89281	26	0.10719	T.89653	10	60	
1	T.78950	17	T.89307	26	0.10693	T.89643	10	59	
2	T.78967	16	T.89333	26	0.10667	T.89633	9	58	28 25
3	T.78983	16	T.89359	26	0.10641	T.89624	10	57	6 2.6 2.5
4	T.78999	16	T.89385	26	0.10615	T.89614	10	56	7 3.0 2.9
5	T.79015	16	T.89411	26	0.10589	T.89604	10	55	8 3.5 3.3
6	T.79031	16	T.89437	26	0.10563	T.89594	10	54	9 3.9 3.8
7	T.79047	16	T.89463	26	0.10537	T.89584	10	53	10 4.3 4.2
8	T.79063	16	T.89489	26	0.10511	T.89574	10	52	20 8.7 8.3
9	T.79079	16	T.89515	26	0.10485	T.89564	10	51	30 13.0 12.5
10	T.79095	16	T.89541	26	0.10459	T.89554	10	50	40 17.3 16.7
									50 21.7 20.8
11	T.79111	17	T.89567	26	0.10433	T.89544	10	49	
12	T.79128	16	T.89593	26	0.10407	T.89534	10	48	
13	T.79144	16	T.89619	26	0.10381	T.89524	10	47	
14	T.79160	16	T.89645	26	0.10355	T.89514	10	46	
15	T.79176	16	T.89671	26	0.10329	T.89504	9	45	17
16	T.79192	16	T.89697	26	0.10303	T.89495	10	44	6 1.7
17	T.79208	16	T.89723	26	0.10277	T.89485	10	43	7 2.0
18	T.79224	16	T.89749	26	0.10251	T.89475	10	42	8 2.3
19	T.79240	16	T.89775	26	0.10225	T.89465	10	41	9 2.6
20	T.79256	16	T.89801	26	0.10199	T.89455	10	40	10 2.8
									20 5.7
21	T.79272	16	T.89827	26	0.10173	T.89445	10	39	30 8.5
22	T.79288	16	T.89853	26	0.10147	T.89435	10	38	40 11.3
23	T.79304	15	T.89879	26	0.10121	T.89425	10	37	50 14.2
24	T.79319	16	T.89905	26	0.10095	T.89415	10	36	
25	T.79335	16	T.89931	26	0.10069	T.89405	10	35	
26	T.79351	16	T.89957	26	0.10043	T.89395	10	34	16 15
27	T.79367	16	T.89983	26	0.10017	T.89385	10	33	6 1.6 1.5
28	T.79383	16	T.90009	26	0.09991	T.89375	11	32	7 1.9 1.8
29	T.79399	16	T.90035	26	0.09965	T.89364	10	31	8 2.1 2.0
30	T.79415	16	T.90061	25	0.09939	T.89354	10	30	9 2.4 2.3
									10 2.7 2.5
31	T.79431	16	T.90086	26	0.09914	T.89344	10	29	20 5.3 5.0
32	T.79447	16	T.90112	26	0.09888	T.89334	10	28	30 8.0 7.5
33	T.79463	15	T.90138	26	0.09862	T.89324	10	27	40 10.7 10.0
34	T.79478	16	T.90164	26	0.09836	T.89314	10	26	50 13.3 12.5
35	T.79494	16	T.90190	26	0.09810	T.89304	10	25	
36	T.79510	16	T.90216	26	0.09784	T.89294	10	24	
37	T.79526	16	T.90242	26	0.09758	T.89284	10	23	
38	T.79542	16	T.90268	26	0.09732	T.89274	10	22	11
39	T.79558	16	T.90294	26	0.09706	T.89264	10	21	6 1.1
40	T.79573	15	T.90320	26	0.09680	T.89254	10	20	7 1.3
		16		26			10		8 1.5
41	T.79589	16	T.90346	25	0.09654	T.89244	11	19	9 1.7
42	T.79605	16	T.90371	26	0.09629	T.89233	10	18	10 1.8
43	T.79621	15	T.90397	26	0.09603	T.89223	10	17	20 3.7
44	T.79636	16	T.90423	26	0.09577	T.89213	10	16	30 5.5
45	T.79652	16	T.90449	26	0.09551	T.89203	10	15	40 7.3
46	T.79668	16	T.90475	26	0.09525	T.89193	10	14	50 9.2
47	T.79684	15	T.90501	26	0.09499	T.89183	10	13	
48	T.79699	16	T.90527	26	0.09473	T.89173	11	12	
49	T.79715	16	T.90553	25	0.09447	T.89162	10	11	
50	T.79731	15	T.90578	26	0.09422	T.89152	10	10	10 9
51	T.79746	16	T.90604	26	0.09396	T.89142	10	9	6 1.0 0.9
52	T.79762	16	T.90630	26	0.09370	T.89132	10	8	7 1.2 1.1
53	T.79778	15	T.90656	26	0.09344	T.89122	10	7	8 1.3 1.2
54	T.79793	16	T.90682	26	0.09318	T.89112	11	6	9 1.5 1.4
55	T.79809	16	T.90708	26	0.09292	T.89101	10	5	10 1.7 1.5
56	T.79825	15	T.90734	25	0.09266	T.89091	10	4	20 3.3 3.0
57	T.79840	16	T.90759	26	0.09241	T.89081	10	3	30 5.0 4.5
58	T.79856	16	T.90785	26	0.09215	T.89071	10	2	40 6.7 6.0
59	T.79872	15	T.90811	26	0.09189	T.89060	11	1	50 8.3 7.5
60	T.79887		T.90837		0.09163	T.89050	10	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.79887	16	T.90837	26	0.09163	T.89050	10	60	
1	T.79903	15	T.90863	26	0.09137	T.89040	10	59	
2	T.79918	16	T.90889	25	0.09111	T.89030	10	58	**28**
3	T.79934	16	T.90914	26	0.09086	T.89020	11	57	6 | 2.6
4	T.79950	15	T.90940	26	0.09060	T.89009	10	56	7 | 3.0
5	T.79965	16	T.90966	26	0.09034	T.88999	10	55	8 | 3.5
6	T.79981	15	T.90992	26	0.09008	T.88989	11	54	9 | 3.9
7	T.79996	16	T.91018	25	0.08982	T.88978	10	53	10 | 4.3
8	T.80012	15	T.91043	26	0.08957	T.88968	10	52	20 | 8.7
9	T.80027	16	T.91069	26	0.08931	T.88958	10	51	30 | 13.0
10	T.80043	15	T.91095	26	0.08905	T.88948	11	50	40 | 17.3
11	T.80058	16	T.91121	26	0.08879	T.88937	10	49	50 | 21.7
12	T.80074	15	T.91147	25	0.08853	T.88927	10	48	
13	T.80089	16	T.91172	26	0.08828	T.88917	11	47	
14	T.80105	15	T.91198	26	0.08802	T.88906	10	46	**25**
15	T.80120	16	T.91224	26	0.08776	T.88896	10	45	6 | 2.5
16	T.80136	15	T.91250	26	0.08750	T.88886	11	44	7 | 2.9
17	T.80151	15	T.91276	25	0.08724	T.88875	10	43	8 | 3.3
18	T.80166	16	T.91301	26	0.08699	T.88865	10	42	9 | 3.8
19	T.80182	15	T.91327	26	0.08673	T.88855	11	41	10 | 4.2
20	T.80197	16	T.91353	26	0.08647	T.88844	10	40	20 | 8.3
21	T.80213	15	T.91379	25	0.08621	T.88834	10	39	30 | 12.5
22	T.80228	16	T.91404	26	0.08596	T.88824	11	38	40 | 16.7
23	T.80244	15	T.91430	26	0.08570	T.88813	10	37	50 | 20.8
24	T.80259	15	T.91456	26	0.08544	T.88803	10	36	
25	T.80274	16	T.91482	25	0.08518	T.88793	11	35	
26	T.80290	15	T.91507	26	0.08493	T.88782	10	34	**16**
27	T.80305	15	T.91533	26	0.08467	T.88772	11	33	6 | 1.6
28	T.80320	16	T.91559	26	0.08441	T.88761	10	32	7 | 1.9
29	T.80336	15	T.91585	26	0.08415	T.88751	10	31	8 | 2.1
30	T.80351	15	T.91610	25	0.08390	T.88741	11	30	9 | 2.4
31	T.80366	16	T.91636	26	0.08364	T.88730	10	29	10 | 2.7
32	T.80382	15	T.91662	26	0.08338	T.88720	11	28	20 | 5.3
33	T.80397	15	T.91688	26	0.08312	T.88709	10	27	30 | 8.0
34	T.80412	16	T.91713	25	0.08287	T.88699	11	26	40 | 10.7
35	T.80428	15	T.91739	26	0.08261	T.88688	10	25	50 | 13.3
36	T.80443	15	T.91765	26	0.08235	T.88678	10	24	
37	T.80458	15	T.91791	26	0.08209	T.88668	11	23	
38	T.80473	16	T.91816	25	0.08184	T.88657	10	22	**15**
39	T.80489	15	T.91842	26	0.08158	T.88647	11	21	6 | 1.5
40	T.80504	15	T.91868	26	0.08132	T.88636	10	20	7 | 1.8
41	T.80519	15	T.91893	25	0.08107	T.88626	11	19	8 | 2.0
42	T.80534	16	T.91919	26	0.08081	T.88615	10	18	9 | 2.3
43	T.80550	15	T.91945	26	0.08055	T.88605	11	17	10 | 2.5
44	T.80565	15	T.91971	26	0.08029	T.88594	10	16	20 | 5.0
45	T.80580	15	T.91996	25	0.08004	T.88584	11	15	30 | 7.5
46	T.80595	15	T.92022	26	0.07978	T.88573	10	14	40 | 10.0
47	T.80610	15	T.92048	26	0.07952	T.88563	11	13	50 | 12.5
48	T.80625	16	T.92073	25	0.07927	T.88552	10	12	
49	T.80641	15	T.92099	26	0.07901	T.88542	11	11	
50	T.80656	15	T.92125	26	0.07875	T.88531	10	10	**11 | 10**
51	T.80671	15	T.92150	25	0.07850	T.88521	10	9	6 | 1.1 | 1.0
52	T.80686	15	T.92176	26	0.07824	T.88510	11	8	7 | 1.3 | 1.2
53	T.80701	15	T.92202	26	0.07798	T.88499	11	7	8 | 1.5 | 1.3
54	T.80716	15	T.92227	25	0.07773	T.88489	10	6	9 | 1.7 | 1.5
55	T.80731	15	T.92253	26	0.07747	T.88478	11	5	10 | 1.8 | 1.7
56	T.80746	15	T.92279	26	0.07721	T.88468	10	4	20 | 3.7 | 3.3
57	T.80762	16	T.92304	25	0.07696	T.88457	11	3	30 | 5.5 | 5.0
58	T.80777	15	T.92330	26	0.07670	T.88447	10	2	40 | 7.3 | 6.7
59	T.80792	15	T.92356	26	0.07644	T.88436	11	1	50 | 9.2 | 8.3
60	T.80807	15	T.92381	25	0.07619	T.88425	11	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.80807		T.92381		0.07619	T.88425		60	
		15		26			10		
1	T.80822		T.92407		0.07593	T.88415	11	59	
		15		26			10		
2	T.80837		T.92433		0.07567	T.88404		58	
		15		25			11		28
3	T.80852		T.92458		0.07542	T.88394		57	6 | 2.6
		15		26			11		
4	T.80867		T.92484		0.07516	T.88383		56	7 | 3.0
		15		26			10		
5	T.80882		T.92510		0.07490	T.88372		55	8 | 3.5
		15		25			11		
6	T.80897		T.92535		0.07465	T.88362		54	9 | 3.9
		15		26			11		
7	T.80912		T.92561		0.07439	T.88351		53	10 | 4.3
		15		26			10		
8	T.80927		T.92587		0.07413	T.88340		52	20 | 8.7
		15		25			11		
9	T.80942		T.92612		0.07388	T.88330		51	30 | 13.0
		15		26			11		
10	T.80957		T.92638		0.07362	T.88319		50	40 | 17.3
		15		25			11		50 | 21.7
11	T.80972		T.92663		0.07337	T.88308		49	
		15		26			10		
12	T.80987		T.92689		0.07311	T.88298	11	48	
		15		26			11		
13	T.81002		T.92715		0.07285	T.88287		47	
		15		25			11		25
14	T.81017		T.92740		0.07260	T.88276		46	
		15		26			10		6 | 2.5
15	T.81032		T.92766		0.07234	T.88266		45	
		15		26			11		7 | 2.9
16	T.81047		T.92792		0.07208	T.88255		44	
		14		25			11		8 | 3.3
17	T.81061		T.92817		0.07183	T.88244		43	
		15		26			10		9 | 3.8
18	T.81076		T.92843		0.07157	T.88234		42	
		15		25			11		10 | 4.2
19	T.81091		T.92868		0.07132	T.88223		41	
		15		26			11		20 | 8.3
20	T.81106		T.92894		0.07106	T.88212		40	30 | 12.5
		15		26			11		40 | 16.7
21	T.81121		T.92920		0.07080	T.88201		39	50 | 20.8
		15		25			10		
22	T.81136		T.92945		0.07055	T.88191		38	
		15		26			11		
23	T.81151		T.92971		0.07029	T.88180		37	
		15		25			11		15
24	T.81166		T.92996		0.07004	T.88169		36	
		14		26			11		
25	T.81180		T.93022		0.06978	T.88158		35	6 | 1.5
		15		26			10		
26	T.81195		T.93048		0.06952	T.88148		34	7 | 1.8
		15		25			11		
27	T.81210		T.93073		0.06927	T.88137		33	8 | 2.0
		15		26			11		
28	T.81225		T.93099		0.06901	T.88126		32	9 | 2.3
		15		25			11		
29	T.81240		T.93124		0.06876	T.88115		31	10 | 2.5
		14		26			10		
30	T.81254		T.93150		0.06850	T.88105		30	20 | 5.0
		15		25			11		
31	T.81269		T.93175		0.06825	T.88094		29	30 | 7.5
		15		26			11		
32	T.81284		T.93201		0.06799	T.88083		28	40 | 10.0
		15		26			11		
33	T.81299		T.93227		0.06773	T.88072		27	50 | 12.5
		15		25			11		
34	T.81314		T.93252		0.06748	T.88061		26	
		14		26			10		
35	T.81328		T.93278		0.06722	T.88051		25	
		15		25			11		
36	T.81343		T.93303		0.06697	T.88040		24	
		15		26			11		
37	T.81358		T.93329		0.06671	T.88029		23	
		14		25			11		14
38	T.81372		T.93354		0.06645	T.88018		22	
		15		26			11		
39	T.81387		T.93380		0.06620	T.88007		21	6 | 1.4
		15		26			11		
40	T.81402		T.93406		0.06594	T.87996		20	7 | 1.6
		15		25			11		
41	T.81417		T.93431		0.06569	T.87985		19	8 | 1.9
		14		26			10		
42	T.81431		T.93457		0.06543	T.87975		18	9 | 2.1
		15		25			11		
43	T.81446		T.93482		0.06518	T.87964		17	10 | 2.3
		15		26			11		
44	T.81461		T.93508		0.06492	T.87953		16	20 | 4.7
		14		25			11		
45	T.81475		T.93533		0.06467	T.87942		15	30 | 7.0
		15		26			11		
46	T.81490		T.93559		0.06441	T.87931		14	40 | 9.3
		15		25			11		
47	T.81505		T.93584		0.06416	T.87920		13	50 | 11.7
		14		26			11		
48	T.81519		T.93610		0.06390	T.87909		12	
		15		26			11		
49	T.81534		T.93636		0.06364	T.87898		11	
		15		25			11		
50	T.81549		T.93661		0.06339	T.87887		10	11 | 10
		14		26			10		6 | 1.1 | 1.0
51	T.81563		T.93687		0.06313	T.87877		9	7 | 1.3 | 1.2
		15		25			11		
52	T.81578		T.93712		0.06288	T.87866		8	8 | 1.5 | 1.3
		14		26			11		
53	T.81592		T.93738		0.06262	T.87855		7	9 | 1.7 | 1.5
		15		25			11		
54	T.81607		T.93763		0.06237	T.87844		6	10 | 1.8 | 1.7
		15		26			11		
55	T.81622		T.93789		0.06211	T.87833		5	20 | 3.7 | 3.3
		14		25			11		
56	T.81636		T.93814		0.06186	T.87822		4	30 | 5.5 | 5.0
		15		26			11		
57	T.81651		T.93840		0.06160	T.87811		3	40 | 7.3 | 6.7
		14		25			11		
58	T.81665		T.93865		0.06135	T.87800		2	50 | 9.2 | 8.3
		15		26			11		
59	T.81680		T.93891		0.06109	T.87789		1	
		14		25			11		
60	T.81694		T.93916		0.06084	T.87778		0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.′
0	T.81694		T.93916		0.06084	T.87778		60	
		15		26			11		
1	T.81709		T.93942		0.06058	T.87767		59	
		14		25			11		
2	T.81723		T.93967		0.06033	T.87756		58	
		15		26			11		28
3	T.81738		T.93993		0.06007	T.87745		57	6 | 2.6
		14		25			11		
4	T.81752		T.94018		0.05982	T.87734		56	7 | 3.0
		15		26			11		
5	T.81767		T.94044		0.05956	T.87723		55	8 | 3.5
		14		25			11		
6	T.81781		T.94069		0.05931	T.87712		54	9 | 3.9
		15		26			11		
7	T.81796		T.94095		0.05905	T.87701		53	10 | 4.3
		14		25			11		
8	T.81810		T.94120		0.05880	T.87690		52	20 | 8.7
		15		26			11		
9	T.81825		T.94146		0.05854	T.87679		51	30 | 13.0
		14		25			11		
10	T.81839		T.94171		0.05829	T.87668		50	40 | 17.3
		15		26			11		50 | 21.7
11	T.81854		T.94197		0.05803	T.87657		49	
		14		25			11		
12	T.81868		T.94222		0.05778	T.87646		48	
		14		26			11		
13	T.81882		T.94248		0.05752	T.87635		47	
		15		25			11		25
14	T.81897		T.94273		0.05727	T.87624		46	
		14		26			11		6 | 2.5
15	T.81911		T.94299		0.05701	T.87613		45	
		15		25			12		7 | 2.9
16	T.81926		T.94324		0.05676	T.87601		44	
		14		26			11		8 | 3.3
17	T.81940		T.94350		0.05650	T.87590		43	
		15		25			11		9 | 3.8
18	T.81955		T.94375		0.05625	T.87579		42	
		14		26			11		10 | 4.2
19	T.81969		T.94401		0.05599	T.87568		41	
		14		25			11		20 | 8.3
20	T.81983		T.94426		0.05574	T.87557		40	
		15		26			11		30 | 12.5
21	T.81998		T.94452		0.05548	T.87546		39	40 | 16.7
		14		25			11		
22	T.82012		T.94477		0.05523	T.87535		38	50 | 20.8
		14		26			11		
23	T.82026		T.94503		0.05497	T.87524		37	
		15		25			11		
24	T.82041		T.94528		0.05472	T.87513		36	
		14		26			12		
25	T.82055		T.94554		0.05446	T.87501		35	
		14		25			11		
26	T.82069		T.94579		0.05421	T.87490		34	15
		15		25			11		
27	T.82084		T.94604		0.05396	T.87479		33	6 | 1.5
		14		26			11		
28	T.82098		T.94630		0.05370	T.87468		32	7 | 1.8
		14		25			11		
29	T.82112		T.94655		0.05345	T.87457		31	8 | 2.0
		14		26			11		
30	T.82126		T.94681		0.05319	T.87446		30	9 | 2.3
		15		25			12		10 | 2.5
31	T.82141		T.94706		0.05294	T.87434		29	20 | 5.0
		14		26			11		
32	T.82155		T.94732		0.05268	T.87423		28	30 | 7.5
		14		25			11		
33	T.82169		T.94757		0.05243	T.87412		27	40 | 10.0
		15		26			11		
34	T.82184		T.94783		0.05217	T.87401		26	50 | 12.5
		14		25			11		
35	T.82198		T.94808		0.05192	T.87390		25	
		14		26			12		
36	T.82212		T.94834		0.05166	T.87378		24	
		14		25			11		
37	T.82226		T.94859		0.05141	T.87367		23	
		14		25			11		
38	T.82240		T.94884		0.05116	T.87356		22	14
		15		26			11		
39	T.82255		T.94910		0.05090	T.87345		21	6 | 1.4
		14		25			11		
40	T.82269		T.94935		0.05065	T.87334		20	7 | 1.6
		14		26			12		8 | 1.9
41	T.82283		T.94961		0.05039	T.87322		19	9 | 2.1
		14		25			11		
42	T.82297		T.94986		0.05014	T.87311		18	10 | 2.3
		14		26			11		
43	T.82311		T.95012		0.04988	T.87300		17	20 | 4.7
		15		25			12		
44	T.82326		T.95037		0.04963	T.87288		16	30 | 7.0
		14		25			11		
45	T.82340		T.95062		0.04938	T.87277		15	40 | 9.3
		14		26			11		
46	T.82354		T.95088		0.04912	T.87266		14	50 | 11.7
		14		25			11		
47	T.82368		T.95113		0.04887	T.87255		13	
		14		26			12		
48	T.82382		T.95139		0.04861	T.87243		12	
		14		25			11		
49	T.82396		T.95164		0.04836	T.87232		11	
		14		26			11		12 | 11
50	T.82410		T.95190		0.04810	T.87221		10	6 | 1.2 | 1.1
		14		25			12		7 | 1.4 | 1.3
51	T.82424		T.95215		0.04785	T.87209		9	8 | 1.6 | 1.5
		15		25			11		
52	T.82439		T.95240		0.04760	T.87198		8	9 | 1.8 | 1.7
		14		26			11		
53	T.82453		T.95266		0.04734	T.87187		7	10 | 2.0 | 1.8
		14		25			12		
54	T.82467		T.95291		0.04709	T.87175		6	20 | 4.0 | 3.7
		14		26			11		
55	T.82481		T.95317		0.04683	T.87164		5	30 | 6.0 | 5.5
		14		25			11		
56	T.82495		T.95342		0.04658	T.87153		4	40 | 8.0 | 7.3
		14		26			12		
57	T.82509		T.95368		0.04632	T.87141		3	50 | 10.0 | 9.2
		14		25			11		
58	T.82523		T.95393		0.04607	T.87130		2	
		14		25			11		
59	T.82537		T.95418		0.04582	T.87119		1	
		14		26			12		
60	T.82551		T.95444		0.04556	T.87107		0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.82551		T.95444		0.04556	T.87107		60	
1	T.82565	14	T.95469	25	0.04531	T.87096	11	59	
2	T.82579	14	T.95495	26	0.04505	T.87085	11	58	26
3	T.82593	14	T.95520	25	0.04480	T.87073	12	57	6 2.6
4	T.82607	14	T.95545	25	0.04455	T.87062	11	56	7 3.0
5	T.82621	14	T.95571	26	0.04429	T.87050	12	55	8 3.5
6	T.82635	14	T.95596	25	0.04404	T.87039	11	54	9 3.9
7	T.82649	14	T.95622	26	0.04378	T.87028	11	53	10 4.3
8	T.82663	14	T.95647	25	0.04353	T.87016	12	52	20 8.7
9	T.82677	14	T.95672	25	0.04328	T.87005	11	51	30 13.0
10	T.82691		T.95698	26	0.04302	T.86993	12	50	40 17.3
		14		25			11		50 21.7
11	T.82705		T.95723		0.04277	T.86982		49	
12	T.82719	14	T.95748	25	0.04252	T.86970	12	48	
13	T.82733	14	T.95774	26	0.04226	T.86959	11	47	
14	T.82747	14	T.95799	25	0.04201	T.86947	12	46	25
15	T.82761	14	T.95825	26	0.04175	T.86936	11	45	6 2.5
16	T.82775	14	T.95850	25	0.04150	T.86924	12	44	7 2.9
17	T.82788	13	T.95875	25	0.04125	T.86913	11	43	8 3.3
18	T.82802	14	T.95901	26	0.04099	T.86902	11	42	9 3.8
19	T.82816	14	T.95926	25	0.04074	T.86890	12	41	10 4.2
20	T.82830	14	T.95952	26	0.04048	T.86879	11	40	20 8.3
				25			12		30 12.5
21	T.82844		T.95977		0.04023	T.86867		39	40 16.7
22	T.82858	14	T.96002	25	0.03998	T.86855	12	38	50 20.8
23	T.82872	14	T.96028	26	0.03972	T.86844	11	37	
24	T.82885	13	T.96053	25	0.03947	T.86832	12	36	
25	T.82899	14	T.96078	25	0.03922	T.86821	11	35	
26	T.82913	14	T.96104	26	0.03896	T.86809	12	34	14
27	T.82927	14	T.96129	25	0.03871	T.86798	11	33	6 1.4
28	T.82941	14	T.96155	26	0.03845	T.86786	12	32	7 1.6
29	T.82955	14	T.96180	25	0.03820	T.86775	11	31	8 1.9
30	T.82968	13	T.96205	25	0.03795	T.86763	12	30	9 2.1
		14		26			11		10 2.3
31	T.82982		T.96231		0.03769	T.86752		29	20 4.7
32	T.82996	14	T.96256	25	0.03744	T.86740	12	28	30 7.0
33	T.83010	14	T.96281	25	0.03719	T.86728	12	27	40 9.3
34	T.83023	13	T.96307	26	0.03693	T.86717	11	26	50 11.7
35	T.83037	14	T.96332	25	0.03668	T.86705	12	25	
36	T.83051	14	T.96357	25	0.03643	T.86694	11	24	
37	T.83065	14	T.96383	26	0.03617	T.86682	12	23	
38	T.83078	13	T.96408	25	0.03592	T.86670	12	22	13
39	T.83092	14	T.96433	25	0.03567	T.86659	11	21	6 1.3
40	T.83106	14	T.96459	26	0.03541	T.86647	12	20	7 1.5
				25			12		8 1.7
41	T.83120		T.96484		0.03516	T.86635		19	9 2.0
42	T.83133	13	T.96510	26	0.03490	T.86624	11	18	10 2.2
43	T.83147	14	T.96535	25	0.03465	T.86612	12	17	20 4.3
44	T.83161	14	T.96560	25	0.03440	T.86600	12	16	30 6.5
45	T.83174	13	T.96586	26	0.03414	T.86589	11	15	40 8.7
46	T.83188	14	T.96611	25	0.03389	T.86577	12	14	50 10.8
47	T.83202	14	T.96636	25	0.03364	T.86565	12	13	
48	T.83215	13	T.96662	26	0.03338	T.86554	11	12	
49	T.83229	14	T.96687	25	0.03313	T.86542	12	11	
50	T.83242	13	T.96712	25	0.03288	T.86530	12	10	
		14		26			12		12 11
51	T.83256		T.96738		0.03262	T.86518		9	6 1.2 1.1
52	T.83270	14	T.96763	25	0.03237	T.86507	11	8	7 1.4 1.3
53	T.83283	13	T.96788	25	0.03212	T.86495	12	7	8 1.6 1.5
54	T.83297	14	T.96814	26	0.03186	T.86483	12	6	9 1.8 1.7
55	T.83310	13	T.96839	25	0.03161	T.86472	11	5	10 2.0 1.8
56	T.83324	14	T.96864	25	0.03136	T.86460	12	4	20 4.0 3.7
57	T.83338	14	T.96890	26	0.03110	T.86448	12	3	30 6.0 5.5
58	T.83351	13	T.96915	25	0.03085	T.86436	12	2	40 8.0 7.3
59	T.83365	14	T.96940	25	0.03060	T.86425	11	1	50 10.0 9.2
60	T.83378	13	T.96966	26	0.03034	T.86413	12	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	7.83378		7.96966		0.03034	7.86413		60	
1	7.83392	14	7.96991	25	0.03009	7.86401	12	59	
2	7.83405	13	7.97016	25	0.02984	7.86389	12	58	26
3	7.83419	14	7.97042	26	0.02958	7.86377	12	57	6 2.6
4	7.83432	13	7.97067	25	0.02933	7.86366	11	56	7 3.0
5	7.83446	14	7.97092	25	0.02908	7.86354	12	55	8 3.5
6	7.83459	13	7.97118	26	0.02882	7.86342	12	54	9 3.9
7	7.83473	14	7.97143	25	0.02857	7.86330	12	53	10 4.3
8	7.83486	13	7.97168	25	0.02832	7.86318	12	52	20 8.7
9	7.83500	14	7.97193	25	0.02807	7.86306	12	51	30 13.0
10	7.83513	13	7.97219	26	0.02781	7.86295	11	50	40 17.3
		14		25			12		50 21.7
11	7.83527		7.97244		0.02756	7.86283		49	
12	7.83540	13	7.97269	25	0.02731	7.86271	12	48	
13	7.83554	14	7.97295	26	0.02705	7.86259	12	47	
14	7.83567	13	7.97320	25	0.02680	7.86247	12	46	25
15	7.83581	14	7.97345	25	0.02655	7.86235	12	45	6 2.5
16	7.83594	13	7.97371	26	0.02629	7.86223	12	44	7 2.9
17	7.83608	14	7.97396	25	0.02604	7.86211	11	43	8 3.3
18	7.83621	13	7.97421	25	0.02579	7.86200	12	42	9 3.8
19	7.83634	13	7.97447	26	0.02553	7.86188	12	41	10 4.2
20	7.83648	14	7.97472	25	0.02528	7.86176	12	40	20 8.3
		13		25			12		30 12.5
21	7.83661		7.97497		0.02503	7.86164		39	40 16.7
22	7.83674	13	7.97523	26	0.02477	7.86152	12	38	50 20.8
23	7.83688	14	7.97548	25	0.02452	7.86140	12	37	
24	7.83701	13	7.97573	25	0.02427	7.86128	12	36	
25	7.83715	14	7.97598	25	0.02402	7.86116	12	35	
26	7.83728	13	7.97624	26	0.02376	7.86104	12	34	14
27	7.83741	13	7.97649	25	0.02351	7.86092	12	33	6 1.4
28	7.83755	14	7.97674	25	0.02326	7.86080	12	32	7 1.6
29	7.83768	13	7.97700	26	0.02300	7.86068	12	31	8 1.9
30	7.83781	13	7.97725	25	0.02275	7.86056	12	30	9 2.1
		14		25			12		10 2.3
31	7.83795		7.97750		0.02250	7.86044		29	20 4.7
32	7.83808	13	7.97776	26	0.02224	7.86032	12	28	30 7.0
33	7.83821	13	7.97801	25	0.02199	7.86020	12	27	40 9.3
34	7.83834	13	7.97826	25	0.02174	7.86008	12	26	50 11.7
35	7.83848	14	7.97851	25	0.02149	7.85996	12	25	
36	7.83861	13	7.97877	26	0.02123	7.85984	12	24	
37	7.83874	13	7.97902	25	0.02098	7.85972	12	23	
38	7.83887	13	7.97927	25	0.02073	7.85960	12	22	13
39	7.83901	14	7.97953	26	0.02047	7.85948	12	21	6 1.3
40	7.83914	13	7.97978	25	0.02022	7.85936	12	20	7 1.5
		13		25			12		8 1.7
41	7.83927		7.98003		0.01997	7.85924		19	9 2.0
42	7.83940	13	7.98029	26	0.01971	7.85912	12	18	10 2.2
43	7.83954	14	7.98054	25	0.01946	7.85900	12	17	20 4.3
44	7.83967	13	7.98079	25	0.01921	7.85888	12	16	30 6.5
45	7.83980	13	7.98104	25	0.01896	7.85876	12	15	40 8.7
46	7.83993	13	7.98130	26	0.01870	7.85864	12	14	50 10.8
47	7.84006	13	7.98155	25	0.01845	7.85851	13	13	
48	7.84020	14	7.98180	25	0.01820	7.85839	12	12	
49	7.84033	13	7.98206	26	0.01794	7.85827	12	11	
50	7.84046	13	7.98231	25	0.01769	7.85815	12	10	12 11
		13		25			12		6 1.2 1.1
51	7.84059		7.98256		0.01744	7.85803		9	7 1.4 1.3
52	7.84072	13	7.98281	25	0.01719	7.85791	12	8	8 1.6 1.5
53	7.84085	13	7.98307	26	0.01693	7.85779	12	7	9 1.8 1.7
54	7.84098	13	7.98332	25	0.01668	7.85766	13	6	10 2.0 1.8
55	7.84112	14	7.98357	25	0.01643	7.85754	12	5	20 4.0 3.7
56	7.84125	13	7.98383	26	0.01617	7.85742	12	4	30 6.0 5.5
57	7.84138	13	7.98408	25	0.01592	7.85730	12	3	40 8.0 7.3
58	7.84151	13	7.98433	25	0.01567	7.85718	12	2	50 10.0 9.2
59	7.84164	13	7.98458	25	0.01542	7.85706	12	1	
60	7.84177	13	7.98484	26	0.01516	7.85693	13	0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

′	log sin	d	log tan	c. d.	log cot	log cos	d		p. p.
0	T.84177	13	T.98484	25	0.01516	T.85693	12	60	
1	T.84190	13	T.98509	25	0.01491	T.85681	12	59	
2	T.84203	13	T.98534	26	0.01466	T.85669	12	58	26
3	T.84216	13	T.98560	25	0.01440	T.85657	12	57	6 | 2.6
4	T.84229	13	T.98585	25	0.01415	T.85645	13	56	7 | 3.0
5	T.84242	13	T.98610	25	0.01390	T.85632	12	55	8 | 3.5
6	T.84255	14	T.98635	26	0.01365	T.85620	12	54	9 | 3.9
7	T.84269	13	T.98661	25	0.01339	T.85608	12	53	10 | 4.3
8	T.84282	13	T.98686	25	0.01314	T.85596	13	52	20 | 8.7
9	T.84295	13	T.98711	26	0.01289	T.85583	12	51	30 | 13.0
10	T.84308	13	T.98737	25	0.01263	T.85571	12	50	40 | 17.3
11	T.84321	13	T.98762	25	0.01238	T.85559	12	49	50 | 21.7
12	T.84334	13	T.98787	25	0.01213	T.85547	13	48	
13	T.84347	13	T.98812	26	0.01188	T.85534	12	47	
14	T.84360	13	T.98838	25	0.01162	T.85522	12	46	25
15	T.84373	12	T.98863	25	0.01137	T.85510	13	45	6 | 2.5
16	T.84385	13	T.98888	25	0.01112	T.85497	12	44	7 | 2.9
17	T.84398	13	T.98913	26	0.01087	T.85485	12	43	8 | 3.3
18	T.84411	13	T.98939	25	0.01061	T.85473	13	42	9 | 3.8
19	T.84424	13	T.98964	25	0.01036	T.85460	12	41	10 | 4.2
20	T.84437	13	T.98989	26	0.01011	T.85448	12	40	20 | 8.3
21	T.84450	13	T.99015	25	0.00985	T.85436	13	39	30 | 12.5
22	T.84463	13	T.99040	25	0.00960	T.85423	12	38	40 | 16.7
23	T.84476	13	T.99065	25	0.00935	T.85411	12	37	50 | 20.8
24	T.84489	13	T.99090	26	0.00910	T.85399	13	36	
25	T.84502	13	T.99116	25	0.00884	T.85386	12	35	
26	T.84515	13	T.99141	25	0.00859	T.85374	13	34	14
27	T.84528	12	T.99166	25	0.00834	T.85361	12	33	6 | 1.4
28	T.84540	13	T.99191	26	0.00809	T.85349	12	32	7 | 1.6
29	T.84553	13	T.99217	25	0.00783	T.85337	13	31	8 | 1.9
30	T.84566	13	T.99242	25	0.00758	T.85324	12	30	9 | 2.1
31	T.84579	13	T.99267	26	0.00733	T.85312	13	29	10 | 2.3
32	T.84592	13	T.99293	25	0.00707	T.85299	12	28	20 | 4.7
33	T.84605	13	T.99318	25	0.00682	T.85287	13	27	30 | 7.0
34	T.84618	13	T.99343	25	0.00657	T.85274	12	26	40 | 9.3
35	T.84630	12	T.99368	26	0.00632	T.85262	12	25	50 | 11.7
36	T.84643	13	T.99394	25	0.00606	T.85250	13	24	
37	T.84656	13	T.99419	25	0.00581	T.85237	12	23	
38	T.84669	13	T.99444	25	0.00556	T.85225	13	22	13
39	T.84682	13	T.99469	26	0.00531	T.85212	12	21	6 | 1.3
40	T.84694	12	T.99495	25	0.00505	T.85200	13	20	7 | 1.5
41	T.84707	13	T.99520	25	0.00480	T.85187	12	19	8 | 1.7
42	T.84720	13	T.99545	25	0.00455	T.85175	13	18	9 | 2.0
43	T.84733	12	T.99570	26	0.00430	T.85162	12	17	10 | 2.2
44	T.84745	13	T.99596	25	0.00404	T.85150	13	16	20 | 4.3
45	T.84758	13	T.99621	25	0.00379	T.85137	12	15	30 | 6.5
46	T.84771	13	T.99646	26	0.00354	T.85125	13	14	40 | 8.7
47	T.84784	12	T.99672	25	0.00328	T.85112	12	13	50 | 10.8
48	T.84796	13	T.99697	25	0.00303	T.85100	13	12	
49	T.84809	13	T.99722	25	0.00278	T.85087	13	11	
50	T.84822	13	T.99747	26	0.00253	T.85074	12	10	12
51	T.84835	12	T.99773	25	0.00227	T.85062	13	9	6 | 1.2
52	T.84847	13	T.99798	25	0.00202	T.85049	12	8	7 | 1.4
53	T.84860	13	T.99823	25	0.00177	T.85037	13	7	8 | 1.6
54	T.84873	12	T.99848	26	0.00152	T.85024	12	6	9 | 1.8
55	T.84885	13	T.99874	25	0.00126	T.85012	13	5	10 | 2.0
56	T.84898	13	T.99899	25	0.00101	T.84999	13	4	20 | 4.0
57	T.84911	13	T.99924	25	0.00076	T.84986	12	3	30 | 6.0
58	T.84923	12	T.99949	26	0.00051	T.84974	13	2	40 | 8.0
59	T.84936	13	T.99975	25	0.00025	T.84961	12	1	50 | 10.0
60	T.84949	13	0.00000		0.00000	T.84949		0	
	log cos	d	log cot	c. d.	log tan	log sin	d	′	p. p.

TABLE

OF

COMMON LOGARITHMS

OF NUMBERS

From 1 to 10,000.

N.	Log.	N.	Log.	N.	Log.	N.	Log.	N.	Log.
0	$-\infty$	**20**	30 103	**40**	60 206	**60**	77 815	**80**	90 309
1	00 000	21	32 222	41	61 278	61	78 533	81	90 849
2	30 103	22	34 242	42	62 325	62	79 239	82	91 381
3	47 712	23	36 173	43	63 347	63	79 934	83	91 908
4	60 206	24	38 021	44	64 345	64	80 618	84	92 428
5	69 897	25	39 794	45	65 321	65	81 291	85	92 942
6	77 815	26	41 497	46	66 276	66	81 954	86	93 450
7	84 510	27	43 136	47	67 210	67	82 607	87	93 952
8	90 309	28	44 716	48	68 124	68	83 251	88	94 448
9	95 424	29	46 240	49	69 020	69	83 885	89	94 939
10	00 000	**30**	47 712	**50**	69 897	**70**	84 510	**90**	95 424
11	04 139	31	49 136	51	70 757	71	85 126	91	95 904
12	07 918	32	50 515	52	71 600	72	85 733	92	96 379
13	11 394	33	51 851	53	72 428	73	86 332	93	96 848
14	14 613	34	53 148	54	73 239	74	86 923	94	97 313
15	17 609	35	54 407	55	74 036	75	87 506	95	97 772
16	20 412	36	55 630	56	74 819	76	88 081	96	98 227
17	23 045	37	56 820	57	75 587	77	88 649	97	98 677
18	25 527	38	57 978	58	76 343	78	89 209	98	99 123
19	27 875	39	59 106	59	77 085	79	89 763	99	99 564
20	30 103	**40**	60 206	**60**	77 815	**80**	90 309	**100**	00 000

N	L. 0	1	2	3	4	5	6	7	8	9
100	00 000	043	087	130	173	217	260	303	346	389
101	432	475	518	561	604	647	689	732	775	817
102	860	903	945	988	*030	*072	*115	*157	*199	*242
103	01 284	326	368	410	452	494	536	578	620	662
104	703	745	787	828	870	912	953	995	*036	*078
105	02 119	160	202	243	284	325	366	407	449	490
106	531	572	612	653	694	735	776	816	857	898
107	938	979	*019	*060	*100	*141	*181	*222	*262	*302
108	03 342	383	423	463	503	543	583	623	663	703
109	743	782	822	862	902	941	981	*021	*060	*100
110	04 139	179	218	258	297	336	376	415	454	493
111	532	571	610	650	689	727	766	805	844	883
112	922	961	999	*038	*077	*115	*154	*192	*231	*269
113	05 308	346	385	423	461	500	538	576	614	652
114	690	729	767	805	843	881	918	956	994	*032
115	06 070	108	145	183	221	258	296	333	371	408
116	446	483	521	558	595	633	670	707	744	781
117	819	856	893	930	967	*004	*041	*078	*115	*151
118	07 188	225	262	298	335	372	408	445	482	518
119	555	591	628	664	700	737	773	809	846	882
120	918	954	990	*027	*063	*099	*135	*171	*207	*243
121	08 279	314	350	386	422	458	493	529	565	600
122	636	672	707	743	778	814	849	884	920	955
123	991	*026	*061	*096	*132	*167	*202	*237	*272	*307
124	09 342	377	412	447	482	517	552	587	621	656
125	691	726	760	795	830	864	899	934	968	*003
126	10 037	072	106	140	175	209	243	278	312	346
127	380	415	449	483	517	551	585	619	653	687
128	721	755	789	823	857	890	924	958	992	*025
129	11 059	093	126	160	193	227	261	294	327	361
130	394	428	461	494	528	561	594	628	661	694
131	727	760	793	826	860	893	926	959	992	*024
132	12 057	090	123	156	189	222	254	287	320	352
133	385	418	450	483	516	548	581	613	646	678
134	710	743	775	808	840	872	905	937	969	*001
135	13 033	066	098	130	162	194	226	258	290	322
136	354	386	418	450	481	513	545	577	609	640
137	672	704	735	767	799	830	862	893	925	956
138	988	*019	*051	*082	*114	*145	*176	*208	*239	*270
139	14 301	333	364	395	426	457	489	520	551	582
140	613	644	675	706	737	768	799	829	860	891
141	922	953	983	*014	*045	*076	*106	*137	*168	*198
142	15 229	259	290	320	351	381	412	442	473	503
143	534	564	594	625	655	685	715	746	776	806
144	836	866	897	927	957	987	*017	*047	*077	*107
145	16 137	167	197	227	256	286	316	346	376	406
146	435	465	495	524	554	584	613	643	673	702
147	732	761	791	820	850	879	909	938	967	997
148	17 026	056	085	114	143	173	202	231	260	289
149	319	348	377	406	435	464	493	522	551	580
150	609	638	667	696	725	754	782	811	840	869
N.	L. 0	1	2	3	4	5	6	7	8	9

P. P.			
	44	**43**	**42**
1	4.4	4.3	4.2
2	8.8	8.6	8.4
3	13.2	12.9	12.6
4	17.6	17.2	16.8
5	22.0	21.5	21.0
6	26.4	25.8	25.2
7	30.8	30.1	29.4
8	35.2	34.4	33.6
9	39.6	38.7	37.8
	41	**40**	**39**
1	4.1	4.0	3.9
2	8.2	8.0	7.8
3	12.3	12.0	11.7
4	16.4	16.0	15.6
5	20.5	20.0	19.5
6	24.6	24.0	23.4
7	28.7	28.0	27.3
8	32.8	32.0	31.2
9	36.9	36.0	35.1
	38	**37**	**36**
1	3.8	3.7	3.6
2	7.6	7.4	7.2
3	11.4	11.1	10.8
4	15.2	14.8	14.4
5	19.0	18.5	18.0
6	22.8	22.2	21.6
7	26.6	25.9	25.2
8	30.4	29.6	28.8
9	34.2	33.3	32.4
	35	**34**	**33**
1	3.5	3.4	3.3
2	7.0	6.8	6.6
3	10.5	10.2	9.9
4	14.0	13.6	13.2
5	17.5	17.0	16.5
6	21.0	20.4	19.8
7	24.5	23.8	23.1
8	28.0	27.2	26.4
9	31.5	30.6	29.7
	32	**31**	**30**
1	3.2	3.1	3.0
2	6.4	6.2	6.0
3	9.6	9.3	9.0
4	12.8	12.4	12.0
5	16.0	15.5	15.0
6	19.2	18.6	18.0
7	22.4	21.7	21.0
8	25.6	24.8	24.0
9	28.8	27.9	27.0

P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
150	17 609	638	667	696	725	754	782	811	840	869	
151	898	926	955	984	*013	*041	*070	*099	*127	*156	
152	18 184	213	241	270	298	327	355	384	412	441	29 28
153	469	498	526	554	583	611	639	667	696	724	1 2.9 2.8
154	752	780	808	837	865	893	921	949	977	*005	2 5.8 5.6
155	19 033	061	089	117	145	173	201	229	257	285	3 8.7 8.4
156	312	340	368	396	424	451	479	507	535	562	4 11.6 11.2
157	590	618	645	673	700	728	756	783	811	838	5 14.5 14.0
158	866	893	921	948	976	*003	*030	*058	*085	*112	6 17.4 16.8
159	20 140	167	194	222	249	276	303	330	358	385	7 20.3 19.6
160	412	439	466	493	520	548	575	602	629	656	8 23.2 22.4
161	683	710	737	763	790	817	844	871	898	925	9 26.1 25.2
162	952	978	*005	*032	*059	*085	*112	*139	*165	*192	
163	21 219	245	272	299	325	352	378	405	431	458	27 26
164	484	511	537	564	590	617	643	669	696	722	1 2.7 2.6
165	748	775	801	827	854	880	906	932	958	985	2 5.4 5.2
166	22 011	037	063	089	115	141	167	194	220	246	3 8.1 7.8
167	272	298	324	350	376	401	427	453	479	505	4 10.8 10.4
168	531	557	583	608	634	660	686	712	737	763	5 13.5 13.0
169	789	814	840	866	891	917	943	968	994	*019	6 16.2 15.6
170	23 045	070	096	121	147	172	198	223	249	274	7 18.9 18.2
171	300	325	350	376	401	426	452	477	502	528	8 21.6 20.8
172	553	578	603	629	654	679	704	729	754	779	9 24.3 23.4
173	805	830	855	880	905	930	955	980	*005	*030	
174	24 055	080	105	130	155	180	204	229	254	279	25
175	304	329	353	378	403	428	452	477	502	527	1 2.5
176	551	576	601	625	650	674	699	724	748	773	2 5.0
177	797	822	846	871	895	920	944	969	993	*018	3 7.5
178	25 042	066	091	115	139	164	188	212	237	261	4 10.0
179	285	310	334	358	382	406	431	455	479	503	5 12.5
180	527	551	575	600	624	648	672	696	720	744	6 15.0
181	768	792	816	840	864	888	912	935	959	983	7 17.5
182	26 007	031	055	079	102	126	150	174	198	221	8 20.0
183	245	269	293	316	340	364	387	411	435	458	9 22.5
184	482	505	529	553	576	600	623	647	670	694	
185	717	741	764	788	811	834	858	881	905	928	24 23
186	951	975	998	*021	*045	*068	*091	*114	*138	*161	1 2.4 2.3
187	27 184	207	231	254	277	300	323	346	370	393	2 4.8 4.6
188	416	439	462	485	508	531	554	577	600	623	3 7.2 6.9
189	646	669	692	715	738	761	784	807	830	852	4 9.6 9.2
190	875	898	921	944	967	989	*012	*035	*058	*081	5 12.0 11.5
191	28 103	126	149	171	194	217	240	262	285	307	6 14.4 13.8
192	330	353	375	398	421	443	466	488	511	533	7 16.8 16.1
193	556	578	601	623	646	668	691	713	735	758	8 19.2 18.4
194	780	803	825	847	870	892	914	937	959	981	9 21.6 20.7
195	29 003	026	048	070	092	115	137	159	181	203	
196	226	248	270	292	314	336	358	380	403	425	22 21
197	447	469	491	513	535	557	579	601	623	645	1 2.2 2.1
198	667	688	710	732	754	776	798	820	842	863	2 4.4 4.2
199	885	907	929	951	973	994	*016	*038	*060	*081	3 6.6 6.3
200	30 103	125	146	168	190	211	233	255	276	298	4 8.8 8.4
											5 11.0 10.5
											6 13.2 12.6
											7 15.4 14.7
											8 17.6 16.8
											9 19.8 18.9
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
200	30 103	125	146	168	190	211	233	255	276	298	
201	320	341	363	384	406	428	449	471	492	514	
202	535	557	578	600	621	643	664	685	707	728	
203	750	771	792	814	835	856	878	899	920	942	
204	963	984	*006	*027	*048	*069	*091	*112	*133	*154	
205	31 175	197	218	239	260	281	302	323	345	366	
206	387	408	429	450	471	492	513	534	555	576	
207	597	618	639	660	681	702	723	744	765	785	
208	806	827	848	869	890	911	931	952	973	994	
209	32 015	035	056	077	098	118	139	160	181	201	
210	222	243	263	284	305	325	346	366	387	408	
211	428	449	469	490	510	531	552	572	593	613	
212	634	654	675	695	715	736	756	777	797	818	
213	838	858	879	899	919	940	960	980	*001	*021	
214	33 041	062	082	102	122	143	163	183	203	224	
215	244	264	284	304	325	345	365	385	405	425	
216	445	465	486	506	526	546	566	586	606	626	
217	646	666	686	706	726	746	766	786	806	826	
218	846	866	885	905	925	945	965	985	*005	*025	
219	34 044	064	084	104	124	143	163	183	203	223	
220	242	262	282	301	321	341	361	380	400	420	
221	439	459	479	498	518	537	557	577	596	616	
222	635	655	674	694	713	733	753	772	792	811	
223	830	850	869	889	908	928	947	967	986	*005	
224	35 025	044	064	083	102	122	141	160	180	199	
225	218	238	257	276	295	315	334	353	372	392	
226	411	430	449	468	488	507	526	545	564	583	
227	603	622	641	660	679	698	717	736	755	774	
228	793	813	832	851	870	889	908	927	946	965	
229	984	*003	*021	*040	*059	*078	*097	*116	*135	*154	
230	36 173	192	211	229	248	267	286	305	324	342	
231	361	380	399	418	436	455	474	493	511	530	
232	549	568	586	605	624	642	661	680	698	717	
233	736	754	773	791	810	829	847	866	884	903	
234	922	940	959	977	996	*014	*033	*051	*070	*088	
235	37 107	125	144	162	181	199	218	236	254	273	
236	291	310	328	346	365	383	401	420	438	457	
237	475	493	511	530	548	566	585	603	621	639	
238	658	676	694	712	731	749	767	785	803	822	
239	840	858	876	894	912	931	949	967	985	*003	
240	38 021	039	057	075	093	112	130	148	166	184	
241	202	220	238	256	274	292	310	328	346	364	
242	382	399	417	435	453	471	489	507	525	543	
243	561	578	596	614	632	650	668	686	703	721	
244	739	757	775	792	810	828	846	863	881	899	
245	917	934	952	970	987	*005	*023	*041	*058	*076	
246	39 094	111	129	146	164	182	199	217	235	252	
247	270	287	305	322	340	358	375	393	410	428	
248	445	463	480	498	515	533	550	568	585	602	
249	620	637	655	672	690	707	724	742	759	777	
250	794	811	829	846	863	881	898	915	933	950	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

	22	21
1	2.2	2.1
2	4.4	4.2
3	6.6	6.3
4	8.8	8.4
5	11.0	10.5
6	13.2	12.6
7	15.4	14.7
8	17.6	16.8
9	19.8	18.9

	20
1	2.0
2	4.0
3	6.0
4	8.0
5	10.0
6	12.0
7	14.0
8	16.0
9	18.0

	19
1	1.9
2	3.8
3	5.7
4	7.6
5	9.5
6	11.4
7	13.3
8	15.2
9	17.1

	18
1	1.8
2	3.6
3	5.4
4	7.2
5	9.0
6	10.8
7	12.6
8	14.4
9	16.2

	17
1	1.7
2	3.4
3	5.1
4	6.8
5	8.5
6	10.2
7	11.9
8	13.6
9	15.3

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
250	39 794	811	829	846	863	881	898	915	933	950	
251	967	985	*002	*019	*037	*054	*071	*088	*106	*123	
252	40 140	157	175	192	209	226	243	261	278	295	**18**
253	312	329	346	364	381	398	415	432	449	466	1 1.8
254	483	500	518	535	552	569	586	603	620	637	2 3.6
255	654	671	688	705	722	739	756	773	790	807	3 5.4
256	824	841	858	875	892	909	926	943	960	976	4 7.2
257	993	*010	*027	*044	*061	*078	*095	*111	*128	*145	5 9.0
258	41 162	179	196	212	229	246	263	280	296	313	6 10.8
259	330	347	363	380	397	414	430	447	464	481	7 12.6
											8 14.4
260	497	514	531	547	564	581	597	614	631	647	9 16.2
261	664	681	697	714	731	747	764	780	797	814	
262	830	847	863	880	896	913	929	946	963	979	
263	996	*012	*029	*045	*062	*078	*095	*111	*127	*144	**17**
264	42 160	177	193	210	226	243	259	275	292	308	1 1.7
265	325	341	357	374	390	406	423	439	455	472	2 3.4
266	488	504	521	537	553	570	586	602	619	635	3 5.1
267	651	667	684	700	716	732	749	765	781	797	4 6.8
268	813	830	846	862	878	894	911	927	943	959	5 8.5
269	975	991	*008	*024	*040	*056	*072	*088	*104	*120	6 10.2
											7 11.9
270	43 136	152	169	185	201	217	233	249	265	281	8 13.6
271	297	313	329	345	361	377	393	409	425	441	9 15.3
272	457	473	489	505	521	537	553	569	584	600	
273	616	632	648	664	680	696	712	727	743	759	**16**
274	775	791	807	823	838	854	870	886	902	917	1 1.6
275	933	949	965	981	996	*012	*028	*044	*059	*075	2 3.2
276	44 091	107	122	138	154	170	185	201	217	232	3 4.8
277	248	264	279	295	311	326	342	358	373	389	4 6.4
278	404	420	436	451	467	483	498	514	529	545	5 8.0
279	560	576	592	607	623	638	654	669	685	700	6 9.6
											7 11.2
280	716	731	747	762	778	793	809	824	840	855	8 12.8
281	871	886	902	917	932	948	963	979	994	*010	9 14.4
282	45 025	040	056	071	086	102	117	133	148	163	
283	179	194	209	225	240	255	271	286	301	317	**15**
284	332	347	362	378	393	408	423	439	454	469	1 1.5
285	484	500	515	530	545	561	576	591	606	621	2 3.0
286	637	652	667	682	697	712	728	743	758	773	3 4.5
287	788	803	818	834	849	864	879	894	909	924	4 6.0
288	939	954	969	984	*000	*015	*030	*045	*060	*075	5 7.5
289	46 090	105	120	135	150	165	180	195	210	225	6 9.0
											7 10.5
290	240	255	270	285	300	315	330	345	359	374	8 12.0
291	389	404	419	434	449	464	479	494	509	523	9 13.5
292	538	553	568	583	598	613	627	642	657	672	
293	687	702	716	731	746	761	776	790	805	820	**14**
294	835	850	864	879	894	909	923	938	953	967	1 1.4
295	982	997	*012	*026	*041	*056	*070	*085	*100	*114	2 2.8
296	47 129	144	159	173	188	202	217	232	246	261	3 4.2
297	276	290	305	319	334	349	363	378	392	407	4 5.6
298	422	436	451	465	480	494	509	524	538	553	5 7.0
299	567	582	596	611	625	640	654	669	683	698	6 8.4
											7 9.8
300	712	727	741	756	770	784	799	813	828	842	8 11.2
											9 12.6
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
300	47 712	727	741	756	770	784	799	813	828	842	
301	857	871	885	900	914	929	943	958	972	986	
302	48 001	015	029	044	058	073	087	101	116	130	
303	144	159	173	187	202	216	230	244	259	273	
304	287	302	316	330	344	359	373	387	401	416	
305	430	444	458	473	487	501	515	530	544	558	**15**
306	572	586	601	615	629	643	657	671	686	700	1 | 1.5
307	714	728	742	756	770	785	799	813	827	841	2 | 3.0
308	855	869	883	897	911	926	940	954	968	982	3 | 4.5
309	996	*010	*024	*038	*052	*066	*080	*094	*108	*122	4 | 6.0
.310	49 136	150	164	178	192	206	220	234	248	262	5 | 7.5
311	276	290	304	318	332	346	360	374	388	402	6 | 9.0
312	415	429	443	457	471	485	499	513	527	541	7 | 10.5
313	554	568	582	596	610	624	638	651	665	679	8 | 12.0
314	693	707	721	734	748	762	776	790	803	817	9 | 13.5
315	831	845	859	872	886	900	914	927	941	955	
316	969	982	996	*010	*024	*037	*051	*065	*079	*092	**14**
317	50 106	120	133	147	161	174	188	202	215	229	1 | 1.4
318	243	256	270	284	297	311	325	338	352	365	2 | 2.8
319	379	393	406	420	433	447	461	474	488	501	3 | 4.2
320	515	529	542	556	569	583	596	610	623	637	4 | 5.6
321	651	664	678	691	705	718	732	745	759	772	5 | 7.0
322	786	799	813	826	840	853	866	880	893	907	6 | 8.4
323	920	934	947	961	974	987	*001	*014	*028	*041	7 | 9.8
324	51 055	068	081	095	108	121	135	148	162	175	8 | 11.2
325	188	202	215	228	242	255	268	282	295	308	9 | 12.6
326	322	335	348	362	375	388	402	415	428	441	
327	455	468	481	495	508	521	534	548	561	574	**13**
328	587	601	614	627	640	654	667	680	693	706	1 | 1.3
329	720	733	746	759	772	786	799	812	825	838	2 | 2.6
330	851	865	878	891	904	917	930	943	957	970	3 | 3.9
331	983	996	*009	*022	*035	*048	*061	*075	*088	*101	4 | 5.2
332	52 114	127	140	153	166	179	192	205	218	231	5 | 6.5
333	244	257	270	284	297	310	323	336	349	362	6 | 7.8
334	375	388	401	414	427	440	453	466	479	492	7 | 9.1
335	504	517	530	543	556	569	582	595	608	621	8 | 10.4
336	634	647	660	673	686	699	711	724	737	750	9 | 11.7
337	763	776	789	802	815	827	840	853	866	879	
338	892	905	917	930	943	956	969	982	994	*007	**12**
339	53 020	033	046	058	071	084	097	110	122	135	1 | 1.2
340	148	161	173	186	199	212	224	237	250	263	2 | 2.4
341	275	288	301	314	326	339	352	364	377	390	3 | 3.6
342	403	415	428	441	453	466	479	491	504	517	4 | 4.8
343	529	542	555	567	580	593	605	618	631	643	5 | 6.0
344	656	668	681	694	706	719	732	744	757	769	6 | 7.2
345	782	794	807	820	832	845	857	870	882	895	7 | 8.4
346	908	920	933	945	958	970	983	995	*008	*020	8 | 9.6
347	54 033	045	058	070	083	095	108	120	133	145	9 | 10.8
348	158	170	183	195	208	220	233	245	258	270	
349	283	295	307	320	332	345	357	370	382	394	
350	407	419	432	444	456	469	481	494	506	518	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
350	54 407	419	432	444	456	469	481	494	506	518	
351	531	543	555	568	580	593	605	617	630	642	
352	654	667	679	691	704	716	728	741	753	765	
353	777	790	802	814	827	839	851	864	876	888	
354	900	913	925	937	949	962	974	986	998	*011	**13**
355	55 023	035	047	060	072	084	096	108	121	133	1 1.3
356	145	157	169	182	194	206	218	230	242	255	2 2.6
357	267	279	291	303	315	328	340	352	364	376	3 3.9
358	388	400	413	425	437	449	461	473	485	497	4 5.2
359	509	522	534	546	558	570	582	594	606	618	5 6.5
360	630	642	654	666	678	691	703	715	727	739	6 7.8
361	751	763	775	787	799	811	823	835	847	859	7 9.1
362	871	883	895	907	919	931	943	955	967	979	8 10.4
363	991	*003	*015	*027	*038	*050	*062	*074	*086	*098	9 11.7
364	56 110	122	134	146	158	170	182	194	205	217	
365	229	241	253	265	277	289	301	312	324	336	
366	348	360	372	384	396	407	419	431	443	455	**12**
367	467	478	490	502	514	526	538	549	561	573	1 1.2
368	585	597	608	620	632	644	656	667	679	691	2 2.4
369	703	714	726	738	750	761	773	785	797	808	3 3.6
370	820	832	844	855	867	879	891	902	914	926	4 4.8
371	937	949	961	972	984	996	*008	*019	*031	*043	5 6.0
372	57 054	066	078	089	101	113	124	136	148	159	6 7.2
373	171	183	194	206	217	229	241	252	264	276	7 8.4
374	287	299	310	322	334	345	357	368	380	392	8 9.6
375	403	415	426	438	449	461	473	484	496	507	9 10.8
376	519	530	542	553	565	576	588	600	611	623	
377	634	646	657	669	680	692	703	715	726	738	
378	749	761	772	784	795	807	818	830	841	852	**11**
379	864	875	887	898	910	921	933	944	955	967	1 1.1
380	978	990	*001	*013	*024	*035	*047	*058	*070	*081	2 2.2
381	58 092	104	115	127	138	149	161	172	184	195	3 3.3
382	206	218	229	240	252	263	274	286	297	309	4 4.4
383	320	331	343	354	365	377	388	399	410	422	5 5.5
384	433	444	456	467	478	490	501	512	524	535	6 6.6
385	546	557	569	580	591	602	614	625	636	647	7 7.7
386	659	670	681	692	704	715	726	737	749	760	8 8.8
387	771	782	794	805	816	827	838	850	861	872	9 9.9
388	883	894	906	917	928	939	950	961	973	984	
389	995	*006	*017	*028	*040	*051	*062	*073	*084	*095	
390	59 106	118	129	140	151	162	173	184	195	207	**10**
391	218	229	240	251	262	273	284	295	306	318	1 1.0
392	329	340	351	362	373	384	395	406	417	428	2 2.0
393	439	450	461	472	483	494	506	517	528	539	3 3.0
394	550	561	572	583	594	605	616	627	638	649	4 4.0
395	660	671	682	693	704	715	726	737	748	759	5 5.0
396	770	780	791	802	813	824	835	846	857	868	6 6.0
397	879	890	901	912	923	934	945	956	966	977	7 7.0
398	988	999	*010	*021	*032	*043	*054	*065	*076	*086	8 8.0
399	60 097	108	119	130	141	152	163	173	184	195	9 9.0
400	206	217	228	239	249	260	271	282	293	304	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
400	60 206	217	228	239	249	260	271	282	293	304	
401	314	325	336	347	358	369	379	390	401	412	
402	423	433	444	455	466	477	487	498	509	520	
403	531	541	552	563	574	584	595	606	617	627	
404	638	649	660	670	681	692	703	713	724	735	
405	746	756	767	778	788	799	810	821	831	842	
406	853	863	874	885	895	906	917	927	938	949	**11**
407	959	970	981	991	*002	*013	*023	*034	*045	*055	1 | 1.1
408	61 066	077	087	098	109	119	130	140	151	162	2 | 2.2
409	172	183	194	204	215	225	236	247	257	268	3 | 3.3
410	278	289	300	310	321	331	342	352	363	374	4 | 4.4
411	384	395	405	416	426	437	448	458	469	479	5 | 5.5
412	490	500	511	521	532	542	553	563	574	584	6 | 6.6
413	595	606	616	627	637	648	658	669	679	690	7 | 7.7
414	700	711	721	731	742	752	763	773	784	794	8 | 8.8
415	805	815	826	836	847	857	868	878	888	899	9 | 9.9
416	909	920	930	941	951	962	972	982	993	*003	
417	62 014	024	034	045	055	066	076	086	097	107	
418	118	128	138	149	159	170	180	190	201	211	
419	221	232	242	252	263	273	284	294	304	315	
420	325	335	346	356	366	377	387	397	408	418	
421	428	439	449	459	469	480	490	500	511	521	**10**
422	531	542	552	562	572	583	593	603	613	624	1 | 1.0
423	634	644	655	665	675	685	696	706	716	726	2 | 2.0
424	737	747	757	767	778	788	798	808	818	829	3 | 3.0
425	839	849	859	870	880	890	900	910	921	931	4 | 4.0
426	941	951	961	972	982	992	*002	*012	*022	*033	5 | 5.0
427	63 043	053	063	073	083	094	104	114	124	134	6 | 6.0
428	144	155	165	175	185	195	205	215	225	236	7 | 7.0
429	246	256	266	276	286	296	306	317	327	337	8 | 8.0
430	347	357	367	377	387	397	407	417	428	438	9 | 9.0
431	448	458	468	478	488	498	508	518	528	538	
432	548	558	568	579	589	599	609	619	629	639	
433	649	659	669	679	689	699	709	719	729	739	
434	749	759	769	779	789	799	809	819	829	839	
435	849	859	869	879	889	899	909	919	929	939	
436	949	959	969	979	988	998	*008	*018	*028	*038	
437	64 048	058	068	078	088	098	108	118	128	137	
438	147	157	167	177	187	197	207	217	227	237	**9**
439	246	256	266	276	286	296	306	316	326	335	1 | 0.9
440	345	355	365	375	385	395	404	414	424	434	2 | 1.8
441	444	454	464	473	483	493	503	513	523	532	3 | 2.7
442	542	552	562	572	582	591	601	611	621	631	4 | 3.6
443	640	650	660	670	680	689	699	709	719	729	5 | 4.5
444	738	748	758	768	777	787	797	807	816	826	6 | 5.4
445	836	846	856	865	875	885	895	904	914	924	7 | 6.3
446	933	943	953	963	972	982	992	*002	*011	*021	8 | 7.2
447	65 031	040	050	060	070	079	089	099	108	118	9 | 8.1
448	128	137	147	157	167	176	186	196	205	215	
449	225	234	244	254	263	273	283	292	302	312	
450	321	331	341	350	360	369	379	389	398	408	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
450	65 321	331	341	350	360	369	379	389	398	408	
451	418	427	437	447	456	466	475	485	495	504	
452	514	523	533	543	552	562	571	581	591	600	
453	610	619	629	639	648	658	667	677	686	696	
454	706	715	725	734	744	753	763	772	782	792	
455	801	811	820	830	839	849	858	868	877	887	
456	896	906	916	925	935	944	954	963	973	982	
457	992	*001	*011	*020	*030	*039	*049	*058	*068	*077	**10**
458	66 087	096	106	115	124	134	143	153	162	172	1 | 1.0
459	181	191	200	210	219	229	238	247	257	266	2 | 2.0
460	276	285	295	304	314	323	332	342	351	361	3 | 3.0
461	370	380	389	398	408	417	427	436	445	455	4 | 4.0
462	464	474	483	492	502	511	521	530	539	549	5 | 5.0
463	558	567	577	586	596	605	614	624	633	642	6 | 6.0
464	652	661	671	680	689	699	708	717	727	736	7 | 7.0
465	745	755	764	773	783	792	801	811	820	829	8 | 8.0
466	839	848	857	867	876	885	894	904	913	922	9 | 9.0
467	932	941	950	960	969	978	987	997	*006	*015	
468	67 025	034	043	052	062	071	080	089	099	108	
469	117	127	136	145	154	164	173	182	191	201	
470	210	219	228	237	247	256	265	274	284	293	
471	302	311	321	330	339	348	357	367	376	385	
472	394	403	413	422	431	440	449	459	468	477	**9**
473	486	495	504	514	523	532	541	550	560	569	1 | 0.9
474	578	587	596	605	614	624	633	642	651	660	2 | 1.8
475	669	679	688	697	706	715	724	733	742	752	3 | 2.7
476	761	770	779	788	797	806	815	825	834	843	4 | 3.6
477	852	861	870	879	888	897	906	916	925	934	5 | 4.5
478	943	952	961	970	979	988	997	*006	*015	*024	6 | 5.4
479	68 034	043	052	061	070	079	088	097	106	115	7 | 6.3
480	124	133	142	151	160	169	178	187	196	205	8 | 7.2
481	215	224	233	242	251	260	269	278	287	296	9 | 8.1
482	305	314	323	332	341	350	359	368	377	386	
483	395	404	413	422	431	440	449	458	467	476	
484	485	494	502	511	520	529	538	547	556	565	
485	574	583	592	601	610	619	628	637	646	655	
486	664	673	681	690	699	708	717	726	735	744	
487	753	762	771	780	789	797	806	815	824	833	**8**
488	842	851	860	869	878	886	895	904	913	922	1 | 0.8
489	931	940	949	958	966	975	984	993	*002	*011	2 | 1.6
490	69 020	028	037	046	055	064	073	082	090	099	3 | 2.4
491	108	117	126	135	144	152	161	170	179	188	4 | 3.2
492	197	205	214	223	232	241	249	258	267	276	5 | 4.0
493	285	294	302	311	320	329	338	346	355	364	6 | 4.8
494	373	381	390	399	408	417	425	434	443	452	7 | 5.6
495	461	469	478	487	496	504	513	522	531	539	8 | 6.4
496	548	557	566	574	583	592	601	609	618	627	9 | 7.2
497	636	644	653	662	671	679	688	697	705	714	
498	723	732	740	749	758	767	775	784	793	801	
499	810	819	827	836	845	854	862	871	880	888	
500	897	906	914	923	932	940	949	958	966	975	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
500	69 897	906	914	923	932	940	949	958	966	975	
501	984	992	*001	*010	*018	*027	*036	*044	*053	*062	
502	70 070	079	088	096	105	114	122	131	140	148	
503	157	165	174	183	191	200	209	217	226	234	
504	243	252	260	269	278	286	295	303	312	321	
505	329	338	346	355	364	372	381	389	398	406	
506	415	424	432	441	449	458	467	475	484	492	
507	501	509	518	526	535	544	552	561	569	578	9
508	586	595	603	612	621	629	638	646	655	663	1 0.9
509	672	680	689	697	706	714	723	731	740	749	2 1.8
510	757	766	774	783	791	800	808	817	825	834	3 2.7
511	842	851	859	868	876	885	893	902	910	919	4 3.6
512	927	935	944	952	961	969	978	986	995	*003	5 4.5
513	71 012	020	029	037	046	054	063	071	079	088	6 5.4
514	096	105	113	122	130	139	147	155	164	172	7 6.3
515	181	189	198	206	214	223	231	240	248	257	8 7.2
516	265	273	282	290	299	307	315	324	332	341	9 8.1
517	349	357	366	374	383	391	399	408	416	425	
518	433	441	450	458	466	475	483	492	500	508	
519	517	525	533	542	550	559	567	575	584	592	
520	600	609	617	625	634	642	650	659	667	675	
521	684	692	700	709	717	725	734	742	750	759	
522	767	775	784	792	800	809	817	825	834	842	8
523	850	858	867	875	883	892	900	908	917	925	1 0.8
524	933	941	950	958	966	975	983	991	999	*008	2 1.6
525	72 016	024	032	041	049	057	066	074	082	090	3 2.4
526	099	107	115	123	132	140	148	156	165	173	4 3.2
527	181	189	198	206	214	222	230	239	247	255	5 4.0
528	263	272	280	288	296	304	313	321	329	337	6 4.8
529	346	354	362	370	378	387	395	403	411	419	7 5.6
530	428	436	444	452	460	469	477	485	493	501	8 6.4
531	509	518	526	534	542	550	558	567	575	583	9 7.2
532	591	599	607	616	624	632	640	648	656	665	
533	673	681	689	697	705	713	722	730	738	746	
534	754	762	770	779	787	795	803	811	819	827	
535	835	843	852	860	868	876	884	892	900	908	
536	916	925	933	941	949	957	965	973	981	989	
537	997	*006	*014	*022	*030	*038	*046	*054	*062	*070	7
538	73 078	086	094	102	111	119	127	135	143	151	1 0.7
539	159	167	175	183	191	199	207	215	223	231	2 1.4
540	239	247	255	263	272	280	288	296	304	312	3 2.1
541	320	328	336	344	352	360	368	376	384	392	4 2.8
542	400	408	416	424	432	440	448	456	464	472	5 3.5
543	480	488	496	504	512	520	528	536	544	552	6 4.2
544	560	568	576	584	592	600	608	616	624	632	7 4.9
545	640	648	656	664	672	679	687	695	703	711	8 5.6
546	719	727	735	743	751	759	767	775	783	791	9 6.3
547	799	807	815	823	830	838	846	854	862	870	
548	878	886	894	902	910	918	926	933	941	949	
549	957	965	973	981	989	997	*005	*013	*020	*028	
550	74 036	044	052	060	068	076	084	092	099	107	
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N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
550	74 036	044	052	060	068	076	084	092	099	107	
551	115	123	131	139	147	155	162	170	178	186	
552	194	202	210	218	225	233	241	249	257	265	
553	273	280	288	296	304	312	320	327	335	343	
554	351	359	367	374	382	390	398	406	414	421	
555	429	437	445	453	461	468	476	484	492	500	
556	507	515	523	531	539	547	554	562	570	578	
557	586	593	601	609	617	624	632	640	648	656	
558	663	671	679	687	695	702	710	718	726	733	
559	741	749	757	764	772	780	788	796	803	811	
560	819	827	834	842	850	858	865	873	881	889	
561	896	904	912	920	927	935	943	950	958	966	8 1 0.8 2 1.6 3 2.4 4 3.2 5 4.0 6 4.8 7 5.6 8 6.4 9 7.2
562	974	981	989	997	*005	*012	*020	*028	*035	*043	
563	75 051	059	066	074	082	089	097	105	113	120	
564	128	136	143	151	159	166	174	182	189	197	
565	205	213	220	228	236	243	251	259	266	274	
566	282	289	297	305	312	320	328	335	343	351	
567	358	366	374	381	389	397	404	412	420	427	
568	435	442	450	458	465	473	481	488	496	504	
569	511	519	526	534	542	549	557	565	572	580	
570	587	595	603	610	618	626	633	641	648	656	
571	664	671	679	686	694	702	709	717	724	732	
572	740	747	755	762	770	778	785	793	800	808	
573	815	823	831	838	846	853	861	868	876	884	
574	891	899	906	914	921	929	937	944	952	959	
575	967	974	982	989	997	*005	*012	*020	*027	*035	
576	76 042	050	057	065	072	080	087	095	103	110	
577	118	125	133	140	148	155	163	170	178	185	
578	193	200	208	215	223	230	238	245	253	260	
579	268	275	283	290	298	305	313	320	328	335	
580	343	350	358	365	373	380	388	395	403	410	
581	418	425	433	440	448	455	462	470	477	485	
582	492	500	507	515	522	530	537	545	552	559	
583	567	574	582	589	597	604	612	619	626	634	
584	641	649	656	664	671	678	686	693	701	708	7 1 0.7 2 1.4 3 2.1 4 2.8 5 3.5 6 4.2 7 4.9 8 5.6 9 6.3
585	716	723	730	738	745	753	760	768	775	782	
586	790	797	805	812	819	827	834	842	849	856	
587	864	871	879	886	893	901	908	916	923	930	
588	938	945	953	960	967	975	982	989	997	*004	
589	77 012	019	026	034	041	048	056	063	070	078	
590	085	093	100	107	115	122	129	137	144	151	
591	159	166	173	181	188	195	203	210	217	225	
592	232	240	247	254	262	269	276	283	291	298	
593	305	313	320	327	335	342	349	357	364	371	
594	379	386	393	401	408	415	422	430	437	444	
595	452	459	466	474	481	488	495	503	510	517	
596	525	532	539	546	554	561	568	576	583	590	
597	597	605	612	619	627	634	641	648	656	663	
598	670	677	685	692	699	706	714	721	728	735	
599	743	750	757	764	772	779	786	793	801	808	
600	815	822	830	837	844	851	859	866	873	880	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
600	77 815	822	830	837	844	851	859	866	873	880	
601	887	895	902	909	916	924	931	938	945	952	
602	960	967	974	981	988	996	*003	*010	*017	*025	
603	78 032	039	046	053	061	068	075	082	089	097	
604	104	111	118	125	132	140	147	154	161	168	
605	176	183	190	197	204	211	219	226	233	240	
606	247	254	262	269	276	283	290	297	305	312	
607	319	326	333	340	347	355	362	369	376	383	**8**
608	390	398	405	412	419	426	433	440	447	455	1 | 0.8
609	462	469	476	483	490	497	504	512	519	526	2 | 1.6
610	533	540	547	554	561	569	576	583	590	597	3 | 2.4
611	604	611	618	625	633	640	647	654	661	668	4 | 3.2
612	675	682	689	696	704	711	718	725	732	739	5 | 4.0
613	746	753	760	767	774	781	789	796	803	810	6 | 4.8
614	817	824	831	838	845	852	859	866	873	880	7 | 5.6
615	888	895	902	909	916	923	930	937	944	951	8 | 6.4
616	958	965	972	979	986	993	*000	*007	*014	*021	9 | 7.2
617	79 029	036	043	050	057	064	071	078	085	092	
618	099	106	113	120	127	134	141	148	155	162	
619	169	176	183	190	197	204	211	218	225	232	
620	239	246	253	260	267	274	281	288	295	302	
621	309	316	323	330	337	344	351	358	365	372	
622	379	386	393	400	407	414	421	428	435	442	**7**
623	449	456	463	470	477	484	491	498	505	511	1 | 0.7
624	518	525	532	539	546	553	560	567	574	581	2 | 1.4
625	588	595	602	609	616	623	630	637	644	650	3 | 2.1
626	657	664	671	678	685	692	699	706	713	720	4 | 2.8
627	727	734	741	748	754	761	768	775	782	789	5 | 3.5
628	796	803	810	817	824	831	837	844	851	858	6 | 4.2
629	865	872	879	886	893	900	906	913	920	927	7 | 4.9
630	934	941	948	955	962	969	975	982	989	996	8 | 5.6
631	80 003	010	017	024	030	037	044	051	058	065	9 | 6.3
632	072	079	085	092	099	106	113	120	127	134	
633	140	147	154	161	168	175	182	188	195	202	
634	209	216	223	229	236	243	250	257	264	271	
635	277	284	291	298	305	312	318	325	332	339	
636	346	353	359	366	373	380	387	393	400	407	
637	414	421	428	434	441	448	455	462	468	475	
638	482	489	496	502	509	516	523	530	536	543	**6**
639	550	557	564	570	577	584	591	598	604	611	1 | 0.6
640	618	625	632	638	645	652	659	665	672	679	2 | 1.2
641	686	693	699	706	713	720	726	733	740	747	3 | 1.8
642	754	760	767	774	781	787	794	801	808	814	4 | 2.4
643	821	828	835	841	848	855	862	868	875	882	5 | 3.0
644	889	895	902	909	916	922	929	936	943	949	6 | 3.6
645	956	963	969	976	983	990	996	*003	*010	*017	7 | 4.2
646	81 023	030	037	043	050	057	064	070	077	084	8 | 4.8
647	090	097	104	111	117	124	131	137	144	151	9 | 5.4
648	158	164	171	178	184	191	198	204	211	218	
649	224	231	238	245	251	258	265	271	278	285	
650	291	298	305	311	318	325	331	338	345	351	
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N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
650	81 291	298	305	311	318	325	331	338	345	351	
651	358	365	371	378	385	391	398	405	411	418	
652	425	431	438	445	451	458	465	471	478	485	
653	491	498	505	511	518	525	531	538	544	551	
654	558	564	571	578	584	591	598	604	611	617	
655	624	631	637	644	651	657	664	671	677	684	
656	690	697	704	710	717	723	730	737	743	750	
657	757	763	770	776	783	790	796	803	809	816	
658	823	829	836	842	849	856	862	869	875	882	
659	889	895	902	908	915	921	928	935	941	948	
660	954	961	968	974	981	987	994	*000	*007	*014	
661	82 020	027	033	040	046	053	060	066	073	079	7
662	086	092	099	105	112	119	125	132	138	145	1 0.7
663	151	158	164	171	178	184	191	197	204	210	2 1.4
664	217	223	230	236	243	249	256	263	269	276	3 2.1
665	282	289	295	302	308	315	321	328	334	341	4 2.8
666	347	354	360	367	373	380	387	393	400	406	5 3.5
667	413	419	426	432	439	445	452	458	465	471	6 4.2
668	478	484	491	497	504	510	517	523	530	536	7 4.9
669	543	549	556	562	569	575	582	588	595	601	8 5.6
670	607	614	620	627	633	640	646	653	659	666	9 6.3
671	672	679	685	692	698	705	711	718	724	730	
672	737	743	750	756	763	769	776	782	789	795	
673	802	808	814	821	827	834	840	847	853	860	
674	866	872	879	885	892	898	905	911	918	924	
675	930	937	943	950	956	963	969	975	982	988	
676	995	*001	*008	*014	*020	*027	*033	*040	*046	*052	
677	83 059	065	072	078	085	091	097	104	110	117	
678	123	129	136	142	149	155	161	168	174	181	
679	187	193	200	206	213	219	225	232	238	245	
680	251	257	264	270	276	283	289	296	302	308	
681	315	321	327	334	340	347	353	359	366	372	
682	378	385	391	398	404	410	417	423	429	436	
683	442	448	455	461	467	474	480	487	493	499	
684	506	512	518	525	531	537	544	550	556	563	
685	569	575	582	588	594	601	607	613	620	626	6
686	632	639	645	651	658	664	670	677	683	689	1 0.6
687	696	702	708	715	721	727	734	740	746	753	2 1.2
688	759	765	771	778	784	790	797	803	809	816	3 1.8
689	822	828	835	841	847	853	860	866	872	879	4 2.4
690	885	891	897	904	910	916	923	929	935	942	5 3.0
691	948	954	960	967	973	979	985	992	998	*004	6 3.6
692	84 011	017	023	029	036	042	048	055	061	067	7 4.2
693	073	080	086	092	098	105	111	117	123	130	8 4.8
694	136	142	148	155	161	167	173	180	186	192	9 5.4
695	198	205	211	217	223	230	236	242	248	255	
696	261	267	273	280	286	292	298	305	311	317	
697	323	330	336	342	348	354	361	367	373	379	
698	386	392	398	404	410	417	423	429	435	442	
699	448	454	460	466	473	479	485	491	497	504	
700	510	516	522	528	535	541	547	553	559	566	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
700	84 510	516	522	528	535	541	547	553	559	566	
701	572	578	584	590	597	603	609	615	621	628	
702	634	640	646	652	658	665	671	677	683	689	
703	696	702	708	714	720	726	733	739	745	751	
704	757	763	770	776	782	788	794	800	807	813	
705	819	825	831	837	844	850	856	862	868	874	
706	880	887	893	899	905	911	917	924	930	936	
707	942	948	954	960	967	973	979	985	991	997	**7**
708	85 003	009	016	022	028	034	040	046	052	058	1 | 0.7
709	065	071	077	083	089	095	101	107	114	120	2 | 1.4
710	126	132	138	144	150	156	163	169	175	181	3 | 2.1
711	187	193	199	205	211	217	224	230	236	242	4 | 2.8
712	248	254	260	266	272	278	285	291	297	303	5 | 3.5
713	309	315	321	327	333	339	345	352	358	364	6 | 4.2
714	370	376	382	388	394	400	406	412	418	425	7 | 4.9
715	431	437	443	449	455	461	467	473	479	485	8 | 5.6
716	491	497	503	509	516	522	528	534	540	546	9 | 6.3
717	552	558	564	570	576	582	588	594	600	606	
718	612	618	625	631	637	643	649	655	661	667	
719	673	679	685	691	697	703	709	715	721	727	
720	733	739	745	751	757	763	769	775	781	788	
721	794	800	806	812	818	824	830	836	842	848	
722	854	860	866	872	878	884	890	896	902	908	**6**
723	914	920	926	932	938	944	950	956	962	968	1 | 0.6
724	974	980	986	992	998	*004	*010	*016	*022	*028	2 | 1.2
725	86 034	040	046	052	058	064	070	076	082	088	3 | 1.8
726	094	100	106	112	118	124	130	136	141	147	4 | 2.4
727	153	159	165	171	177	183	189	195	201	207	5 | 3.0
728	213	219	225	231	237	243	249	255	261	267	6 | 3.6
729	273	279	285	291	297	303	308	314	320	326	7 | 4.2
730	332	338	344	350	356	362	368	374	380	386	8 | 4.8
731	392	398	404	410	415	421	427	433	439	445	9 | 5.4
732	451	457	463	469	475	481	487	493	499	504	
733	510	516	522	528	534	540	546	552	558	564	
734	570	576	581	587	593	599	605	611	617	623	
735	629	635	641	646	652	658	664	670	676	682	
736	688	694	700	705	711	717	723	729	735	741	
737	747	753	759	764	770	776	782	788	794	800	**5**
738	806	812	817	823	829	835	841	847	853	859	1 | 0.5
739	864	870	876	882	888	894	900	906	911	917	2 | 1.0
740	923	929	935	941	947	953	958	964	970	976	3 | 1.5
741	982	988	994	999	*005	*011	*017	*023	*029	*035	4 | 2.0
742	87 040	046	052	058	064	070	075	081	087	093	5 | 2.5
743	099	105	111	116	122	128	134	140	146	151	6 | 3.0
744	157	163	169	175	181	186	192	198	204	210	7 | 3.5
745	216	221	227	233	239	245	251	256	262	268	8 | 4.0
746	274	280	286	291	297	303	309	315	320	326	9 | 4.5
747	332	338	344	349	355	361	367	373	379	384	
748	390	396	402	408	413	419	425	431	437	442	
749	448	454	460	466	471	477	483	489	495	500	
750	506	512	518	523	529	535	541	547	552	558	
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N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
750	87 506	512	518	523	529	535	541	547	552	558	
751	564	570	576	581	587	593	599	604	610	616	
752	622	628	633	639	645	651	656	662	668	674	
753	679	685	691	697	703	708	714	720	726	731	
754	737	743	749	754	760	766	772	777	783	789	
755	795	800	806	812	818	823	829	835	841	846	
756	852	858	864	869	875	881	887	892	898	904	
757	910	915	921	927	933	938	944	950	955	961	
758	967	973	978	984	990	996	*001	*007	*013	*018	
759	88 024	030	036	041	047	053	058	064	070	076	
760	081	087	093	098	104	110	116	121	127	133	
761	138	144	150	156	161	167	173	178	184	190	
762	195	201	207	213	218	224	230	235	241	247	
763	252	258	264	270	275	281	287	292	298	304	
764	309	315	321	326	332	338	343	349	355	360	
765	366	372	377	383	389	395	400	406	412	417	
766	423	429	434	440	446	451	457	463	468	474	
767	480	485	491	497	502	508	513	519	525	530	
768	536	542	547	553	559	564	570	576	581	587	
769	593	598	604	610	615	621	627	632	638	643	
770	649	655	660	666	672	677	683	689	694	700	
771	705	711	717	722	728	734	739	745	750	756	
772	762	767	773	779	784	790	795	801	807	812	
773	818	824	829	835	840	846	852	857	863	868	
774	874	880	885	891	897	902	908	913	919	925	
775	930	936	941	947	953	958	964	969	975	981	
776	986	992	997	*003	*009	*014	*020	*025	*031	*037	
777	89 042	048	053	059	064	070	076	081	087	092	
778	098	104	109	115	120	126	131	137	143	148	
779	154	159	165	170	176	182	187	193	198	204	
780	209	215	221	226	232	237	243	248	254	260	
781	265	271	276	282	287	293	298	304	310	315	
782	321	326	332	337	343	348	354	360	365	371	
783	376	382	387	393	398	404	409	415	421	426	
784	432	437	443	448	454	459	465	470	476	481	
785	487	492	498	504	509	515	520	526	531	537	
786	542	548	553	559	564	570	575	581	586	592	
787	597	603	609	614	620	625	631	636	642	647	
788	653	658	664	669	675	680	686	691	697	702	
789	708	713	719	724	730	735	741	746	752	757	
790	763	768	774	779	785	790	796	801	807	812	
791	818	823	829	834	840	845	851	856	862	867	
792	873	878	883	889	894	900	905	911	916	922	
793	927	933	938	944	949	955	960	966	971	977	
794	982	988	993	998	*004	*009	*015	*020	*026	*031	
795	90 037	042	048	053	059	064	069	075	080	086	
796	091	097	102	108	113	119	124	129	135	140	
797	146	151	157	162	168	173	179	184	189	195	
798	200	206	211	217	222	227	233	238	244	249	
799	255	260	266	271	276	282	287	293	298	304	
800	309	314	320	325	331	336	342	347	352	358	
N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.

6	
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3	1.8
4	2.4
5	3.0
6	3.6
7	4.2
8	4.8
9	5.4

5	
1	0.5
2	1.0
3	1.5
4	2.0
5	2.5
6	3.0
7	3.5
8	4.0
9	4.5

N.	L. 0	1	2	3	4	5	6	7	8	9	P. P.
800	90 309	314	320	325	331	336	342	347	352	358	
801	363	369	374	380	385	390	396	401	407	412	
802	417	423	428	434	439	445	450	455	461	466	
803	472	477	482	488	493	499	504	509	515	520	
804	526	531	536	542	547	553	558	563	569	574	
805	580	585	590	596	601	607	612	617	623	628	
806	634	639	644	650	655	660	666	671	677	682	
807	687	693	698	703	709	714	720	725	730	736	
808	741	747	752	757	763	768	773	779	784	789	
809	795	800	806	811	816	822	827	832	838	843	
810	849	854	859	865	870	875	881	886	891	897	
811	902	907	913	918	924	929	934	940	945	950	
812	956	961	966	972	977	982	988	993	998	*004	
813	91 009	014	020	025	030	036	041	046	052	057	
814	062	068	073	078	084	089	094	100	105	110	
815	116	121	126	132	137	142	148	153	158	164	
816	169	174	180	185	190	196	201	206	212	217	
817	222	228	233	238	243	249	254	259	265	270	
818	275	281	286	291	297	302	307	312	318	323	
819	328	334	339	344	350	355	360	365	371	376	
820	381	387	392	397	403	408	413	418	424	429	
821	434	440	445	450	455	461	466	471	477	482	
822	487	492	498	503	508	514	519	524	529	535	
823	540	545	551	556	561	566	572	577	582	587	
824	593	598	603	609	614	619	624	630	635	640	
825	645	651	656	661	666	672	677	682	687	693	
826	698	703	709	714	719	724	730	735	740	745	
827	751	756	761	766	772	777	782	787	793	798	
828	803	808	814	819	824	829	834	840	845	850	
829	855	861	866	871	876	882	887	892	897	903	
830	908	913	918	924	929	934	939	944	950	955	
831	960	965	971	976	981	986	991	997	*002	*007	
832	92 012	018	023	028	033	038	044	049	054	059	
833	065	070	075	080	085	091	096	101	106	111	
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835	169	174	179	184	189	195	200	205	210	215	
836	221	226	231	236	241	247	252	257	262	267	
837	273	278	283	288	293	298	304	309	314	319	
838	324	330	335	340	345	350	355	361	366	371	
839	376	381	387	392	397	402	407	412	418	423	
840	428	433	438	443	449	454	459	464	469	474	
841	480	485	490	495	500	505	511	516	521	526	
842	531	536	542	547	552	557	562	567	572	578	
843	583	588	593	598	603	609	614	619	624	629	
844	634	639	645	650	655	660	665	670	675	681	
845	686	691	696	701	706	711	716	722	727	732	
846	737	742	747	752	758	763	768	773	778	783	
847	788	793	799	804	809	814	819	824	829	834	
848	840	845	850	855	860	865	870	875	881	886	
849	891	896	901	906	911	916	921	927	932	937	
850	942	947	952	957	962	967	973	978	983	988	
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852	93 044	049	054	059	064	069	075	080	085	090	
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862	551	556	561	566	571	576	581	586	591	596	4 2.4
863	601	606	611	616	621	626	631	636	641	646	5 3.0
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866	752	757	762	767	772	777	782	787	792	797	8 4.8
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868	852	857	862	867	872	877	882	887	892	897	
869	902	907	912	917	922	927	932	937	942	947	
870	952	957	962	967	972	977	982	987	992	997	
871	94 002	007	012	017	022	027	032	037	042	047	
872	052	057	062	067	072	077	082	086	091	096	5
873	101	106	111	116	121	126	131	136	141	146	1 0.5
874	151	156	161	166	171	176	181	186	191	196	2 1.0
875	201	206	211	216	221	226	231	236	240	245	3 1.5
876	250	255	260	265	270	275	280	285	290	295	4 2.0
877	300	305	310	315	320	325	330	335	340	345	5 2.5
878	349	354	359	364	369	374	379	384	389	394	6 3.0
879	399	404	409	414	419	424	429	433	438	443	7 3.5
880	448	453	458	463	468	473	478	483	488	493	8 4.0
881	498	503	507	512	517	522	527	532	537	542	9 4.5
882	547	552	557	562	567	571	576	581	586	591	
883	596	601	606	611	616	621	626	630	635	640	
884	645	650	655	660	665	670	675	680	685	689	
885	694	699	704	709	714	719	724	729	734	738	
886	743	748	753	758	763	768	773	778	783	787	
887	792	797	802	807	812	817	822	827	832	836	
888	841	846	851	856	861	866	871	876	880	885	4
889	890	895	900	905	910	915	919	924	929	934	1 0.4
890	939	944	949	954	959	963	968	973	978	983	2 0.8
891	988	993	998	*002	*007	*012	*017	*022	*027	*032	3 1.2
892	95 036	041	046	051	056	061	066	071	075	080	4 1.6
893	085	090	095	100	105	109	114	119	124	129	5 2.0
894	134	139	143	148	153	158	163	168	173	177	6 2.4
895	182	187	192	197	202	207	211	216	221	226	7 2.8
896	231	236	240	245	250	255	260	265	270	274	8 3.2
897	279	284	289	294	299	303	308	313	318	323	9 3.6
898	328	332	337	342	347	352	357	361	366	371	
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900	424	429	434	439	444	448	453	458	463	468	
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900	95 424	429	434	439	444	448	453	458	463	468	
901	472	477	482	487	492	497	501	506	511	516	
902	521	525	530	535	540	545	550	554	559	564	
903	569	574	578	583	588	593	598	602	607	612	
904	617	622	626	631	636	641	646	650	655	660	
905	665	670	674	679	684	689	694	698	703	708	
906	713	718	722	727	732	737	742	746	751	756	
907	761	766	770	775	780	785	789	794	799	804	
908	809	813	818	823	828	832	837	842	847	852	
909	856	861	866	871	875	880	885	890	895	899	
910	904	909	914	918	923	928	933	938	942	947	**5**
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915	142	147	152	156	161	166	171	175	180	185	5 | 2.5
916	190	194	199	204	209	213	218	223	227	232	6 | 3.0
917	237	242	246	251	256	261	265	270	275	280	7 | 3.5
918	284	289	294	298	303	308	313	317	322	327	8 | 4.0
919	332	336	341	346	350	355	360	365	369	374	9 | 4.5
920	379	384	388	393	398	402	407	412	417	421	
921	426	431	435	440	445	450	454	459	464	468	
922	473	478	483	487	492	497	501	506	511	515	
923	520	525	530	534	539	544	548	553	558	562	
924	567	572	577	581	586	591	595	600	605	609	
925	614	619	624	628	633	638	642	647	652	656	
926	661	666	670	675	680	685	689	694	699	703	
927	708	713	717	722	727	731	736	741	745	750	
928	755	759	764	769	774	778	783	788	792	797	
929	802	806	811	816	820	825	830	834	839	844	
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932	942	946	951	956	960	965	970	974	979	984	
933	988	993	997	*002	*007	*011	*016	*021	*025	*030	
934	97 035	039	044	049	053	058	063	067	072	077	**4**
935	081	086	090	095	100	104	109	114	118	123	1 | 0.4
936	128	132	137	142	146	151	155	160	165	169	2 | 0.8
937	174	179	183	188	192	197	202	206	211	216	3 | 1.2
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939	267	271	276	280	285	290	294	299	304	308	5 | 2.0
940	313	317	322	327	331	336	340	345	350	354	6 | 2.4
941	359	364	368	373	377	382	387	391	396	400	7 | 2.8
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943	451	456	460	465	470	474	479	483	488	493	9 | 3.6
944	497	502	506	511	516	520	525	529	534	539	
945	543	548	552	557	562	566	571	575	580	585	
946	589	594	598	603	607	612	617	621	626	630	
947	635	640	644	649	653	658	663	667	672	676	
948	681	685	690	695	699	704	708	713	717	722	
949	727	731	736	740	745	749	754	759	763	768	
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951	818	823	827	832	836	841	845	850	855	859	
952	864	868	873	877	882	886	891	896	900	905	
953	909	914	918	923	928	932	937	941	946	950	
954	955	959	964	968	973	978	982	987	991	996	
955	98 000	005	009	014	019	023	028	032	037	041	
956	046	050	055	059	064	068	073	078	082	087	
957	091	096	100	105	109	114	118	123	127	132	
958	137	141	146	150	155	159	164	168	173	177	
959	182	186	191	195	200	204	209	214	218	223	
960	227	232	236	241	245	250	254	259	263	268	
961	272	277	281	286	290	295	299	304	308	313	
962	318	322	327	331	336	340	345	349	354	358	**5**
963	363	367	372	376	381	385	390	394	399	403	1 0.5
964	408	412	417	421	426	430	435	439	444	448	2 1.0
965	453	457	462	466	471	475	480	484	489	493	3 1.5
966	498	502	507	511	516	520	525	529	534	538	4 2.0
967	543	547	552	556	561	565	570	574	579	583	5 2.5
968	588	592	597	601	605	610	614	619	623	628	6 3.0
969	632	637	641	646	650	655	659	664	668	673	7 3.5
970	677	682	686	691	695	700	704	709	713	717	8 4.0
971	722	726	731	735	740	744	749	753	758	762	9 4.5
972	767	771	776	780	784	789	793	798	802	807	
973	811	816	820	825	829	834	838	843	847	851	
974	856	860	865	869	874	878	883	887	892	896	
975	900	905	909	914	918	923	927	932	936	941	
976	945	949	954	958	963	967	972	976	981	985	
977	989	994	998	*003	*007	*012	*016	*021	*025	*029	
978	99 034	038	043	047	052	056	061	065	069	074	
979	078	083	087	092	096	100	105	109	114	118	
980	123	127	131	136	140	145	149	154	158	162	
981	167	171	176	180	185	189	193	198	202	207	
982	211	216	220	224	229	233	238	242	247	251	**4**
983	255	260	264	269	273	277	282	286	291	295	1 0.4
984	300	304	308	313	317	322	326	330	335	339	2 0.8
985	344	348	352	357	361	366	370	374	379	383	3 1.2
986	388	392	396	401	405	410	414	419	423	427	4 1.6
987	432	436	441	445	449	454	458	463	467	471	5 2.0
988	476	480	484	489	493	498	502	506	511	515	6 2.4
989	520	524	528	533	537	542	546	550	555	559	7 2.8
990	564	568	572	577	581	585	590	594	599	603	8 3.2
991	607	612	616	621	625	629	634	638	642	647	9 3.6
992	651	656	660	664	669	673	677	682	686	691	
993	695	699	704	708	712	717	721	726	730	734	
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995	782	787	791	795	800	804	808	813	817	822	
996	826	830	835	839	843	848	852	856	861	865	
997	870	874	878	883	887	891	896	900	904	909	
998	913	917	922	926	930	935	939	944	948	952	
999	957	961	965	970	974	978	983	987	991	996	
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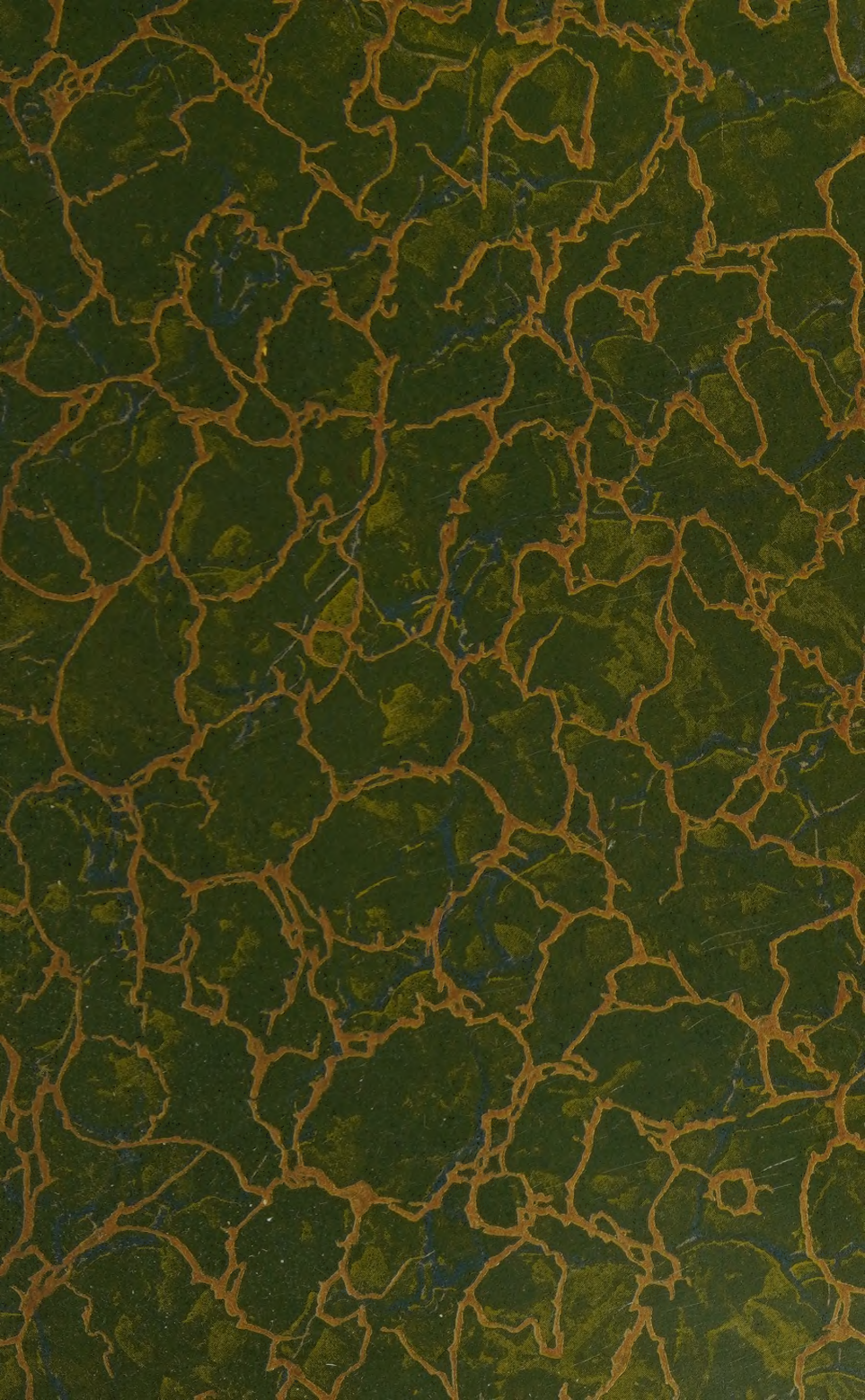
TABLE OF USEFUL NUMBERS.

	Symbol.	Numerical Value.	Logarithm.
Diagonal of square whose side is 1	$\sqrt{2}$	1.4142	0.15052
Side of square whose diagonal is 1	$\frac{1}{2}\sqrt{2}$	0.70710	1.84948
Diagonal of cube whose edge is 1	$\sqrt{3}$	1.7321	0.23856
Edge of cube whose diagonal is 1	$\frac{1}{3}\sqrt{3}$	0.57735	$\bar{1}$.76144
Circumference of circle whose diameter is 1	π	3.1416	0.49715
Diameter of circle whose circumference is 1	$\frac{1}{\pi}$	0.31831	$\bar{1}$.50285
Area of circle whose diameter is 1	$\frac{1}{4}\pi$	0.7854	$\bar{1}$.89509
Volume of sphere whose diameter is 1 . . .	$\frac{1}{6}\pi$	0.5236	$\bar{1}$.71900
One meter in yards		1.0936	0.03887
One yard in meters		0.91439	$\bar{1}$.96113
One centimeter in inches		0.39370	$\bar{1}$.59517
One inch in centimeters		2.5400	0.40483
One square meter in square feet		10.764	1.03198
One square foot in square meters		0.092901	$\bar{2}$.96802
One cubic centimeter in cubic inches . . .		0.061025	$\bar{2}$.78551
One cubic inch in cubic centimeters		16.387	1.21449
One liter in gallons		0.26418	$\bar{1}$.42190
One gallon in liters		3.7853	0.57810
One kilogram in pounds		2.2046	0.34333
One pound in kilograms		0.45359	$\bar{1}$.65667